ECE 544: Digital Comm. (Session 27)

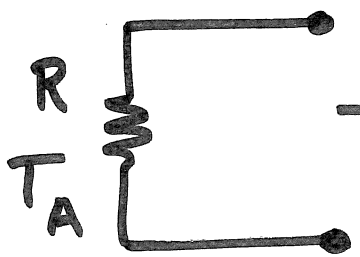
Antenna Noise Temperature antenna noise comes from two sources

① external environment (outside of designer's control)

② thermal noise due to losses in antenna itself.

e.g.
- cosmic background noise
- sun + stars
- thermal noise from ground
- lightening
- high voltage lines
- auto ignition, building lighting, computers
- other radio transmissions

Model for these background sources, antenna losses, etc:



$\longrightarrow N_{\text{background, etc}} = k T_A B$ → one-sided BW

$\triangleq$ power input to the rest of the receiver due to antenna.

T_A	3–5 K	antenna	pointed	toward	zenith
	50–100 K	"	"	"	horizon
	290–300 K	"	"	"	ground

G/T Figure of Merit for a Receive Antenna

$$\frac{G_R}{T_A}\,(\mathrm{dB}) \triangleq 10\log_{10}\left(\frac{G_R}{T_A}\right) \quad \mathrm{dBK}^{-1}$$

SNR at input to receiver is proportional to G_R/T_A .... hence consider as figure of merit for antenna.

Friis Eqn: Sig. power delivered by receive antenna to matched receiver input is

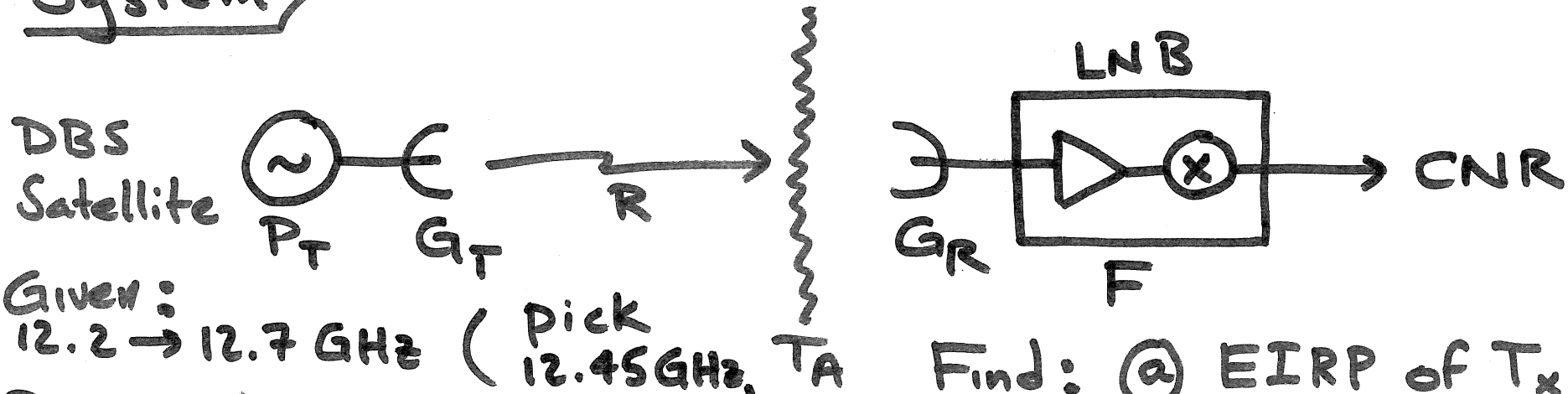
$$S_i = \frac{G_R\, G_T\, P_T\, \lambda^2}{(4\pi R)^2}$$

Noise input to receiver: $N_i = kT_A B$

$$SNR = \frac{S_i}{N_i} = \frac{G_R G_T P_T \lambda^2}{kT_A B (4\pi R)^2}$$

$$= \underbrace{\left(\frac{G_R}{T_A}\right)}_{\text{only parameter controllable at receiver}} \frac{G_T P_T \lambda^2}{kB (4\pi R)^2}$$

Example Calculation for Direct Broadcast System



Given:

$12.2 \rightarrow 12.7$ GHz (pick 12.45 GHz, $\lambda = .0241$ m)

$P_T = 120$ W

$G_T = 34$ dB $\rightarrow = 2512$

$B = $ IF BW $= 20$ MHz

$R = 39\,000$ km

$G_R = 33.5$ dB $\rightarrow 2239$ (18" dish)

$T_A = 50$K

$F = 1.1$ dB $\rightarrow 1.29$

Find: (a) EIRP of T_x

(b) G/T for combo of receive antenna + low noise block

(c) received carrier power @ receive ant. terms.

(d) CNR @ output of LNB

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ⓐ $EIRP = P_T G_T = (120W)(2512) = 3.01 \times 10^5 \, W$
$= 54.8 \, dBW$

ⓑ To find G/T for antenna + LNB combo, find noise temp of cascade: antenna → LNB, but referenced to LNB input

$$T_e = T_A + T_{LNB} = T_A + (F-1)290K$$
$$= 50K + (1.29-1)290K = 134K$$

$$\therefore \frac{G}{T} = 10 \log_{10}\left(\frac{2239}{134}\right) = 12.2 \, dBK^{-1}$$

(we do not include the LNB gain, G_{LNB} in this calculation)

(c) $$P_R = \frac{P_T G_T G_R \lambda^2}{(4\pi R)^2} = 1.63 \times 10^{-12}\ \text{W}$$

$$= -117.9\ \text{dBW}$$

(d) $$CNR = \frac{P_R \cancel{G_{LNB}}}{kT_e B \cancel{G_{LNB}}} = \frac{1.63 \times 10^{-12}\ \text{W}}{(1.38 \times 10^{-23})(134)(20 \times 10^6)}$$

$$= 44.1$$

$$= 16.4\ \text{dB}$$

Recall Signal Space

$H_i : Y(t) = s_i(t) + X(t) \qquad i = 0, 1, 2 \ldots M-1 \qquad 0 \le t \le T$

- $s_i(t)$ of finite energy
- $X(t)$ AWGN; $N_0/2$

$$\text{Signal Space} \triangleq \text{Span}\{s_0(t), s_1(t), \ldots, s_{M-1}(t)\}$$

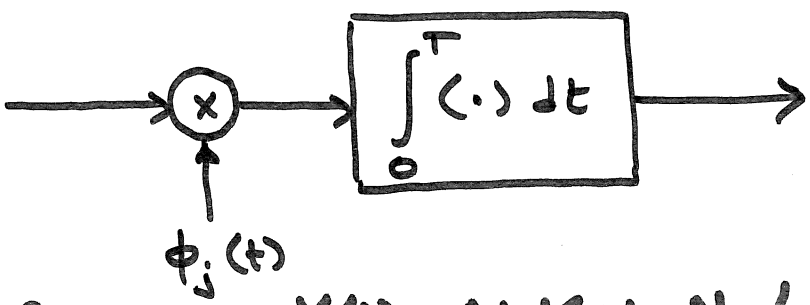
$$= \text{Span}\{\phi_0(t), \phi_1(t), \ldots, \phi_{J-1}(t)\}$$

$J \le M$, $\{\phi_i(t)\}$ are orthonormal, can be constructed from original signals using Gram–Schmidt process.

Notes:

- Set of J O.N. sigs $\{\phi_0 \cdots \phi_{J-1}\}$ can be extended to make C.O.N.S.

$$= \{ \underbrace{\phi_0, \phi_1, \cdots \phi_{J-1}}_{\text{span signal space}}, \underbrace{\phi_J, \phi_{J+1} \cdots}_{\text{span a space orthogonal to sig. space}} \}$$

- 

* If input $X(t)$, AWGN, $N_0/2$ then $X_j = \langle X, \phi_j \rangle$ is Gaussian, mean-zero, var $N_0/2$
* $X_j \perp\!\!\!\perp X_k \quad j \neq k$

Create equiv. HT problem

Define signal samples $s_{ij} = \langle s_i, \phi_j \rangle$ $\quad j = 0, 1, \ldots J-1$, $\quad i = 0, 1 \ldots M-1$

$$\underline{s_i} = [s_{i0}, s_{i1}, \ldots, s_{i,J-1}]$$

and also noise samples

$$X_j = \langle X, \phi_j \rangle \qquad j = 0, 1, \ldots J-1$$

$$\underline{X} = [X_0, X_1 \ldots X_{J-1}] \longrightarrow$$ a Gaussian r.v. with mean zero and iid components $\sim N(0, N_0/2)$

Equivalent H.T.

$$H_i : \underline{Y} = \underline{s_i} + \underline{X} \qquad i = 0, 1, \ldots M-1$$

$\longrightarrow$ J dimensional pdfs; but usual framework.

$$H_i : \quad f_i(\underline{y}) \quad N\left(\underline{s}_i, \frac{N_0}{2} I_{J\times J}\right)$$

$$= \prod_{j=0}^{J-1} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(y_j - s_{i,j})^2/2\sigma^2} \qquad \sigma^2 = N_0/2$$

$$= (2\pi\sigma^2)^{-J/2} \exp\left\{-\frac{1}{2\sigma^2}\sum_{j=0}^{J-1}(y_j - s_{i,j})^2\right\}$$

$$= (2\pi\sigma^2)^{-J/2} \exp\left\{-\frac{1}{2\sigma^2}\|\underline{y} - \underline{s}_i\|^2\right\}$$

MAP estimate (ie Bayes for unif. costs & possibly unequal priors) computes

$$\pi_i f_i(\underline{y}) = \pi_i (2\pi\sigma^2)^{-J/2} \exp\left\{-\frac{1}{2\sigma^2}\|\underline{y} - \underline{s}_i\|^2\right\}$$

and pick hyp. corresp. to largest one

$$k = \underset{0 \le i \le M-1}{\operatorname{argmax}} \; \pi_i f_i(\underline{y})$$

Can simplify MAP by taking $\ln(\cdot)$, etc.

$$k = \underset{0 \le i \le M-1}{\operatorname{argmin}} \left\{ \|\underline{y} - \underline{s}_i\|^2 - 2\sigma^2 \ln \pi_i \right\}$$

When priors are equal ($\pi_i = 1/M$) then MAP becomes maximum likelihood (ML)

$$k = \underset{0 \le i \le M-1}{\operatorname{argmin}} \|\underline{y} - \underline{s}_i\|^2 \longrightarrow$$

ie pick the signal closest to the observation in Euclidean distance.

<u>Def</u> A "plot" of signal points $\underline{s}_i$ $0 \le i \le M-1$ in J-dim space is called a <u>signal constellation</u>

Example Constellation + Decision Regions for Min Dist. (ML) Receiver

$J = 2$, $M = 3$ signals of equal energy E