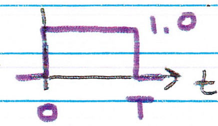
Intersymbol Interference + Nyquist's Criterion

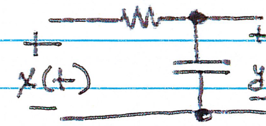
Pulse train modulated by random ± 1 digits $\{a_k\}$

$$x(t) = \sum_k a_k\, p(t-kT) \quad \ldots \quad p(t) =$$



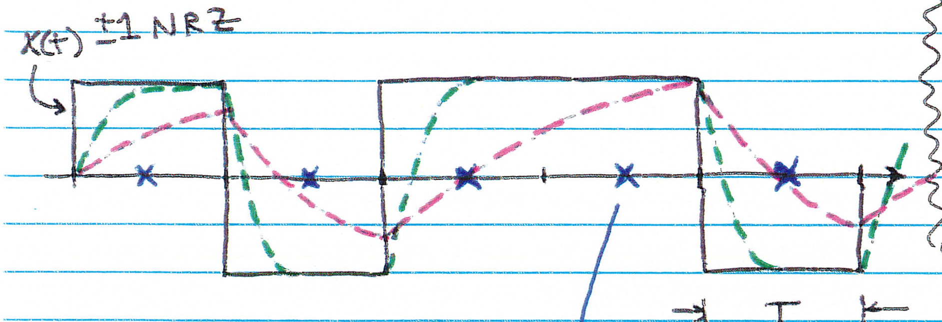
Imagine filtering with a low-pass filter

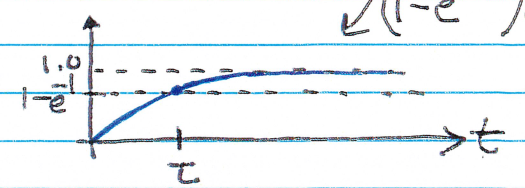
$\tau = RC$



unit step resp.

$(1-e^{-t/\tau})u(t)$





- - - $\tau \ll T$ $y(t)$

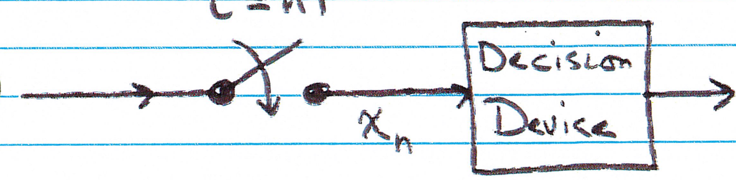
- - - $\tau \approx T$ $y(t)$

sample half-way is ok for $\tau \ll T$, but may make bit errors with no noise if $\tau \approx T$.

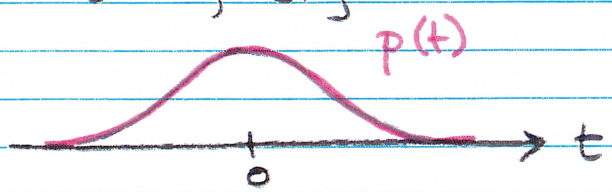
1.

Sampling Receiver

$$x(t) = \sum_k a_k\, p(t-kT)$$



Assume for simplicity that pulses are non-causal & centered at $t=0$, e.g.



$$x_n = x(t)\Big|_{t=nT} = \sum_k a_k\, p((n-k)T)$$

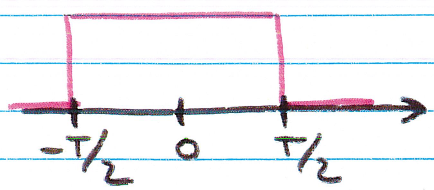
$$= a_n\, p(0) + \underbrace{\sum_{k\neq n} a_k\, p((n-k)T)}_{\text{ISI}}$$

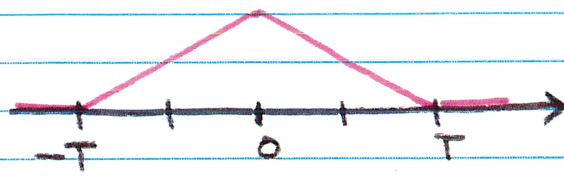
Time-domain condition for Zero-ISI

$$p(0) = 1$$

$$p(mT) = 0 \quad \text{for } m \neq 0$$

Trivial Zero-ISI Pulses





These take large bandwidth ...

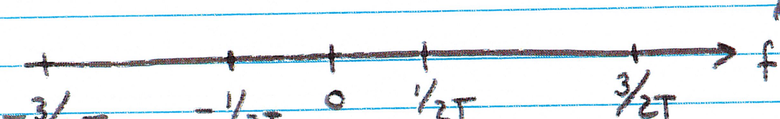
$\boxed{\text{Spectral-Domain Condition for Zero-ISI}}$

$p(t) \longleftrightarrow P(f) \implies p(t) = \int_{-\infty}^{\infty} P(f)\, e^{j2\pi f t}\, df$

Sample @ $t = mT$

$p(mT) = \int_{-\infty}^{\infty} P(f)\, e^{j2\pi f m T}\, df$

Then note that the exponential terms $e^{j2\pi f m T}$ are all periodic in f of period $1/T$ (some ... $1/mT = \frac{1}{m} \cdot \frac{1}{T}$)



$\therefore\ p(mT) = \sum_k \int_{\frac{2k-1}{2T}}^{\frac{2k+1}{2T}} P(f)\, e^{j2\pi f m T}\, df \implies$ C.O.V. $f' = f - \frac{k}{T}$

$$p(mT) = \sum_k \int_{-1/2T}^{1/2T} P\left(f' + \frac{k}{T}\right) e^{j2\pi mT(f' + k/T)} \, df'$$

$$= e^{j2\pi mTf'} e^{j2\pi mk} \rightarrow 1$$

$$= \int_{-1/2T}^{1/2T} \sum_k P\left(f' + \frac{k}{T}\right) e^{j2\pi mTf'} \, df'$$

Define
$\lambda = f'T$
$d\lambda = df'T$

$$p(mT) = \int_{-1/2}^{1/2} \frac{1}{T} \sum_k P\left(\frac{\lambda + k}{T}\right) e^{j2\pi \lambda m} \, d\lambda$$

5.

Now apply the time-domain Nyquist Criterion
i.e. desire

$$p(mT) = \begin{cases} 1 & m = 0 \\ 0 & m \neq 0 \end{cases}$$

Observe that the inverse DTFT of the const. freq. function: $1 \quad \forall \lambda \in [-1/2, 1/2)$ is

$$\int_{-1/2}^{1/2} 1 \, e^{j2\pi \lambda m} \, d\lambda = \begin{cases} 1 & m = 0 \\ 0 & m \neq 0 \end{cases}$$

Then, since DTFT pairs are unique ...

6.

$$p(t) \longleftrightarrow P(f)$$

is a Nyquist pulse if and only if

$$1 = \frac{1}{T} \sum_{k=-\infty}^{\infty} P\left(\frac{\lambda + k}{T}\right) \qquad |\lambda| \leq 1/2$$

$\Longleftrightarrow$

$$1 = \frac{1}{T} \sum_{k=-\infty}^{\infty} P\left(f + \frac{k}{T}\right) \qquad |f| \leq 1/2T$$