ECE 440 **Exam 1** **2/11/22**

Name: Solution

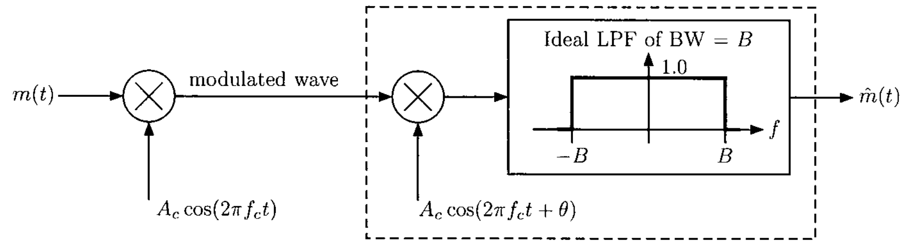
General Instructions:

- **You have 50 minutes to complete the exam.**
- Write your name on every page of the exam.
- Please do not write on the backs of pages.
- The exam is closed book. Calculators are not allowed. You are allowed both sides of one 8.5 by 11 inch sheet of paper for your personal notes in addition to the instructor supplied formula sheet.
- Your work must be explained to receive full credit. All plots must be carefully drawn with axes labeled.
- Point values for each problem are as indicated. The exam totals 100 points.
- Please do not leave early as it is disruptive to those working around you.

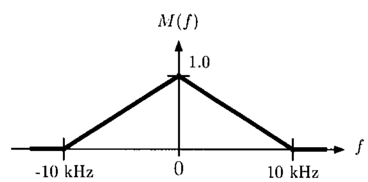
Do not open the exam until you are told to begin.

Name: ____________________

Problem 1. *Related to DSB Modulation and Demodulation.* [50 pts. total]



Suppose that a deterministic message $m(t)$ is applied to the block diagram above. Assume that the message has a Fourier transform $M(f)$ with the triangular spectral shape shown below. In the parts below you will solve for the spectrum of the output $\hat{M}(f)$ for various values of the free parameters A_c, f_c, θ, and B. When plotting $\hat{M}(f)$ plot the real and imaginary parts separately if it is complex-valued. If it is real-valued you may plot only the real part. Note that $M(f)$ is real-valued.



(a) [20 pts.] Solve for the general form of $\hat{M}(f)$, i.e., give a formula for $\hat{M}(f)$ that would hold for any choices of the free parameters in the problem: A_c, f_c, θ, and B. Use $H(f)$ to represent the lowpass filter in the block diagram.

The input to the LPF is

$$m(t)\, A_c^2 \cos 2\pi f_c t \cos(2\pi f_c t + \theta)$$

and so the output $\hat{m}(t)$ is simply

$$\hat{m}(t) = h(t) * \left[m(t) A_c^2 \cos(2\pi f_c t) \cos(2\pi f_c t + \theta) \right]$$

To get the desired formula we will want to take the Fourier transform. But first use a trig identity to write

$$\cos(2\pi f_c t)\cos(2\pi f_c t + \theta) = \frac{1}{2}\cos\theta + \frac{1}{2}\cos(2\pi 2 f_c t + \theta)$$

Problem 1. (cont'd.) Name: ____________________

(a) (cont'd) ...

$\therefore$ LPF input is

$$\frac{A_c^2}{2} m(t) \cos\theta + \frac{A_c^2}{2} m(t) \cos(2\pi 2f_c t + \theta)$$

From Euler: $\cos(2\pi f_c t + \theta) = \dfrac{e^{j\theta} e^{j2\pi 2f_c t} + e^{-j\theta} e^{-j2\pi 2f_c t}}{2}$

Then

$$\hat{M}(f) = H(f) \cdot \left\{ \begin{array}{l} \text{the FT of the} \\ \text{input above} \end{array} \right\}$$

$$= H(f) \left\{ \frac{A_c^2}{2} \cos\theta \, M(f) + \frac{A_c^2}{4} e^{j\theta} M(f-2f_c) + \frac{A_c^2}{4} e^{-j\theta} M(f+2f_c) \right\}$$

$$= \frac{A_c^2}{2} \cos\theta \, H(f) M(f) + \frac{A_c^2}{4} e^{j\theta} H(f) M(f-2f_c) + \frac{A_c^2}{4} e^{-j\theta} H(f) M(f+2f_c)$$

... to plot we need to pick some parameter values.

Problem 1. (cont'd.) Name: ____________

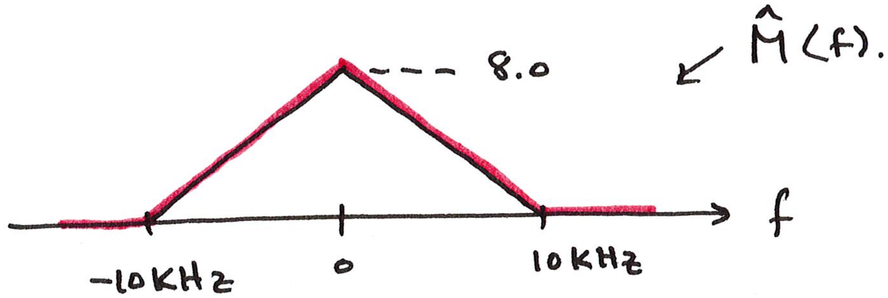
(b) [5 pts.] If $A_c = 4$, $f_c = 100$ kHz, $\theta = 0$, and $B = 20$ kHz solve for $\hat{M}(f)$ and plot.

For $B = 20\text{KHz}$ and $f_c = 100\text{KHz}$

$$H(f)M(f-2f_c) = 0 = H(f)M(f+2f_c)$$

$$\therefore \hat{M}(f) = \frac{A_c^2}{2}\cos\theta \, H(f)M(f)$$

$$= 8\,M(f)$$

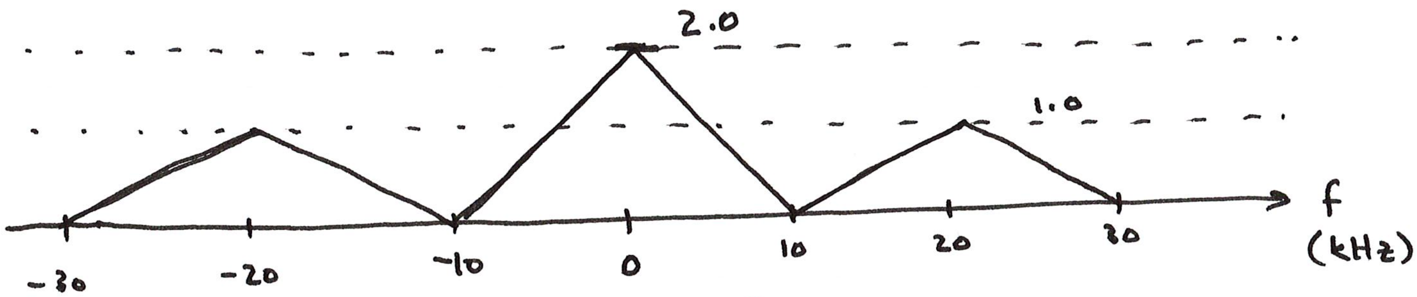


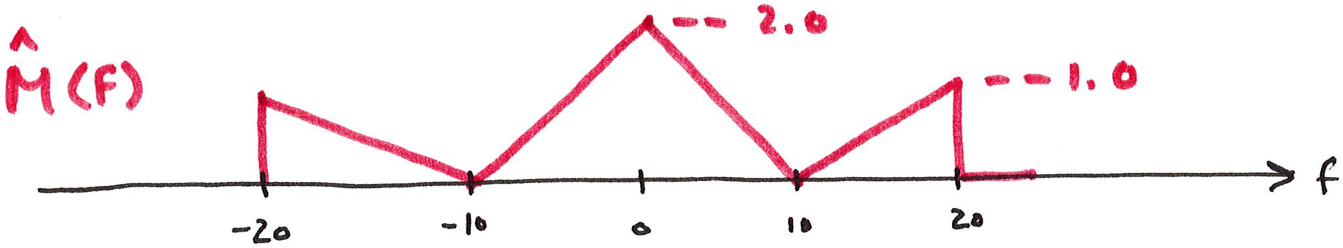
Problem 1. (cont'd.) Name: ______________________

(c) [5 pts.] If $A_c = 2$, $f_c = 10$ kHz, $\theta = 0$, and $B = 20$ kHz solve for $\hat{M}(f)$ and plot.

$$\hat{M}(f) = 2H(f)M(f) + H(f)M(f - 20\text{kHz}) + H(f)M(f + 20\text{kHz})$$

$$= H(f)\left[2M(f) + M(f - 20\text{kHz}) + M(f + 20\text{ kHz})\right]$$



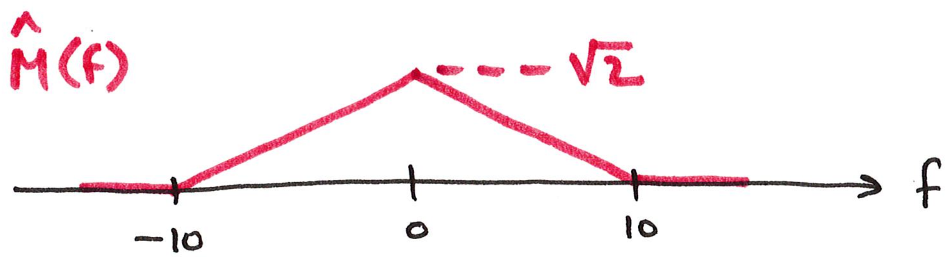


(d) [5 pts.] If $A_c = 2$, $f_c = 10$ kHz, $\theta = 45°$, and $B = 10$ kHz solve for $\hat{M}(f)$ and plot.

Two things change from previous part $B \to 10$ kHz from 20 kHz and

$$\cos\theta \longrightarrow \cos 45° = \sqrt{2}/2 \approx 0.71$$

Therefore only the center term in the previous spectrum makes it through $H(f)$ and the amplitude is reduced by 0.71 ...



Problem 1. (cont'd.) Name: ______________________

(e) [5 pts.] If $A_c = 2$, $f_c = 10$ kHz, $\theta = 90°$, and $B = 10$ kHz solve for $\hat{M}(f)$ and plot.

The only change from the previous part is that

$\cos\theta \longrightarrow \cos 90° = 0$

$\therefore \hat{M}(f) = 0$

Problem 1. (cont'd.) Name: ______________

(f) [10 pts.] If $A_c = 2$, $f_c = 10$ kHz, $\theta = 45°$, and $B = 20$ kHz solve for $\hat{M}(f)$ and plot.

This is the same as part (d) except

$$B \longrightarrow 20\text{KHz}$$

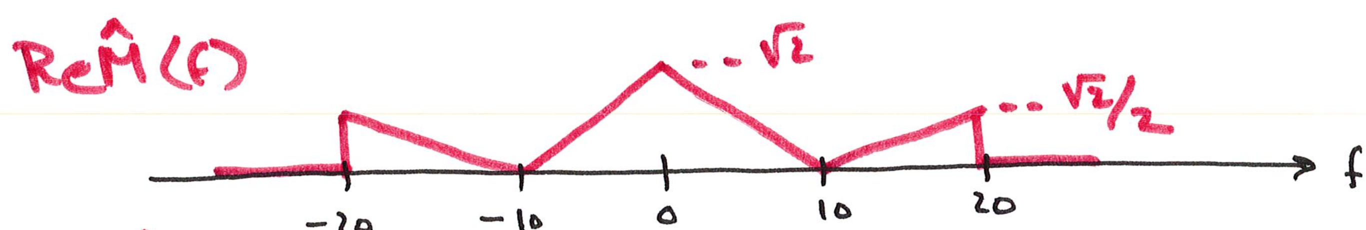
which lets part of the parts centered at $f = \pm 20\text{KHz}$ to get through the filter.

$$\hat{M}(f) = H(f)\left[2\cos 45° M(f) + e^{j45°} M(f - 20\text{KHz}) + e^{-j45°} M(f + 20\text{kHz})\right]$$

$$= H(f)\left[\sqrt{2} M(f) + \frac{\sqrt{2}}{2} M(f - 20\text{kHz}) + j\frac{\sqrt{2}}{2} M(f - 20\text{kHz}) + \frac{\sqrt{2}}{2} M(f + 20\text{kHz}) - j\frac{\sqrt{2}}{2} M(f + 20\text{kHz})\right]$$

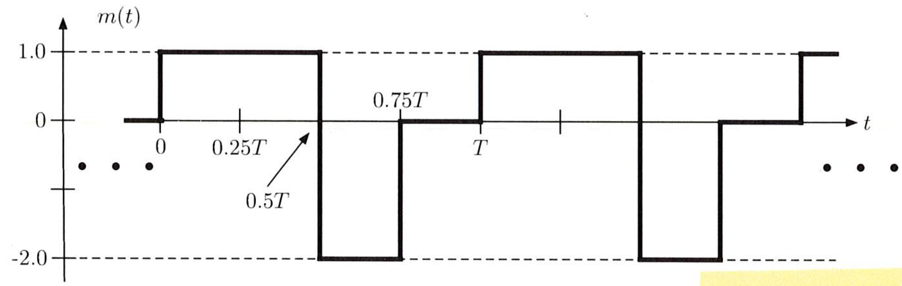
$$\therefore \text{Re}\,\hat{M}(f) = H(f)\left[\sqrt{2} M(f) + \frac{\sqrt{2}}{2} M(f - 20\text{kHz}) + \frac{\sqrt{2}}{2} M(f + 20\text{kHz})\right]$$

$$\text{Im}\,\hat{M}(f) = H(f)\left[\frac{\sqrt{2}}{2} M(f - 20\text{kHz}) - \frac{\sqrt{2}}{2} M(f + 20\text{kHz})\right]$$



Name: ______________________

Problem 2. *AM-LC Computations and Plots.* [50 pts. total]



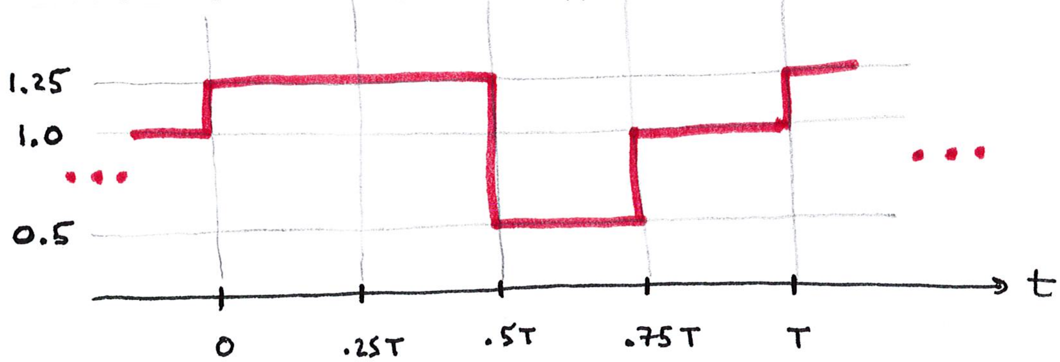
An AM-LC wave

$$x(t) = 8[1 + k_a m(t)]\cos(2\pi f_c t)$$

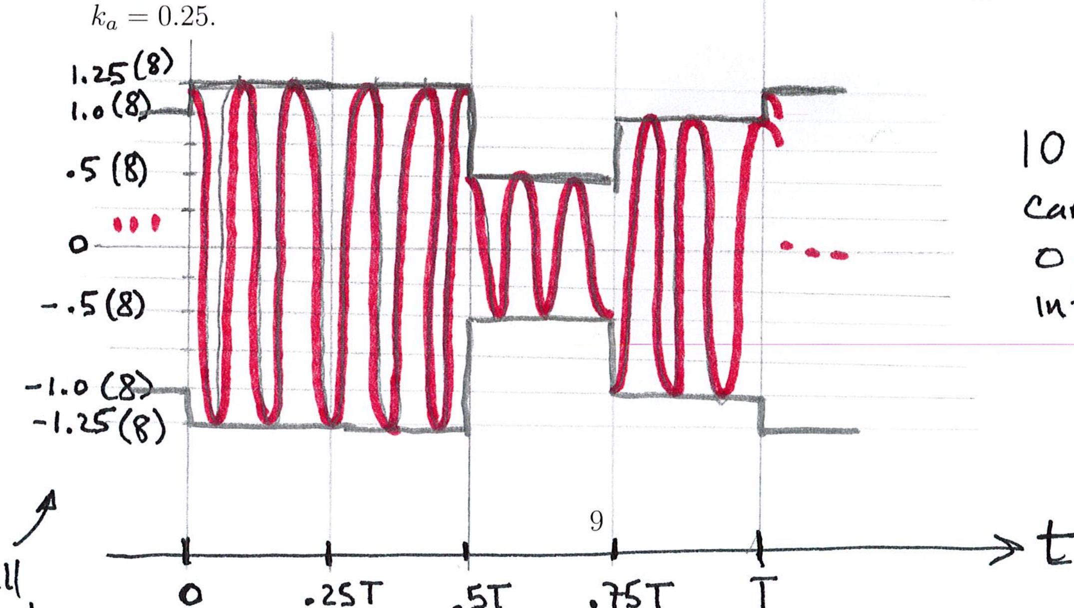
is formed from the periodic message $m(t)$ shown above.

$$1 + 0.25m(t) = \begin{cases} 1 & t = 0^- \\ 1.25 & 0^+ < t < .5T^- \\ 0.5 & .5T^+ < t < .75T^- \\ 1 & .75T^+ < t < T^- \end{cases}$$

(a) [10 pts.] Sketch the signal $1 + k_a m(t)$ for $k_a = 0.25$ and $0 < t < T$.



(b) [10 pts.] Sketch the time domain wave $x(t)$ for $0 < t < T$ showing important features such as envelope and number of cycles of the carrier. Do this for $f_c = 10/T$ and $k_a = 0.25$.



10 cycles of carrier in the $0<t<T$ interval.

overall amplitude factor of 8.

Problem 2. (cont'd.) Name: ____________________

(c) [10 pts.] What is the largest value of k_a such that envelope detection can be used? Explain.

The envelope magnitude cannot have phase reversals ie,

$$1 + k_a m(t) \geq 0 \quad \forall t$$

This is governed by the minimum value of m(t), which is −2 :

$$1 + k_a(-2) \geq 0$$

$$k_a(-2) \geq -1$$

$$k_a \leq 0.5$$

(d) [10 pts.] For $k_a = 0.25$ find the efficiency η, defined as

$$\eta = \frac{\text{sideband power}}{\text{carrier power + sideband power}}.$$

Write $x(t) = 8 \cos 2\pi f_c t + 8\frac{1}{4} m(t) \cos 2\pi f_c t$

$= 8 \cos 2\pi f_c t + 2 m(t) \cos 2\pi f_c t$

Carrier power $= \frac{1}{2} 8^2 = 32$

Sideband power $= \frac{1}{2} 2^2 \langle m^2(t) \rangle = 2 P_m$

$$P_m = \frac{1}{T}\int_0^T m^2(t)\,dt = \frac{1}{T}\left\{1 \cdot 0.5T + 4 \cdot 0.25T + 0\right\}$$

$$= 0.5 + 1 = 1.5$$

$$\therefore \eta = \frac{3}{32 + 3} = \frac{3}{35}$$

Problem 2. (cont'd.) Name: ____________

(e) [10 pts.] For $k_a = 0.25$, $f_c = 10/T$, approximately sketch the spectrum $X(f)$. On the last two pages a matlab code and the plot it produces are given, which will be useful here.

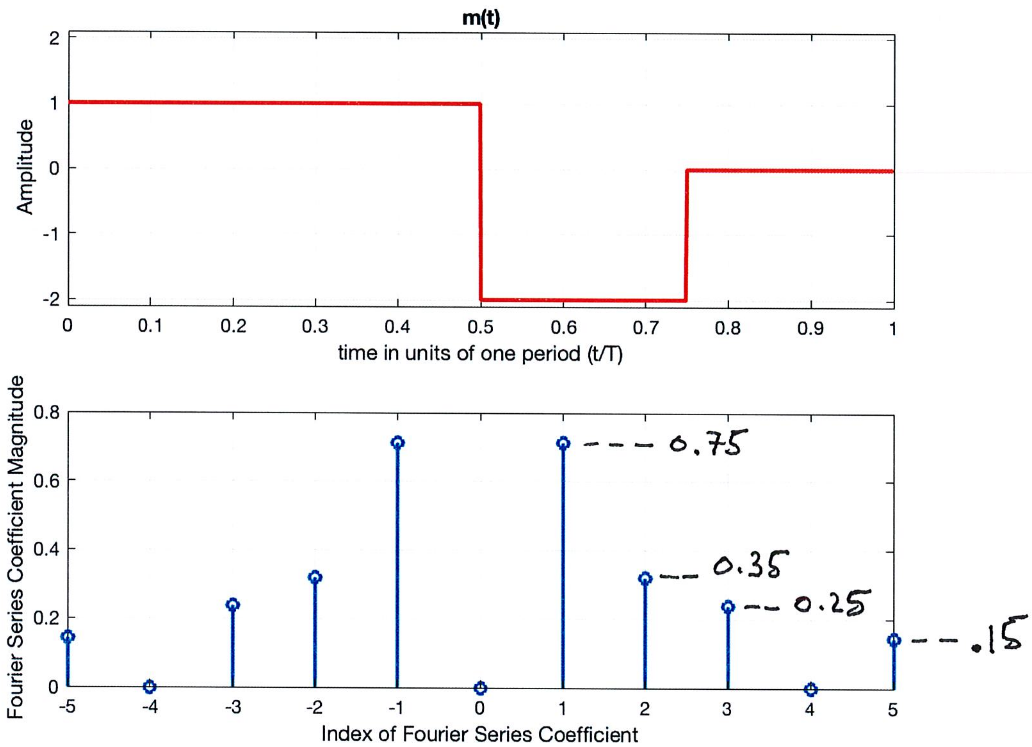
$$X(f) = 4\delta(f-f_c) + 4\delta(f+f_c)$$
$$+ 0.5\cdot 8\cdot 0.25\, M(f-f_c) + 0.5\cdot 8\cdot 0.25\, M(f+f_c)$$
$$= 4\delta(f-f_c) + 4\delta(f+f_c) + M(f-f_c) + M(f+f_c)$$

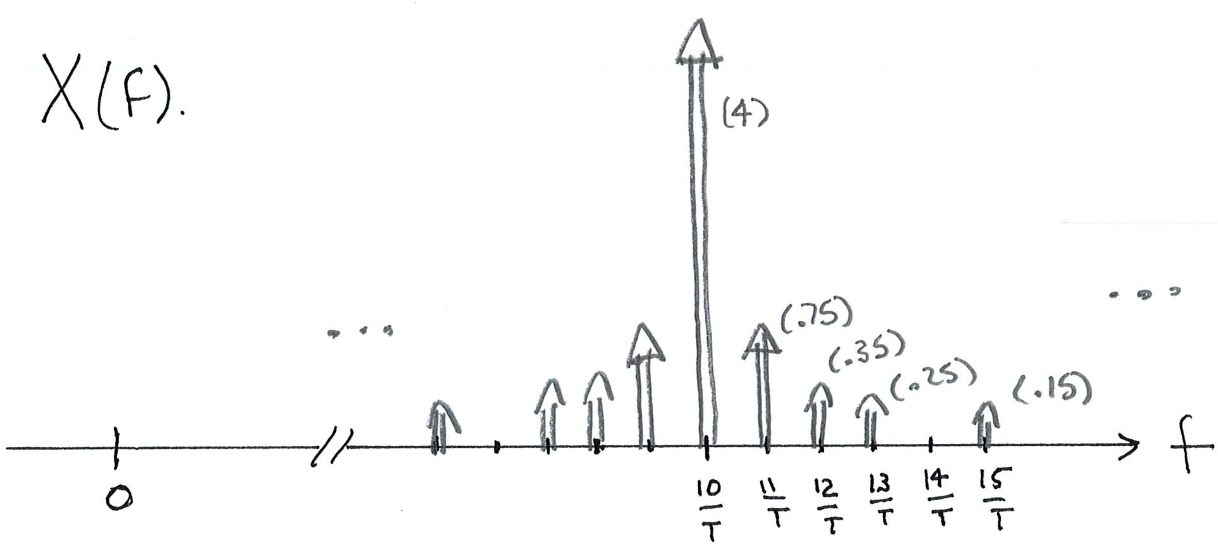
where
$$M(f) = \sum_{k=-\infty}^{\infty} M_k\, \delta(f - k/T)$$

Fourier Series Coeffs. From plot and code

See next page.

Problem 2. (Code and Plots) Name: ______________________





Problem 2. (Code and Plots) Name: ______________________

```
%% ECE 440 Spring 2022
% Using FFT to compute Fourier Series coeffs of a square-like periodic
% wave

K = 1024;                           %K pt. DFT
M = 5;                              %Compute 2*M+1 FS coeffs

% Enter the definition of one period of the periodic waveform. Sampled
% to K samples.

N = K/4;

% This defines the signal over the first period
x1 = ones(1,(2*N));
x2 = -2*ones(1,N);
x3 = zeros(1,N);

x = [x1, x2, x3];

% Make the time index
t = (0:(K-1))/K;

figure(1)
subplot(2,1,1); plot(t,x,'r')
set(gca,'xlim',[0 1.0],'ylim',[-2.1 2.1])
title('m(t)')
xlabel('time in units of one period (t/T)')
ylabel('Amplitude')
grid on

X = fft(x)/K;                       %Create FS coeffs
X_coeffs = [X(K-M+1:K) X(1:M+1)];   %Find the actual FS coeff ests
index = -M:1:M;                     %For plotting

figure(1)
subplot(2,1,2)
stem(index,abs(X_coeffs))
ylabel('Fourier Series Coefficient Magnitude')
xlabel('Index of Fourier Series Coefficient')
grid on
```