ECE 544 Fall 2014
Problem Set 9
Due December 5, 2014

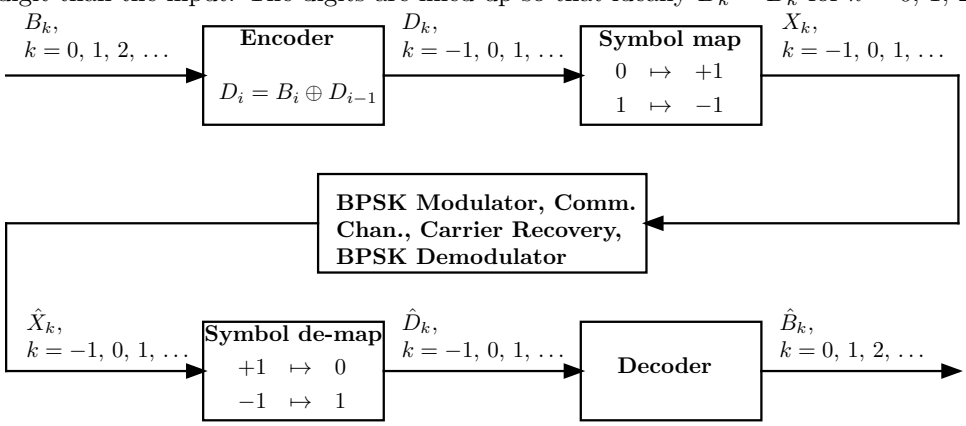
1. Read Chapter 8 of M. B. Pursley, *Introduction to Digital Communications* (MBP).

2. MBP Problems 7.6, 7.9, 7.13, 7.17, 7.20, 7.23

3. The block diagram shown is a possible implementation of a differentially encoded communication system.

   The encoder accepts a binary (i.e., 0 or 1) string B_k at the input and produces a binary string D_k at the output. The output string contains an extra digit D_{-1}, which is set as an initial condition of the encoder. The symbol "$\oplus$" in the encoder denotes modulo 2 binary addition (i.e., exclusive or).

   The decoder takes a binary string input $\hat{D}_k$ and produces a binary string output $\hat{B}_k$ containing one fewer digit than the input. The digits are lined up so that ideally $\hat{B}_k = B_k$ for $k = 0, 1, 2, \ldots$.



   (a) The table below gives an example input string B_k. Assuming that the encoder initial condition is $D_{-1} = 0$ as shown, fill in the values for the encoded bits D_k in the indicated row of the table.

	Time index k										
	-1	0	1	2	3	4	5	6	7	8	9
B_k	–	0	1	1	0	1	0	1	0	0	1
D_k	0										
$\hat{D}_k$											
$\hat{B}_k$	–										

   (b) The bits D_k starting from $k = -1$ are mapped to symbols, sent through the channel, and then de-mapped to produce the estimated bit sequence $\hat{D}_k$. Assuming that no bit errors occur in transmission, fill in the row in the above table corresponding to $\hat{D}_k$.

   Then give a mathmatical formula for the decoder and write down the estimated bit sequence $\hat{B}_k$ in the above table.

   (c) Repeat part (b) for the table shown below but now assume that the channel is such that the bits $\hat{D}_k$ are the complements of the corresponding bits D_k. Use exactly the same decoder as you found in part (b). The table is repeated below. You will need to fill the D_k row in with the same values as found in part (a).

   Explain why the result you find is important in BPSK systems, which use either the squaring loop or the Costas loop for carrier phase recovery.

	Time index k										
	-1	0	1	2	3	4	5	6	7	8	9
B_k	–	0	1	1	0	1	0	1	0	0	1
D_k	0										
$\hat{D}_k$											
$\hat{B}_k$	–										

(d) Repeat part (b) only now assuming that a single bit error is made in the channel at time index $k = 3$, i.e., $\hat{D}_k = D_k$ for $k \neq 3$ and $\hat{D}_3 \neq D_3$. The position of the bit error is indicated in the table below with a small box.

Comment on the result in light of what was found in the homework problem about the probability of error performance of differentially encoded BPSK in comparison to regular BPSK.

	Time index k										
	-1	0	1	2	3	4	5	6	7	8	9
B_k	–	0	1	1	0	1	0	1	0	0	1
D_k	0										
$\hat{D}_k$					$\boxed{}$						
$\hat{B}_k$	–										

4. This problem concerns just the encoder of the DBPSK system given before. Suppose that the input bit string B_k is independent and identically distributed (i.i.d.) with

$$P(B_k = 1) = p \quad \text{and} \quad P(B_k = 0) = 1 - p.$$

(a) Assuming that the encoder initial state is $D_{-1} = 0$ find the marginal probability distribution of the encoder output for all time k, i.e., find

$$q_k \stackrel{\text{def}}{=} P(D_k = 1)$$

for $k \geq 0$. *Hint:* Find a first order difference equation for q_k and solve it.

(b) For general p, are the random variables $\{D_k : k \geq 0\}$ identically distributed? Are they statistically independent? You must prove or give a counter example.

(c) For the special case of $p = 1/2$, are the random variables $\{D_k : k \geq 0\}$ identically distributed? Are they statistically independent? You must prove or give a counter example.

5. Phase locked loop problems:

- Z&T Computer Exercise 3.9.

> **3.9.** Modify the simulation program given in Computer Example 3.4 to allow the sampling frequency to be entered interactively. Examine the effect of using different sampling frequencies by executing the simulation with a range of sampling frequencies. Be sure that you start with a sampling frequency that is clearly too low and gradually increase the sampling frequency until you reach a sampling frequency that is clearly higher than is required for an accurate simulation result. Comment on the results. How do you know that the sampling frequency is sufficiently high?

- Modify the simulation program in Computer Example 3.4 to replace the perfect loop filter with the imperfect loop filter of Z&T Problem 3.47 where $\lambda = 0.1$.