Antenna Noise Temperature

Antenna noise comes from two sources:

1. **External environment (outside of designer's control)**
2. **Thermal noise due to losses in antenna itself.**

   e.g.
   - Cosmic background noise
   - Sun + stars
   - Thermal noise from ground
   - Lightening
   - High voltage lines
   - Auto ignition, building lighting, computers
   - Other radio transmissions
Model for these background sources, antenna losses, etc:

\[ R = \frac{N_{\text{background}} \text{ etc}}{k T_A B} \Rightarrow \text{one-sided BW} \]

\[ \Delta = \text{power input to the rest of the receiver due to antenna.} \]

\[ T_A \]

3-5 K antenna pointed toward Zenith
50-100 K " " " horizon
290-300 K " " " ground
G/T Figure of Merit for a Receive Antenna

\[
\frac{G_R}{T_A} \text{ (dB)} \triangleq 10 \log_{10} \left( \frac{G_R}{T_A} \right) \text{ dBK}^{-1}
\]

SNR at input to receiver is proportional to \( G_R/T_A \) .... hence consider as figure of merit for antenna.

Fris Eqn: Sig. power delivered by receive antenna to matched receiver input is

\[
S_i = \frac{G_R G_T P_T \lambda^2}{(4\pi R)^2}
\]
Noise input to receiver: 

\[ N_i = kT_A B \]

\[
SNR = \frac{S_i}{N_i} = \frac{Gr G_T P_T \lambda^2}{kT_A B (4\pi R)^2} = \left(\frac{Gr}{T_A}\right) \frac{G_T P_T \lambda^2}{kB (4\pi R)^2}
\]

Only parameter controllable at receiver
Example Calculation for Direct Broadcast System

Given:
- $\lambda = 0.0241 \text{ m}$
- $P_T = 120 \text{ W}$
- $G_T = 34 \text{ dB} \rightarrow 2512$
- $B = \text{IF BW} = 20 \text{ MHz}$
- $R = 39 \text{,000 km}$
- $G_R = 33.5 \text{ dB} \rightarrow 2239$ ($18''$ dish)
- $T_A = 50\text{k}$
- $F = 1.1 \text{ dB} \rightarrow 1.29$

Find:
- @ EIRP of Tx
- $G/T$ for combo of receive antenna + low noise block
- Received carrier power @ receive antenna terms.
- CNR @ output of LNB
a) \[ \text{EIRP} = P_T G_T = (120\text{W})(2512) = 3.01 \times 10^5 \text{W} = 54.8 \text{ dBW} \]

b) To find G/T for antenna + LNB combo, find noise temp of cascade: antenna \rightarrow LNB, but referenced to LNB input

\[ T_e = T_A + T_{\text{LNB}} = T_A + (F-1)290 \text{K} \]
\[ = 50K + (1.29-1)290K = 134 \text{K} \]

\[ \therefore \frac{G}{T} = 10 \log_{10} \left( \frac{2239}{134} \right) = 12.2 \text{ dBK}^{-1} \]

(we do not include the LNB gain, \( G_{\text{LNB}} \) in this calculation)
\[ \text{Pr} = \frac{P_T G_T G_R \lambda^2}{(4\pi R)^2} \]

\[ \begin{align*}
\text{Pr} & = 1.63 \times 10^{-12} \text{ W} \\
\text{Pr} & = -117.9 \text{ dBW}
\end{align*} \]

\[ \text{CNR} = \frac{P_R G_{LB}}{kT_e B G_{LB}} \]

\[ \begin{align*}
\text{CNR} & = \frac{1.63 \times 10^{-12} \text{ W}}{(1.38 \times 10^{-23})(134)(20 \times 10^6)} \\
\text{CNR} & = 44.1 \\
\text{CNR} & = 16.4 \text{ dB}
\end{align*} \]
Recall Signal Space

\[ H_i : Y(t) = s_i(t) + X(t) \quad i = 0, 1, 2 \ldots M-1 \quad 0 \leq t \leq T \]

- \( s_i(t) \) of finite energy
- \( X(t) \) AWGN; \( \mathcal{N}_0/2 \)

Signal Space \( \equiv \text{Span} \{ s_0(t), s_1(t), \ldots, s_{M-1}(t) \} \)

\[ = \text{Span} \{ \phi_0(t), \phi_1(t), \ldots, \phi_{M-1}(t) \} \]

\( J \leq M \), \( \{ \phi_i(t) \} \) are orthonormal, can be constructed from original signals using Gram-Schmidt process.
Notes:

- Set of $J$ o.n. sigs $\{\phi_0 \ldots \phi_{J-1}\}$ can be extended to make C.O.N.S.

\[
\begin{align*}
\{ &\phi_0, \phi_1, \ldots \phi_{J-1}, \phi_J, \phi_{J+1} \ldots \} \\
\text{span signal space} \quad \text{span a space orthogonal to sig. space}
\end{align*}
\]

* If input $X(t)$, AWGN, $N_0/2$ then $X_j = \langle X, \phi_j \rangle$

* $X_j \perp X_k$, $j \neq k$

* $X$ is Gaussian, mean-zero, var $N_0/2$
Create equiv. HT problem

Define signal samples \( S_i j = \langle S_i, \phi_j \rangle \) \( j = 0, 1, \ldots, J-1 \)
\( i = 0, 1, \ldots, M-1 \)

\( S_i = [S_{i0}, S_{i1}, \ldots, S_{iJ-1}] \)

and also noise samples

\( X_j = \langle X, \phi_j \rangle \) \( j = 0, 1, \ldots, J-1 \)

\( X = [X_0, X_1, \ldots, X_{J-1}] \) \( \sim \) a Gaussian r.v. with mean zero and iid components

Equivalent H.T.

\( H_i : Y = S_i + X \) \( i = 0, 1, \ldots, M-1 \)

\( \rightarrow \) \( J \) dimensional pdfs\( \phi_j \) but usual framework.
\[ H_i : f_i(y) \sim N(s_i, \frac{\lambda}{J} \mathbb{I}_{J \times J}) \]

\[ = \prod_{j=0}^{J-1} \frac{1}{\sqrt{2\pi} \sigma_j} \exp \left\{ \frac{1}{2\sigma_j^2} (y_j - s_{i,j})^2 \right\} \]

\[ = (2\pi \sigma^2)^{-J/2} \exp \left\{ -\frac{1}{2\sigma^2} \sum_{j=0}^{J-1} (y_j - s_{i,j} \mid \gamma^2 \right\} \]

\[ = (2\pi \sigma^2)^{-J/2} \exp \left\{ -\frac{1}{2\sigma^2} \| y - s_i \|^2 \right\} \]

MAP estimate (i.e. Bayes for uniform costs & possibly unequal priors) computes

\[ \pi_i f_i(y) = \pi_i (2\pi \sigma^2)^{-J/2} \exp \left\{ -\frac{1}{2\sigma^2} \| y - s_i \|^2 \right\} \]

and pick hyp. corresponding to largest one

\[ k = \arg \max_{0 \leq i \leq N-1} \pi_i f_i(y) \]
Can simplify MAP by taking $\ln(\cdot)$, etc.

$$k = \arg \min_{0 \leq i \leq M-1} \left\{ ||y - s_i||^2 - 2 \sigma^2 \ln \pi_i \right\}$$

When priors are equal ($\pi_i = \frac{1}{M}$) then MAP becomes maximum likelihood (ML)

$$k = \arg \min_{0 \leq i \leq M-1} ||y - s_i||^2 \quad \Rightarrow \quad \text{ie pick the signal closest to the observation in Euclidean distance.}$$

Def A "plot" of signal points $S_i \quad 0 \leq i \leq M-1$
in J-dim space is called a signal constellation
Example Constellation + Decision Regions for Min Dist. (ML) Receiver

\[ J = 2, \ M = 3 \ \text{signals of equal energy } E \]