Friday 2/19 L13

Discrete Time Detection

H_0 : $Y_k = N_k + S_{0k}$ $1 \le k \le n$

vs

H_1 : $Y_k = N_k + S_{1k}$ $1 \le k \le n$

$Y = [Y_1 \dots Y_n]^T$

$N = [N_1 \dots N_n]^T$

$S_0 =$ ....

$S_1 =$

Signal characterization is one of 3 types:

1) S_0 & S_1 Known (deterministic)
2) S_0 & S_1 known except for some params.
3) S_0 & S_1 random and only know prob. distributions

Noise characterization

1) Noise indep of sigs under either hyp.
2) Noise distribution does not dep. on hyp.
3) Noise dist. given by a density or pmf

$f_N(\cdot)$ domain $\mathbb{R}^n$

Likelihood Ratio

H_0: pdf of Y given $S_0 = s_0$ $\quad f_0(y \mid S_0 = s_0)$

$$= f_N(y - s_0)$$

$\Longrightarrow$ uncond. dist. of Y is

$$f_0(y) = E\{f_N(y - S_0)\}$$

Similarly under H_1 $\Longrightarrow$ $f_1(y \mid S_1 = s_1) = f_N(y - s_1)$

$$f_1(y) = E\{f_N(y - S_1)\}$$

$\Longrightarrow$ Likelihood Ratio

$$L(y) = \frac{f_1(y)}{f_0(y)} = \frac{E\{f_N(y - S_1)\}}{E\{f_N(y - S_0)\}}$$

Thresholds still found from

$$\tau = \frac{\pi_0}{\pi_1} \frac{c_{10} - c_{00}}{c_{01} - c_{11}} \quad \text{for Bayes}$$

or

$\eta \longrightarrow$ found from $P_{FA} = \alpha$ for NP.

Adding Assumptions to Get more specific detectors ...

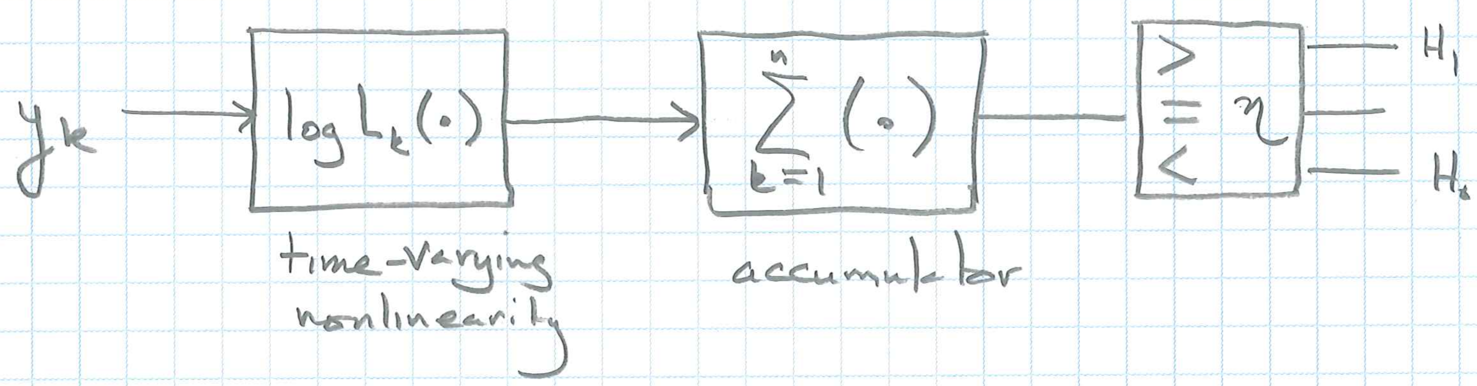
<u>Det. Sigs in Indep Noise</u>

$$L(y) = \frac{f_N(y - s_1)}{f_N(y - s_0)} = \frac{f_N(y_1 - s_{11}, y_2 - s_{12}, \ldots, y_n - s_{1n})}{f_N(y_1 - s_{01}, y_2 - s_{02}, \ldots, y_n - s_{0n})}$$

If indep. then factors $\longrightarrow$

$$= \prod_{k=1}^{n} \frac{f_{N_k}(y_k - s_{1k})}{f_{N_k}(y_k - s_{0k})} \longrightarrow L_k(y_k).$$

Take Log get a detector structure

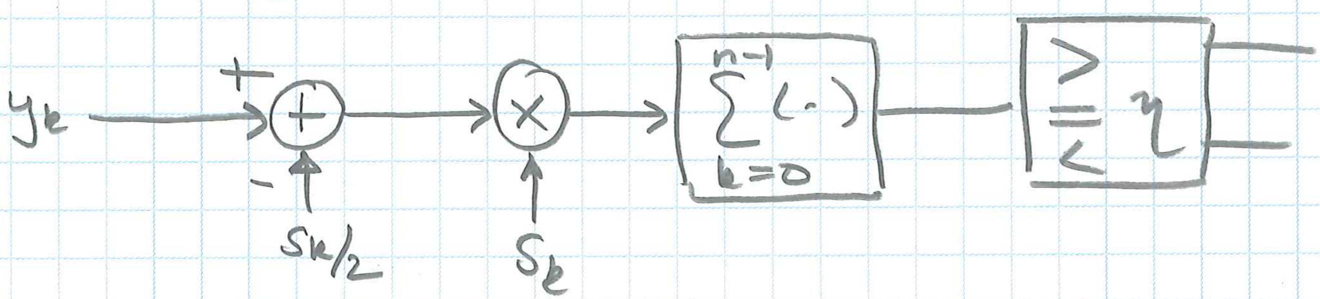


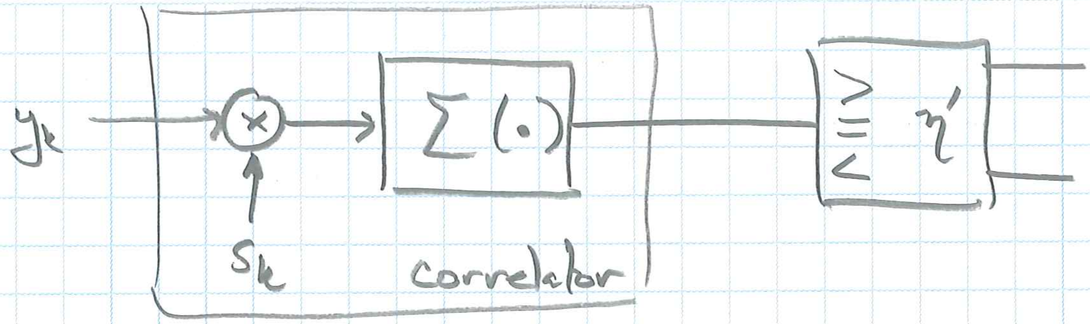
<u>Coherent Detection in IID Noise</u> $\longrightarrow$ $H_i: Y_k = N_k + S_{ik} \quad k = 0, 1, 2 .. n-1$

$N_k \sim N(0, \sigma^2)$ iid

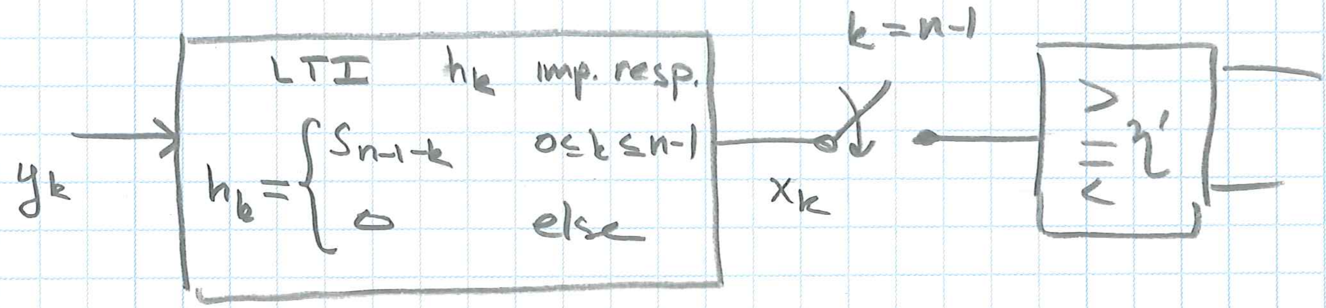
$S_0 = 0 \qquad S_1 = s$

$$\Rightarrow \log L_k(y_k) = s_k(y_k - s_k/2)/\sigma^2$$

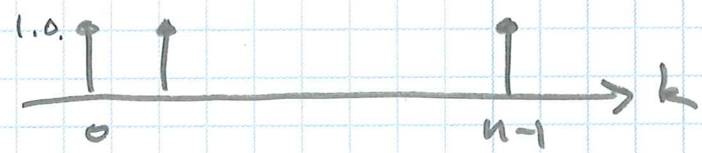




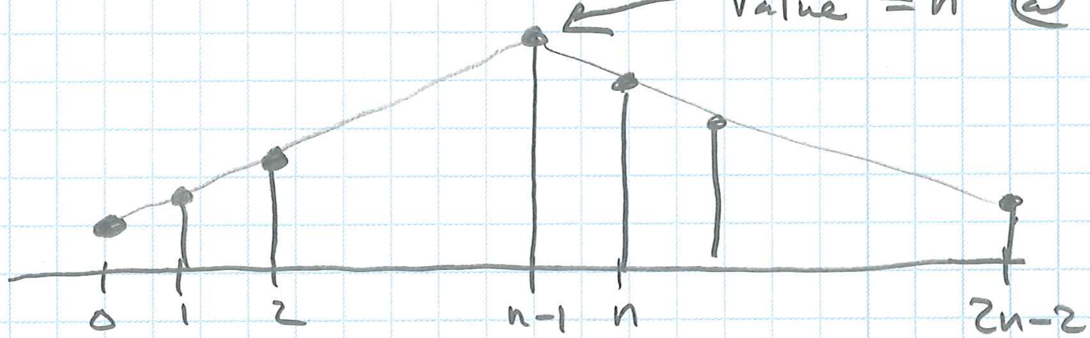
This can be seen as a matched filter.



Look @ $s_k =$



Value $= n$ @ time $k = n-1$.



Max SNR Prop

$$X_{n-1} = \sum_{\ell=0}^{n-1} h_{n-1-\ell} \underbrace{Y_\ell}_{S_\ell + N_\ell}$$

SNR @ filter @ Sampling Time

$$= \frac{\left(E_1\{X_{n-1}\}\right)^2}{Var\{X_{n-1}\}} = \frac{\left(\sum_{\ell=0}^{n-1} h_{n-1-\ell}\, s_\ell\right)^2}{Var\left\{\sum_{\ell=0}^{n-1} h_{n-1-\ell} N_\ell\right\}}.$$

Can show: $SNR_{max} = \sum_{\ell=0}^{n-1} s_\ell^2 \Big/ \sigma^2$

is achieve for $h_m = s_{n-1-m}$ $\quad m=0,1\ldots n-1.$

LMP Detection in IID Noise $\longrightarrow$ HW?

(To fill in yet.)

Deterministic Signals in Gaussian Noise

- Model:

$$
\begin{aligned}
H_0 : \quad & Y_k = N_k + s_{0k} \;\; 1 \le k \le n \\
\text{vs.} & \\
H_1 : \quad & Y_k = N_k + s_{1k} \;\; 1 \le k \le n
\end{aligned}
$$

where:

- s_0 and s_1 are known $n \times 1$ deterministic vectors
- $N \sim \mathcal{N}(0, \Sigma)$ is a zero mean Gaussian noise of dimension $n \times 1$.

- Likelihood ratio is

$$
\begin{aligned}
L(y) &= \frac{\frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(y - s_1)^T \Sigma^{-1} (y - s_1)\right\}}{\frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(y - s_0)^T \Sigma^{-1} (y - s_0)\right\}} \\
&= \exp\left\{(s_1 - s_0)^T \Sigma^{-1} \left(y - \frac{s_0 + s_1}{2}\right)\right\}
\end{aligned}
$$

for $y \in \mathcal{R}^n$.

- Detector structure is the same as for the i.i.d. noise case since the decision rule is of the form

$$\delta(y) = \begin{cases} 1 & \\ \gamma & \overbrace{(s_1 - s_0)^T \Sigma^{-1}}^{\tilde{s}^T} y \\ 0 & \end{cases} \begin{array}{c} > \\ = \\ < \end{array} \eta$$

where η is picked either to satisfy a false alarm constraint or via the Bayes formulation[2].

- If define *pseudo signal* $\tilde{s}_k$ via $\tilde{s} = \Sigma^{-1}(s_1 - s_0)$ then have same correlator as in i.i.d. case (but for a modified signal).

[2]For Bayes η would be chosen

$$\eta = \log \left[\frac{\pi_0 (c_{10} - c_{00})}{\pi_1 (c_{01} - c_{11})} \right] + (s_1 - s_0)^T \Sigma^{-1} \left(\frac{s_1 + s_0}{2} \right).$$