ECE 544 Dig. Comm.

Noise Modeling + System Noise Char.

references: Davenport + Root
Freeman
Engberg + Larson
Pozar "Microwave + RF Design of Wireless Systems"

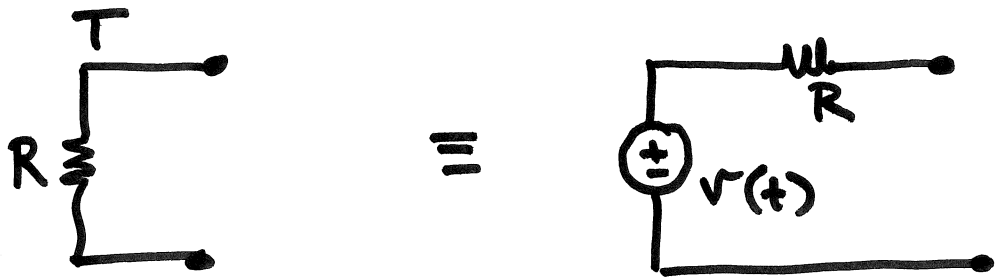
Nyquist 1928: Mean sq. voltage app. across terms of R Ω @ temp T Kelvin

is $4kTRB$ V^2

k → Boltzman const.

1.38×10^{-23} J/K

Model for a Noisy Resistor



noisy R

Thevenin equiv.

- $v(t)$ is a WSS zero mean r.p. with 2 sided psd

$$S_v(f) = 2kTR \ V^2/Hz$$

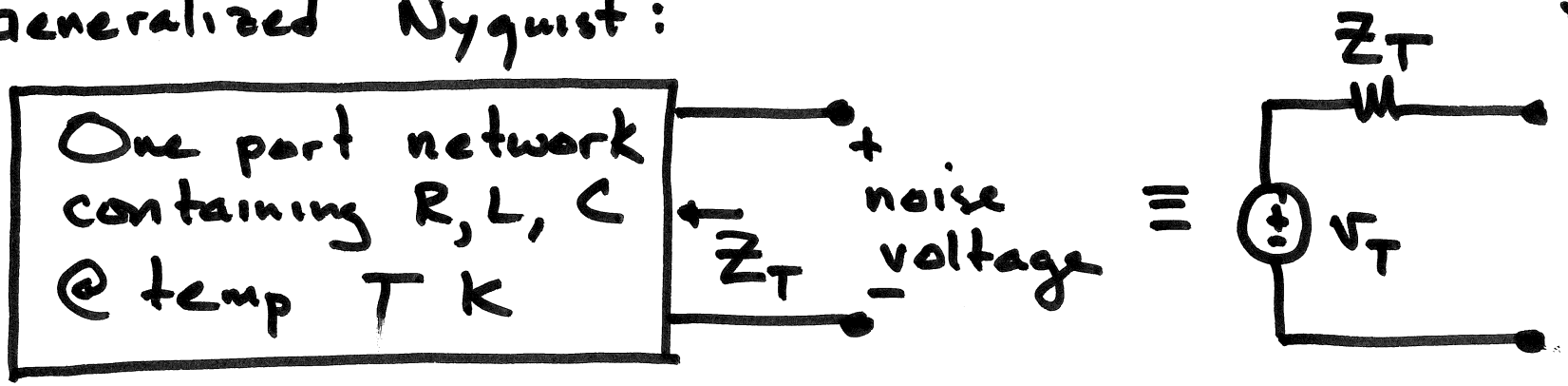
- R noiseless.

* Model valid for freqs up to μwaves.
* Optical comms req new model.
* $v(t)$ is Gaussian
* Com. Model comes from Stat. physics $\rightarrow$ rand motion of charge carriers

- ☐ resistors, lossy trans. lines
- ☐ atten. through atmosp.

3.

Generalized Nyquist:



Here $Z_T = R_T + j X_T$

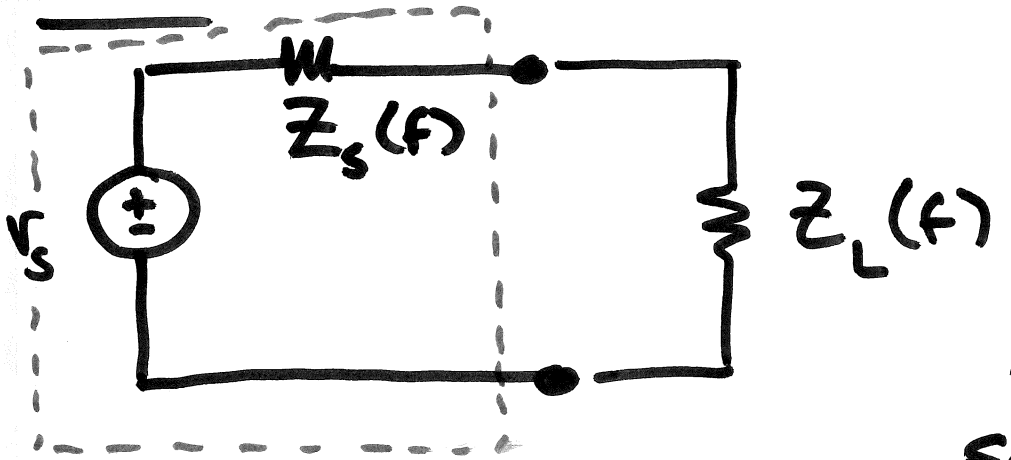
The noise source v_T is WSS, zero mean, Gaussian with

$$S_{v_T}(f) = 2kT R_T(f) \quad V^2/Hz$$

Available Noise Power

Refers to the max. power that can be delivered from source to a load.

Max Power Transf. Thm for Rand. Proc.



Thev. Eq. Mod. of Noise Source

$v_s(t)$ has $S_{v_s}(f)$ as psd.

To prov. max pow. to load set $Z_L(f) = Z_s^*(f)$

Then the resulting max power is

$$P_L^{max} = \int_{-\infty}^{\infty} \frac{S_{v_s}(f)}{4\,R_s(f)}\,df$$

$$S_{v_s}^{avail}(f) = S_{v_s}(f) / 4R_s(f) \quad W/Hz = J$$

<u>Special Case:</u> Noisy resistor (R, T)

$$S^{avail}(f) = \frac{2kTR}{4R} = \frac{kT}{2}$$

This is the $N_0/2$ is our model for AWGN.

Sample Numbers

$$N_0 = kT = (1.38 \times 10^{-23})(290\ K)$$

$$= 4.0 \times 10^{-21}\ J \quad (W/Hz)$$

$$= -203.98 \quad dBJ$$

Available noise power in a one-sided bw of B Hz is

$$P_o = \frac{N_o}{2}(B2) = N_o B$$

$$P_{dBW} = (N_o)_{dBJ} + (B)_{dBHz}$$

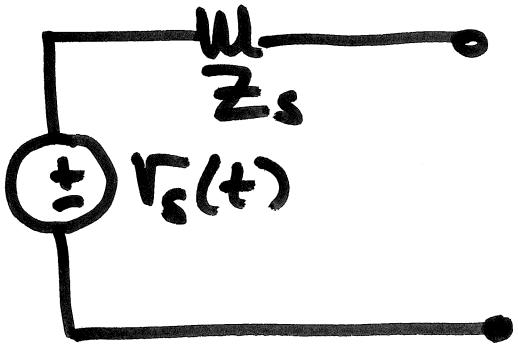
$$dB(W\text{-}Hz)$$

$$= -203.98 + 10\log_{10} B$$

$$(dBW)$$

B	P_{dBW}	P (W)
1 MHz	−143.98	4.0×10^{-15}
10 MHz	−133.98	4×10^{-14}
100 MHz	−123.98	4×10^{-13}
1 GHz	−113.98	4×10^{-12}

Effective Noise Temperature of a Source



$$S_{v_s}^{avail}(f) = \frac{S_{v_s}(f)}{4R_s(f)}$$

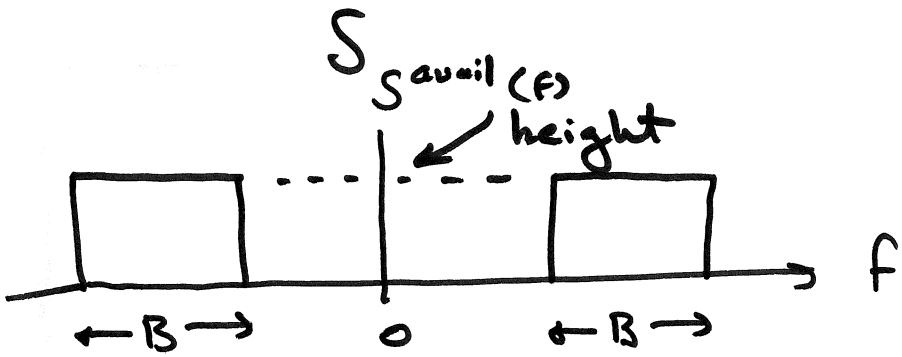
Set

$$\frac{kT_e(f)}{2}$$

$$T_e(f) = \frac{2\, S_{v_s}^{avail}(f)}{k}$$

Most common application is to noise sources that are either white or white over a restricted band.

$\Rightarrow$ Often ignore the freq. dependence of $T_e \,//\, S_{v_s}^{avail}$.



Suppose have a noise source which delivers a total noise power P_{noise} into a matched load over a one-sided BW B.

$$P_{noise} = 2B\,(\text{height})$$

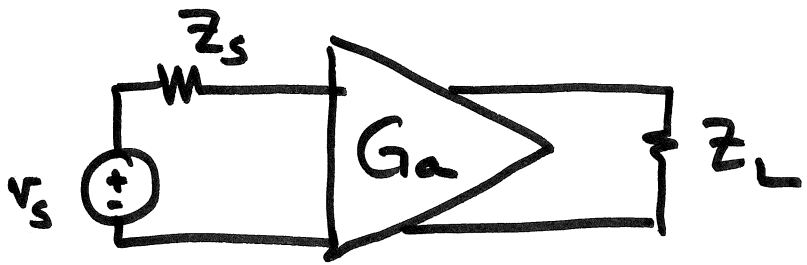
$$\Rightarrow \text{height} = P_{noise}/2B$$

$$\Rightarrow S_{v_s}^{avail}(f) = \begin{cases} P_{noise}/2B & f \in \text{the Band.} \\ 0 & f \notin \text{the Band} \end{cases}$$

Then over the band of interest

$$T_e = \frac{2\,S_{v_s}^{avail}}{k} = \frac{P_{noise}}{kB}$$

Available Power Gain of an Amp

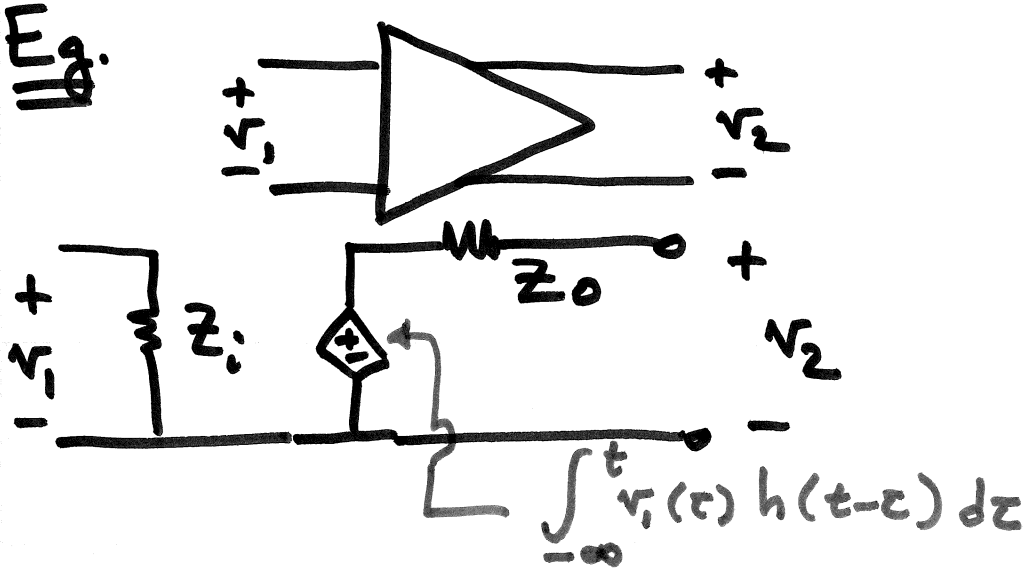


$$G_a(f) = \frac{S_{out}^{avail}(f)}{S_{in}^{avail}(f)}.$$

$S_{in}^{avail}(f)$ is that assoc. with v_s, Z_s.

S_{out}^{avail} is the avail psd of amp. output.

Eg.



For this unilat. model

$$G_a = \frac{S^a_{out}(f)}{S^a_{in}(f)} = \frac{R_s(f)\, S_{out}(f)}{R_o(f)\, S_{in}(f)}$$

$$v_s \mapsto v_1 \quad \text{with} \quad \frac{Z_i}{Z_s + Z_i}$$

$$v_1 \longmapsto \int_{-\infty}^{t} v_1(\tau)\, h(t-\tau)\, d\tau \quad \text{is} \quad H(f)$$

$$S_o(f) = \left| \frac{Z_i}{Z_s + Z_i} H \right|^2 S_{in}(f)$$

$$G_a = \frac{R_s}{R_o} \left| \quad \downarrow \quad \right|^2$$