ECE 44000 **HW #7 (Probs. 31 – 37)** **Due 11/14/22**

Problem 31: [Fall 1999 Final Exam] A binary baseband data transmission system uses the signal set $s_0(t) = -s(t)$ and $s_1(t) = s(t)$ where

$$s(t) = \begin{cases} 2At/T & 0 \le t < T/2 \\ A(2t-T)/T & T/2 \le t < T \\ 0 & \text{otherwise} \end{cases} .$$

The channel is an additive white Gaussian noise channel with psd height $N_0/2$. The minimax criterion is to be used.

(a) What is the minimum probability of error for this system? Give your answer in terms of A, T, N_0, and the function $Q(\cdot)$.

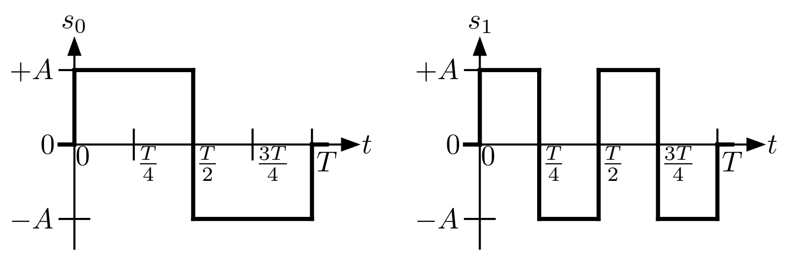
(b) Give the impulse response of the filter which achieves the minimum error probability (i.e., the matched filter). Simplify as much as possible.

(c) Give the optimum sampling time and optimum (minimax) threshold for the receiver which uses the filter of part (b).

(d) Suppose that the filter in the receiver is not the filter of part (b) but is instead a filter with impulse response $h(t) = p_T(t)$. Given an expression for the output $\hat{s}(t)$ when the input is $s(t)$. Find the maximum value of $\hat{s}(t)$.

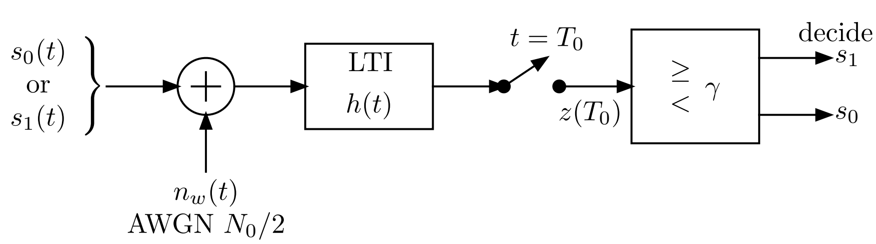
(e) Find the degradation in SNR incurred by using the filter $p_T(t)$ instead of the matched filter.

Problem 32: [Spring 2010 Final Exam] A binary baseband data transmission system uses the signal set $s_0(t)$ and $s_1(t)$ shown below.



The channel is an additive white Gaussian noise channel with psd height $N_0/2$. The Bayes criterion is to be used with equally likely priors, i.e., $\pi_0 = \Pr[s_0 \text{ is trans.}] = \pi_1 = \Pr[s_1 \text{ is trans.}] = 0.5$. Find the average probability of error in terms of A, T, N_0, and the function $Q(\cdot)$. Assume that the receiver has been optimally designed. Explain your work and show any calculations needed.

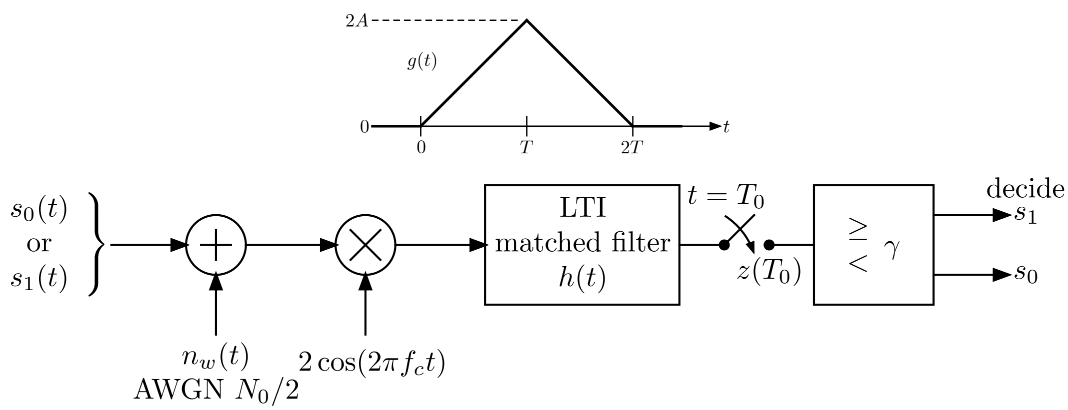
Problem 33: [Spring 2010 Final Exam] Consider again the scenario described in Problem 43. As discussed in class the optimal receiver has the architecture shown in the block diagram below.



(a) Find and plot the impulse response of a filter $h(t)$ which achieves the minimum average error probability (i.e., the matched filter). Simplify as much as possible and explain your work.

(b) Find the optimum sampling time and optimum (Bayes) threshold for the receiver which uses the filter of part (a). Explain your work.

Problem 34: [Fall 2011 Final Exam] The purpose of this problem is step through the development of the optimal receiver for BPSK. The signals $s_0(t) = -g(t)\cos(2\pi f_c t)$, $s_1(t) = +g(t)\cos(2\pi f_c t)$, where $g(t)$ is the triangularly shaped pulse:



Assume that $f_c \gg 1/T$ (which will suggest a certain approximation simplifying the results below).

(a) For the receiver shown above and the assumed signals $s_0(t)$ and $s_1(t)$ choose the impulse response $h(t)$ of the matched filter and specify the sampling time T_0. Note that the downconversion via multiplication by $2\cos(2\pi f_c t)$ occurs before the matched filter in this architecure.

(b) Assuming that the transmitted signal is actually $s_0(t)$ find:

- The message-related part of $z(T_0)$.
- The noise-related part of $z(T_0)$: Specify its distribution and its mean and variance.
- What is the distribution of the random variable $z(T_0)$ conditioned on $s_0(t)$ being transmitted?

(c) Repeat part (b) assuming that $s_1(t)$ is transmitted. You can do this by inspection given your derivation from (b) if you give the proper justification.

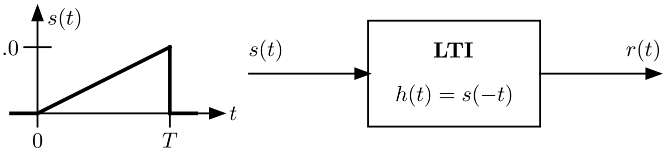
(d) Assuming that the prior probabilities of $s_0(t)$ and $s_1(t)$ are 1/2 choose the threshold γ for minimum average probability of error.

(e) Find the average probability of error.

Problem 35: [Spring 2012 Final Exam] Consider the LTI system shown below where the impulse response is "matched" to the input as indicated. Find:

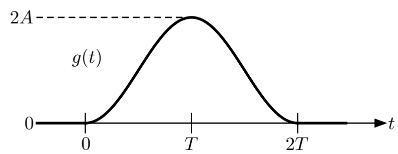
(a1) The range of t for which the output $r(t)$ is nonzero.

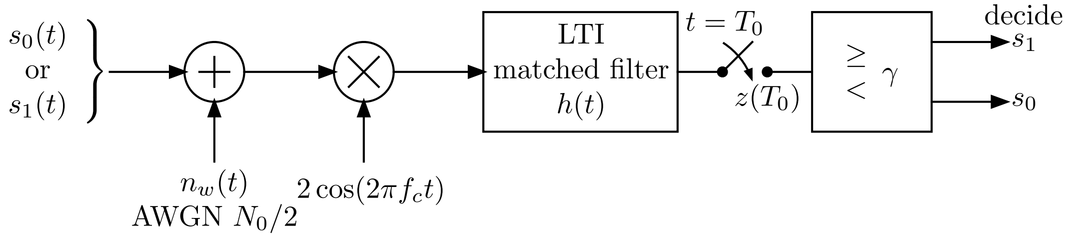
(a2) The maximum value of the output $r(t)$ and the time t_* where the maximum occurs.



Problem 36: [Spring 2012 Final Exam] The purpose of this problem is step through the development of the optimal receiver for ASK. The signals $s_0(t) = 0$, $s_1(t) = +g(t)\cos(2\pi f_c t)$, where $g(t)$ is the time-domain raised cosine shaped pulse:

$$g(t) = \begin{cases} A(1 + \cos(\pi(t - T)/T) & 0 \le t \le 2T \\ 0 & \text{else} \end{cases} .$$





Assume that $f_c \gg 1/T$ (which will suggest a certain approximation simplifying the results below).

(a) For the receiver shown above and the assumed signals $s_0(t)$ and $s_1(t)$ choose the impulse response $h(t)$ of the matched filter and specify the sampling time T_0. Note that the downconversion via multiplication by $2\cos(2\pi f_c t)$ occurs before the matched filter in this architecure.

(b) Assuming that the transmitted signal is actually $s_1(t)$ find:

(b1) The message-related part of $z(T_0)$.

(b2) The noise-related part of $z(T_0)$: Specify its distribution and its mean and variance.

(b3) What is the distribution of the random variable $z(T_0)$ conditioned on $s_1(t)$ being transmitted?

(c) Repeat part (b) assuming that $s_0(t)$ is transmitted. You can do this by inspection given your derivation from (b) if you give the proper justification.

(d) Assuming that the prior probabilities of $s_0(t)$ and $s_1(t)$ are 1/2 choose the threshold γ for minimum average probability of error.

(e) Find the average probability of error.

Problem 37: [Spring 2010 Final Exam] Calculate the mean and variance of the random variable Z in the block diagram below and write down its pdf.