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ECE 645

J. V. Krogmeier

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Random Variable (rv)

A real val. rv. $X : \Omega \to \mathbb{R}$ st.

$$\{\omega \in \Omega : X(\omega) \leq x\} \in \mathcal{F} \qquad \text{for all } x \in \mathbb{R}.$$

$\Rightarrow$ ie $P(\{X \leq x\})$ are defined, $P(X \leq x)$.

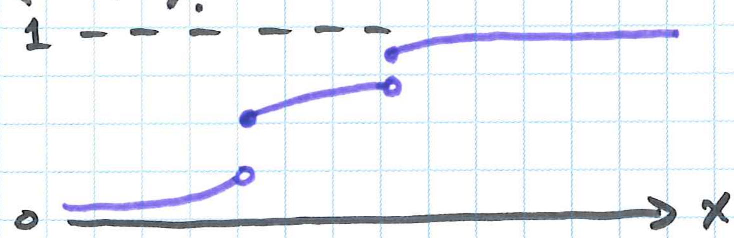
- complex-valued rv.
- random vectors $\mathbb{R}^n$, $\mathbb{C}^n$
- discrete rv. $X : \Omega \to \left\{ \begin{matrix} \text{countable or} \\ \text{finite set} \end{matrix} \right\}$

$$\{X = x\} \in \mathcal{F} \quad \forall x$$

Assoc. Functions

cdf $\quad F_X(x) \triangleq P(X \leq x)$. $\quad$ defined $\forall x \in \mathbb{R}$

- right continuous
- non-decreasing
- left hand limits exist



pdf is the kernel st

$$F_X(x) = \int_{-\infty}^{x} f_X(\alpha)\, d\alpha \qquad f_X(x) = \frac{d}{dx} F_X(x).$$

pmf (prob. mass function)

$$p_X(x) = P(X = x) \qquad x \in \text{domain}$$

Mixed rvs with both cont. & discrete are possible.

Moments, Mean, Var.

$$E[X] = \begin{cases} \sum_x x\, p_X(x) \\ \int x\, f_X(x)\, dx \end{cases} \Longrightarrow E[X^k]$$

$$E[g(X)] = \int g(x) f_X(x)\, dx$$

$$\mathrm{Var}(X) = E\left[(X - E[X])^2\right]$$

Discrete RVs

Bernoulli: $p_X(1) = P(X=1) = p$

$p_X(0) = P(X=0) = 1-p$

Binomial

Poisson rv. $$p_X(k) = e^{-\lambda} \frac{\lambda^k}{k!} \qquad k = 0, 1, 2 \ldots$$

Continuous RVs

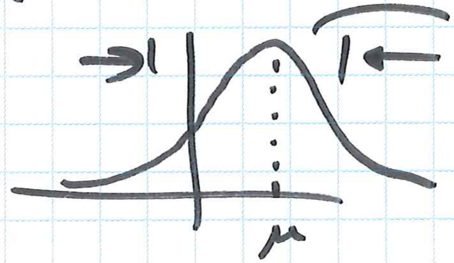
Uniform rv.

Exponential $$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0. \end{cases}$$

Rayleigh.

Gaussian

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2} \qquad x \in \mathbb{R}$$



prop. to σ.

$Var(X) = \sigma^2$

$E[X] = \mu.$

Jointly Dist. RVs.

$\{X \le x\} \cap \{Y \le y\}$.

$$F_{X,Y}(x,y) = P(X \le x, Y \le y).$$

$p_{X,Y}(x,y)$

$f_{X,Y}(x,y)$.

marginal via

$$F_X(x) = F_{X,Y}(x, +\infty)$$

Independence

X, Y are indep. $\Longleftrightarrow$ $F_{X,Y}(x,y) = F_X(x) F_Y(y)$

Joint Moments

Correlation $E[XY]$

Covariance $E[(X - E[X])(Y - E[Y])]$

Conditional Distributions

$$p_{X|Y}(x|y) \triangleq P(X = x \mid Y = y)$$
$$= p_{XY}(x,y) / p_Y(y)$$

Joint Gaussian X, Y are bivariate Gaussian

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)} \left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 + \left(\frac{y-\mu_y}{\sigma_y}\right)^2 - 2\rho\frac{(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} \right] \right\}$$

$$\mathrm{Cov}(X,Y) = \rho\,\sigma_x\sigma_y \qquad -1 \le \rho \le +1.$$