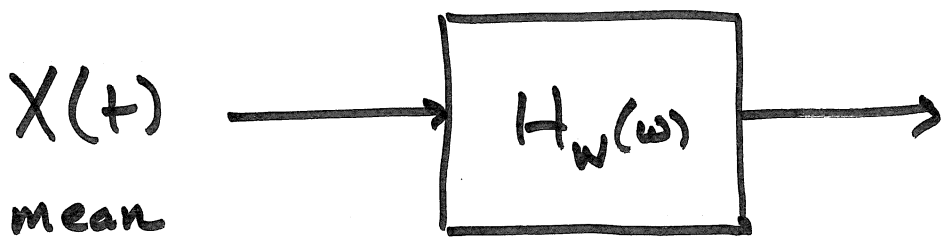
EE 544 Dig. Comm. (Session 10)

MF Filtering for Colored Noise.

Approach : Whitening Filter



Desire a white r.p. $W(t)$

0 mean Gaussian

$S_X(\omega)$ not necc. flat

Standard on LTI filters:

$$EX(t) = 0 \implies EW(t) = 0$$

$$X(t) \text{ Gauss.} \implies W(t) \text{ is Gauss.}$$

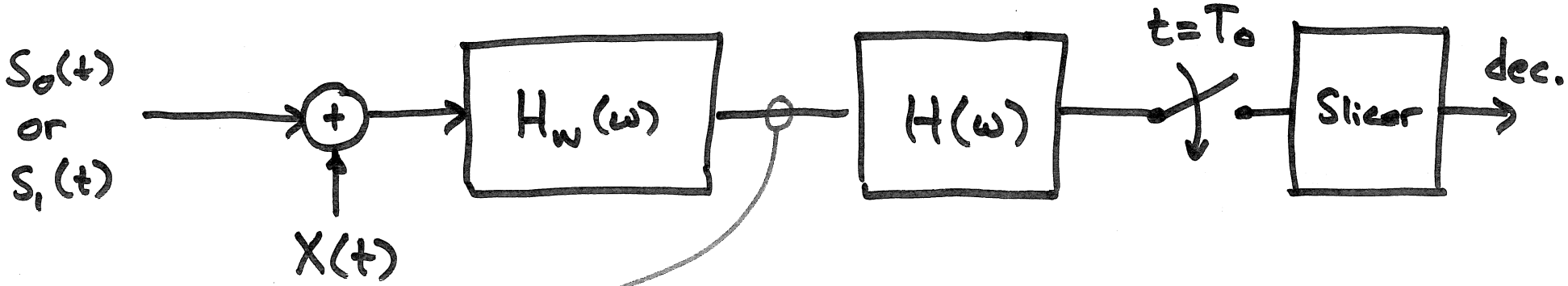
$$S_W(\omega) = |H_W(\omega)|^2 S_X(\omega)$$

I seek a filter H_W s.t.

$$1 = |H_W(\omega)|^2 S_x(\omega) \Longrightarrow |H_W(\omega)|^2 = 1/S_x(\omega)$$

Suppose there is such a filter $h_W(t) \longleftrightarrow H_W(\omega)$

Then consider orig. prob but with h_W inserted:



$$\left.\begin{aligned} u_0(t) &\triangleq s_0 * h_W(t) \\ &\text{or} \\ u_1(t) &\triangleq s_1 * h_W(t) \end{aligned}\right\} + W(t) \longrightarrow \text{white with } S_W(\omega) = 1$$

This is already solved. Know that MF is

$$h(t) = c\tilde{u}_{T_0}(t) = c\left[u_0(T_0-t) - u_1(T_0-t)\right]$$

$$\updownarrow$$

$$H(\omega) = c e^{-j\omega T_0}\left[U_0^*(\omega) - U_1^*(\omega)\right]$$

$$= c e^{-j\omega T_0} H_W^*(\omega)\left[S_0^*(\omega) - S_1^*(\omega)\right]$$

<u>The overall filter</u>

$$H_W(\omega)H(\omega) = c e^{-j\omega T_0}\frac{S_0^*(\omega) - S_1^*(\omega)}{S_X(\omega)}$$

<u>Questions</u> ① Existence of whitening filter

② Optimality of approach.

General Decision Rules / General Channels (MBP Sec 5.4 pp 224 - 234).

Let Z be a r.v. with two possible statistical descriptions ("states" of nature)

$$H_0 : \quad Z \sim f_0(z)$$

vs.

$$H_1 : \quad Z \sim f_1(z)$$

- Either H_0 or H_1 must occur, but not both.
- May have (or maybe not) "side" or "prior" information about H_0, H_1

$$\pi_0 = P(H_0 \text{ is true}) \; ; \; \pi_1 = P(H_1 \text{ is true})$$

$$\pi_0 + \pi_1 = 1$$

A <u>general</u> <u>decision</u> <u>rule</u> is a two-valued function of the observations

$$Z = z$$

$$T(z) = \begin{cases} 0 & \longrightarrow \text{ dec. rule chooses } H_0 \\ \text{or} & \\ 1 & \longrightarrow \text{ " " " } H_1 \end{cases}$$

Let Γ be the space of all poss. values taken by Z. Then specifying T is the same as specifying a partition

$$\Gamma = \Gamma_0 \cup \Gamma_1 \quad \text{st} \quad \Gamma_0 \cap \Gamma_1 = \phi$$

$$z \in \Gamma_0 \iff T(z) = 0$$
$$z \in \Gamma_1 \iff T(z) = 1$$

$T(z) =$ "indicator" of set Γ_1
$= 1_{\Gamma}(z)$

Every possible partition gives a decision rule.
Our prev. threshold tests

$$Z \begin{matrix} > \\ \leq \end{matrix} \gamma \qquad \begin{matrix} \text{decide } 0 \\ \text{"} \quad 1 \end{matrix}$$

corresponds to $\Gamma_0 = (\gamma, +\infty)$; $\Gamma_1 = (-\infty, \gamma]$

<u>Optimizing Decision Rules</u> $\Gamma = \Gamma_0 \cup \Gamma_1$; $\Gamma_0 \cap \Gamma_1 = \phi$

$P_{e,0} = P(\text{say } H_1 \mid H_0 \text{ true})$ $\qquad$ $P_{e,1} = P(\text{say } H_0 \mid H_1 \text{ true})$

$= P(Z \in \Gamma_1 \mid H_0 \text{ true})$ $\qquad$ $= P(Z \in \Gamma_0 \mid H_1 \text{ true})$

$= \int_{\Gamma_1} f_0(z)\, dz$ $\qquad$ $= \int_{\Gamma_0} f_1(z)\, dz$

If we have priors then can use Bayes criterion

$$\overline{P}_e = \pi_0 P_{e,0} + \pi_1 P_{e,1}$$

$$= \pi_0 \int_{\Gamma_1} f_0(z)\,dz + \pi_1 \int_{\Gamma_0} f_1(z)\,dz$$

$$= \pi_0 \left\{ 1 - \int_{\Gamma_0} f_0(z)\,dz \right\} + \pi_1 \int_{\Gamma_0} f_1(z)\,dz$$

$$= \pi_0 + \int_{\Gamma_0} \left[\pi_1 f_1(z) - \pi_0 f_0(z) \right] dz$$

<u>note:</u>

$$1 = \int_{\Gamma = \Gamma_0 \cup \Gamma_1} f_0(z)\,dz$$

$$= \int_{\Gamma_1} f_0(z)\,dz + \int_{\Gamma_0} f_0(z)\,dz$$