Nyquist 1928: Mean sq. voltage app. across terms of resistance at temp $T$ Kelvin is $4kTRB V^2$

$\kappa$

Boltzmann constant

$1.38 \times 10^{-23} \text{ J/K}$
Model for a Noisy Resistor

\[ V(t) = \frac{V_R}{R_s} \]

- Model valid for freq up to \( \mu \) waves.
- Optical comms req new model.
- \( V(t) \) is Gaussian
- Com. Model comes from Stat. physics \( \rightarrow \) rand motion of charge carriers
  - Resistor, lossy trans. lines
  - Attenu. through atmosp.

Thevenin equiv.
- \( V(t) \) is a WSS zero mean r.p.
  - with 2 sided pdf
  \[ S_v(f) = 2kT R \frac{V^2}{\text{Hz}} \]
- R noiseless.
Generalized Nyquist:

One port network containing $R, L, C$ at temp $T$ K

$Z_T$ and noise voltage

\[ Z_T + \text{noise voltage} = V_T \]

Here $Z_T = R_T + jX_T$

The noise source $V_T$ is WSS, zero mean, Gaussian with

\[ S_{v_T}(f) = 2kT R_T(f) \quad V^2/Hz \]

Available Noise Power

Refers to the max. power that can be delivered from source to a load.
Max Power Transfer. Thru for Rand.

\[ V_s(f) \text{ has } S_y, \text{ as psd.} \]

To prove max pow. to load

\[ Z_L(f) = Z_s^*(f) \]

Then max power is

\[ P_L = \int_{-\infty}^{\infty} \frac{S_y(f)}{4R_s^2(f)} \]
\[ S_{\text{avail}}(f) = \frac{S_{\text{rs}}(f)}{4R_s(f)} \text{ W/Hz} \]

Special Case: Noisy resistor \((R, T)\)

\[ S_{\text{avail}}(f) = \frac{2kTR}{4R} = \frac{kT}{2} \]

This is the No/2 is our model for AWGN.
Sample Numbers

\[ N_0 = kT = (1.38 \times 10^{-23}) (290 \text{ K}) \]
\[ = 4.0 \times 10^{-21} \text{ J (W/Hz)} \]
\[ = -203.98 \text{ dBJ} \]

Available noise power in a one-sided bw of B Hz is

\[ P_0 = \frac{N_0}{2} (B2) = N_0 B \]

\[ P_{\text{dBW}} = (N_0)_{\text{dBJ}} + (B)_{\text{dBHz}} \]
\[ = -203.98 + 10 \log_{10} B \text{ (dBW)} \]
<table>
<thead>
<tr>
<th>B</th>
<th>P_{dbw}</th>
<th>P\ (\text{W})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 MHz</td>
<td>-143.98</td>
<td>4.0 \times 10^{-15}</td>
</tr>
<tr>
<td>10 MHz</td>
<td>-133.98</td>
<td>4 \times 10^{-14}</td>
</tr>
<tr>
<td>100 MHz</td>
<td>-123.98</td>
<td>4 \times 10^{-13}</td>
</tr>
<tr>
<td>1 GHz</td>
<td>-113.98</td>
<td>4 \times 10^{-12}</td>
</tr>
</tbody>
</table>

**Effective Noise Temperature of a Source**

\[
\text{Set} \quad k \cdot T_e(f) = \frac{S_{V_e}(f)}{4R_s(f)}
\]

\[
W = \frac{Z_s}{Z_s}
\]

\[
S_{V_s}(f) = \text{avail}
\]
\[ T_c(f) = \frac{2 S_{\text{avail}}(f)}{k} \]

Most common application is to noise sources that are either white or white over a restricted band.

\( \Rightarrow \) Often ignore the freq. dependence of \( T_e / S_{\text{avail}} \).
Suppose we have a noise source which delivers a total noise power $P_{\text{noise}}$ into a matched load over a one-sided BW $B$.

Then over the band of interest

$$T_c = 2 \frac{S_{\text{avail}}}{k} = \frac{P_{\text{noise}}}{kB}$$
Available Power Gain of an Amp

\[ G_a(f) = \frac{S_{\text{out}}(f)}{S_{\text{in}}(f)} \]

\( S_{\text{in}}(f) \) is that assoc. with \( v_s, Z_s \).

\( S_{\text{out}} \) is the avail psd of amp. output.

Equation:

\[ v_1^+ - v_1^- = v_2^+ - v_2^- \]
For this unilat. model

\[ G_a = \frac{S_{\text{out}}(f)}{S_{\text{in}}(f)} = \frac{R_s(f) S_{\text{out}}(f)}{R_o(f) S_{\text{in}}(f)} \]

\[ v_s \rightarrow v_i \quad \text{with} \quad \frac{Z_i}{Z_s + Z_i} \]

\[ v_i \rightarrow \int_{-\infty}^{+} v_i(t) h(c + t - z) \, dz \quad \text{is} \quad H(f) \]

\[ S_0(f) = \left| \frac{Z_i}{Z_s + Z_i} H \right|^2 S_{\text{in}}(f) \]

\[ G_a = \frac{R_s}{R_o} \left| \downarrow \right|^2 \]