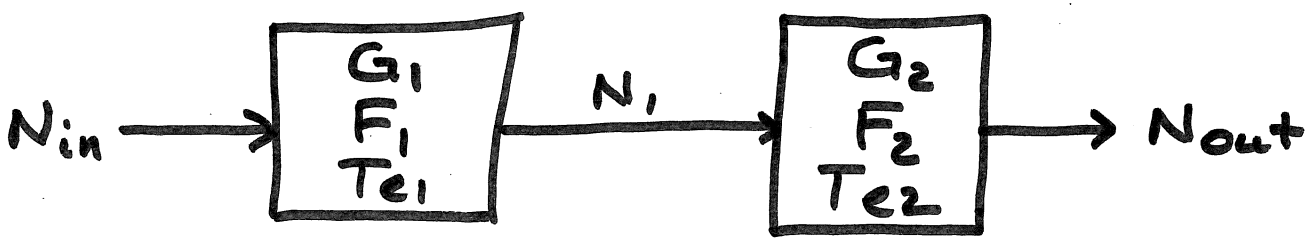
ECE 544: Digital Communications (Session 25)

Noise Figure of Cascaded Components

Let B be a (one-sided) band over which all powers are measured.



$\equiv$ (find equiv. such that)

* noise figs relative to $T_0 = 290\,K$
* N's represent total avail. powers over B.
* $N_{in} = kT_0B$

$$N_1 = G_1 \cdot \left(\begin{matrix}\text{power at}\\ \text{input}\end{matrix}\right) + \left(\begin{matrix}\text{amplifier internal}\\ \text{noise as appears}\\ \text{in output}\end{matrix}\right)$$

$$= G_1 N_{in} + G_1 k T_{e1} B \quad \left(\begin{matrix}\text{amplifier interal}\\ \text{noise is referred to}\\ \text{input}\end{matrix}\right)$$

$$N_{out} = G_2 N_1 + G_2 k T_{e2} B$$

$$= G_1 G_2 N_{in} + G_1 G_2 k T_{e1} B + G_2 k T_{e2} B$$

$$= G_1 G_2 \left[N_{in} + k \left(T_{e1} + \frac{T_{e2}}{G_1} \right) B \right]$$

↳ This from 1st block diagram

From the 2nd block diagram:

$$N_{out} = G_{12} N_{in} + G_{12} k T_{e12} B$$

Comparing 1st and 2nd, want to solve for G_{12}, T_{e12} so they give same result:

$$G_{12} = G_1 G_2$$

$$T_{e12} = T_{e1} + \frac{T_{e2}}{G_1}$$

For any amplifier the relationship between noise figure and equiv. noise temp is:

$$T_e = T_o (F-1)$$

$T_o \hookrightarrow 290K$

$\therefore$ In the case under consideration

$$T_{e12} = T_o (F_{12} - 1)$$

and

$$T_{e1} = T_o (F_1 - 1)$$

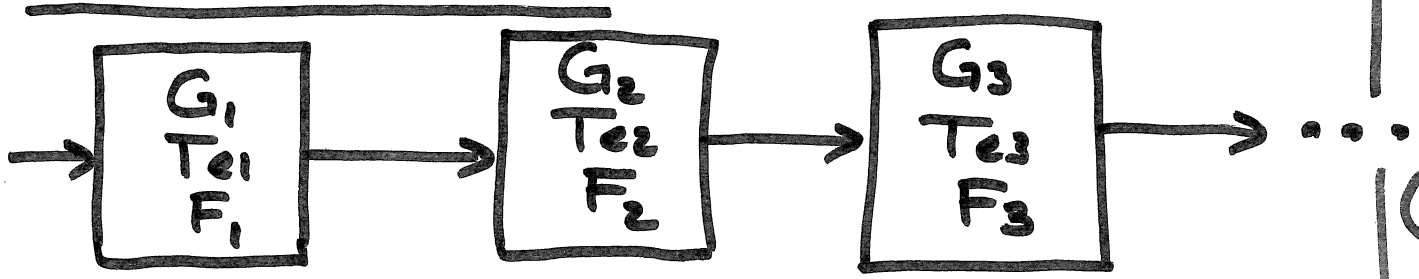
$$T_{e2} = T_o (F_2 - 1)$$

Substituting:

$$T_0(F_{12}-1) = T_0(F_1-1) + T_0\frac{F_2-1}{G_1}$$

$$\Rightarrow \quad F_{12} = F_1 + \frac{F_2-1}{G_1}$$

It Generalizes:



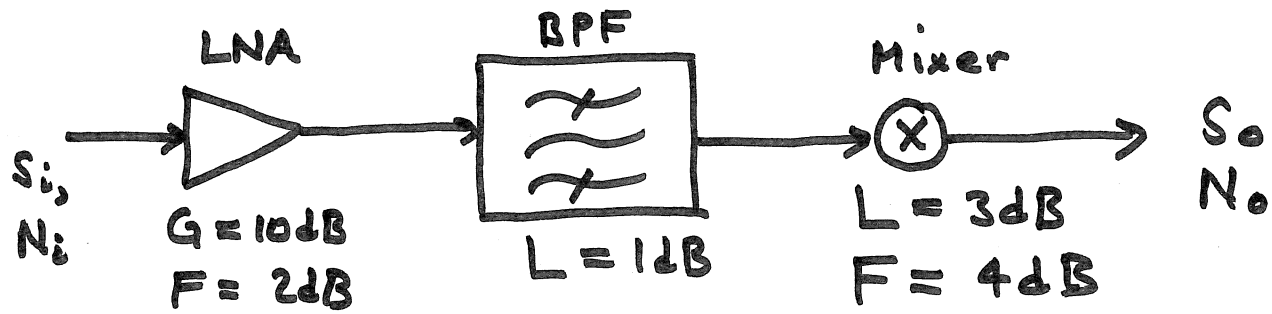
$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \cdots$$

$$F = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1 G_2} + \cdots$$

Conclusions:

1. Noise perf. dominated by first stages.
2. In general, 1st stage of receiver should have low noise fig. and at least moder. gain

Example (Front End of a Rx)



Assume: Input noise from antenna feed with $T_a = 15K$
IF Bandwidth $B = 10$ MHz
System @ $T_o = 290K$
Syst. comp. all matched to $50\,\Omega$

Find:
1. Overall noise figure.
2. Output noise power in dBm
3. Find 2 sided psd height of output noise.
4. If need SNR of 20 dB at output, what is min. sig. voltage at receiver input?

<u>Prelim calcs</u> $F_{dB} = 10 \log_{10} F$; $10^{F_{dB}/10} = F$

LNA: $G = 10\,dB \longrightarrow 10$

$F = 2\,dB \longrightarrow 1.58 \longrightarrow T_{e,LNA} = (F-1)\underbrace{T_0}_{290K} = 0.58\,T_0$

BPF: $G = -1\,dB \longrightarrow 0.79$

model as a passive attenuator, matched and at the syst. temp of 290K

Then $F = L = 1\,dB$

$\hookrightarrow 1.26 \longrightarrow T_{e,BPF} = .26\,T_0$

Mixer: $G = -3\,dB \longrightarrow .5$

$F = 4\,dB \longrightarrow 2.51 \longrightarrow T_{e,mixer} = 1.51\,T_0$

Overall noise equiv. temp

$$T_e = T_{e,LNA} + \frac{T_{e,BPF}}{G_{LNA}} + \frac{T_{e,mixer}}{G_{LNA} G_{BPF}}$$

$$= .58 T_o + \frac{.26 T_o}{10} + \frac{1.51 T_o}{10 \cdot 0.79}$$

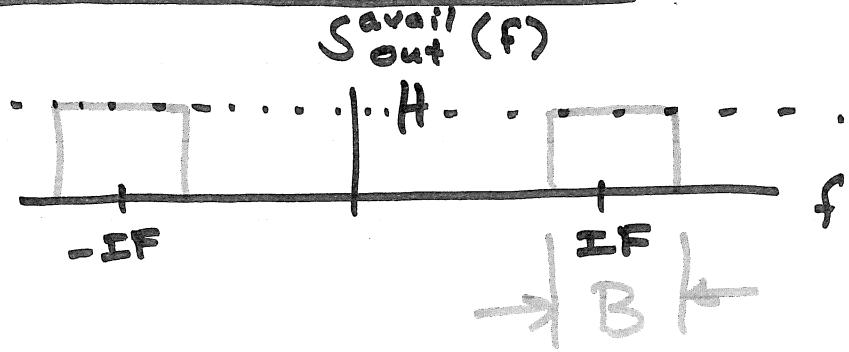
$$= 0.8 T_o = 232 K$$

$$F = 1 + \frac{T_e}{T_o} = 1.8 \longrightarrow 2.55 \text{ dB} \quad (1).$$

<u>Output Noise Power</u>

$$N_o = k(T_a + T_e)BG = \begin{cases} 1.35 \times 10^{-13} \text{ W} \\ -98.7 \text{ dBm} \end{cases} \quad (2)$$

Two-sided psd height



Total noise power

$$H \cdot 2B = 1.35 \times 10^{-13}\ \text{W}$$

$$H = \frac{1.35 \times 10^{-13}\ \text{W}}{2(10 \times 10^{6}\ \text{s}^{-1})}$$

$$= 6.75 \times 10^{-21}\ \text{W} \cdot \text{s} \quad (3)$$

Output SNR = 20dB = 100

$$S_i = \frac{S_o}{G}\frac{N_o}{N_o} = \frac{S_o}{N_o} \cdot \frac{N_o}{G} = 100 \cdot \frac{1.35 \times 10^{-13}\ \text{W}}{3.95}$$

$$= 3.42 \times 10^{-12}\ \text{W} \quad (-84.7\ \text{dBm})$$

$$S_i = V_i^2 / 50\Omega \longrightarrow V_i = 13.1\ \mu\text{V (rms)}.$$

Antennas + Propagation (D.M. Pozar, Microwave + RF Design of Wireless).

Take a systems viewpoint. Antenna params