Noise Figure of Cascaded Components

Let $B$ be a (one-sided) band over which all powers are measured.

- Noise figs relative to $T_0 = 290\, K$
- $N$'s represent total available powers over $B$
- $N_{in} = kT_0 B$
\[ N_i = G_1 \cdot \text{(power at input)} + \text{(amplifier internal noise as appears in output)} \]
\[ = G_1 N_{in} + G_1 kT e_1 B \text{ (amplifier internal noise is referred to input)} \]

\[ N_{out} = G_2 N_i + G_2 kT e_2 B \]
\[ = G_1 G_2 N_{in} + G_1 G_2 kT e_1 B + G_2 kT e_2 B \]
\[ = G_1 G_2 \left[ N_{in} + k \left( T e_1 + \frac{T e_2}{G_1} \right) B \right] \]

\[ \rightarrow \text{This from 1st block diagram} \]
From the 2nd block diagram:

\[ N_{\text{out}} = G_{12} N_{\text{in}} + G_{12} k T_{e_{12}} B \]

Comparing 1st and 2nd, want to solve for \( G_{12}, T_{e_{12}} \) so they give same result:

\[ G_{12} = G_1 G_2 \]

\[ T_{e_{12}} = T_{e_1} + \frac{T_{e_2}}{G_1} \]
For any amplifier the relationship between noise figure and equiv. noise temp is:

\[ T_e = T_0 (F - 1) \]
\[ \Rightarrow 290K \]

\[ \therefore \text{In the case under consideration} \]
\[ T_{e1z} = T_0 (F_{1z} - 1) \]

and

\[ T_{e1} = T_0 (F_1 - 1) \]
\[ T_{e2} = T_0 (F_2 - 1) \]
Substituting:
\[ T_0 (F_{i2} - 1) = T_0 (F_i - 1) + T_0 \frac{F_{2} - 1}{G_1} \]

\[ \Rightarrow F_{i2} = F_i + \frac{F_{2} - 1}{G_1} \]

It Generalizes:

\[ T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1G_2} + \ldots \]

\[ F = F_i + \frac{F_{2} - 1}{G_1} + \frac{F_{3} - 1}{G_1G_2} + \ldots \]

Conclusions:
1. Noise perf. dominated by first stages.
2. In general, 1st stage of receiver should have low noise fig. and at least moderate gain.
Example (Front End of a Rx)

LNA

BPF

Mixer

$S_i$

$N_i$

$G = 10\,\text{dB}$

$L = 1\,\text{dB}$

$L = 3\,\text{dB}$

$F = 2\,\text{dB}$

$F = 4\,\text{dB}$

$S_o$

$N_o$

Assume: Input noise from antenna feed with $T_e = 15\,\text{K}$

IF Bandwidth $B = 10\,\text{MHz}$

System @ $T_e = 290\,\text{K}$

Syst. comp. all matched to 50$\,\Omega$

Find:
1. Overall noise figure.
2. Output noise power in dBm.
3. Find 2-sided psd height of output noise.
4. If needed SNR of 20 dB at output, what is min. sig. voltage at receiver input?
Prelim calc  $F_{dB} = 10 \log_{10} F$  \[ F_{dB} \% = F \]

LNA:  $G = 10 \text{dB} \rightarrow 10$

$F = 2 \text{dB} \rightarrow 1.58 \rightarrow T_{c, \text{LNA}} = (F-1)T_0 = 0.58 T_0$

BPF:  $G = -1 \text{dB} \rightarrow 0.79$

Model as a passive attenuator, matched and at the syst. temp. of 290K

Then  $F = L = 1 \text{dB}$

$\rightarrow 1.26 \rightarrow T_{c, \text{BPF}} = .26 T_0$

Mixer:  $G = -3 \text{dB} \rightarrow .5$

$F = 4 \text{dB} \rightarrow 2.51 \rightarrow T_{c, \text{mixer}} = 1.51 T_0$
Overall noise equiv. temp

\[ T_e = T_{e,LNA} + \frac{T_{e,BPF}}{G_{LNA}} + \frac{T_{e, mixer}}{G_{LNA} G_{BPF}} \]

\[ = 0.58 T_0 + \frac{0.26 T_0}{10} + \frac{1.51 T_0}{10 \cdot 0.79} \]

\[ = 0.8 T_0 = 232 \text{K} \]

\[ F = 1 + \frac{T_e}{T_0} = 1.8 \rightarrow 2.55 \text{dB} \]

Output Noise Power

\[ N_0 = k(T_e + T_c)BG = 1.35 \times 10^{-13} \text{W} \]

\[ -98.7 \text{ dBm} \]
Two-sided psd height

\[
\frac{S_{\text{avail}}(f)}{S_{\text{out}}(f)}
\]

Output SNR = 20dB = 100

\[
S_i = \frac{S_o}{G} \cdot \frac{N_o}{N_0} \cdot \frac{N_o}{G} = 100 \cdot \frac{1.35 \times 10^{-13}}{3.95}
\]

\[
= 3.42 \times 10^{-12} \text{ W} \quad (-84.7 \text{ dBm})
\]

\[
S_i = \frac{V_i^2}{50 \Omega} \quad \rightarrow \quad V_i = 13.1 \mu \text{V (rms)}.
\]

Total noise power

\[
H \cdot 2B = 1.35 \times 10^{-13} \text{ W}
\]

\[
H = \frac{1.35 \times 10^{-13}}{2(10 \times 10^6 \text{ s}^{-1})}
\]

\[
= 6.75 \times 10^{-21} \text{ W s}
\]
Antennas + Propagation (D.M. Pozar, Microwave + RF Design of Wireless).

Take a systems viewpoint. Antenna params.