ECE 44000 (Fall 2022)
HW 6 (Probs. 25–30)
Due: 11/2/2022

Problem 25: Z&T Problem 7.8.

7.8. Assume that an AM system operates with an index of 0.6 and that the message signal is $12\cos(8\pi)$. Compute the efficiency, the detection gain in dB, and the output SNR in decibels relative to the baseband performance P_T/N_0W. Determine the improvement (in decibels) in the output SNR that results if the modulation index is increased from 0.6 to 0.9.

Problem 26:

(a) Use the code given in the m-file `AM_dsb_ideal.m` to experiment with AM DSB. First get the code running, make the plots, and explain the code in your own words (printouts of the code are included at the end of these problems).

(b) Then modify the code to change the message and make new plots.

(c) Finally, modify the code to include the addition of noise in the IF signal. Explain how you calibrate the noise power and how to calculate the SNR numerically. Experiment with various SNRs.

Problem 27:

(a) Use the code given in the m-file `Test_fm_and_pm.m` to experiment with FM (you can remove the PM parts of the code if desired). First get the code running, make the plots, and explain the code in your own words (printouts of the code are included at the end of these problems). Note that you can select various messages.

(b) Modify the code to include the addition of noise in the IF signal. Explain how you calibrate the noise power and how to calculate the SNR numerically. Experiment with various SNRs.

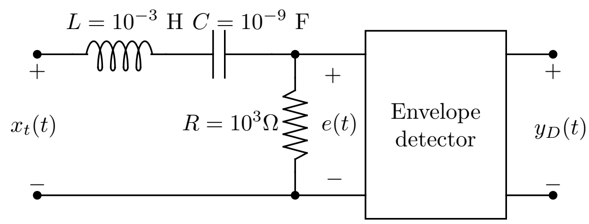
Problem 28: [Fall 2010 Exam 2] An FM modulator is followed by an ideal bandpass filter having a center frequency of 100 kHz and a bandwidth of 100 Hz, that is, the passband of the bandpass filter extends from 99.95 kHz to 100.05 kHz. The gain of the filter is assumed to be 1 in the passband. The unmodulated carrier is given by $10\cos(2\pi 10^5 t)$ and the message signal is $m(t) = A_m \cos(2\pi 20t)$.

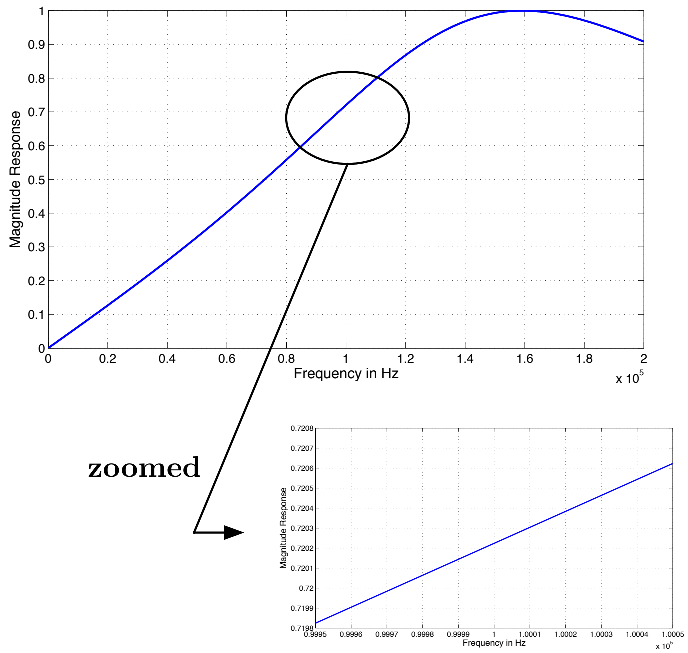
(a) The amplitude A_m of the modulating wave is picked to set the peak frequency deviation $\Delta_f = 60$ Hz. Find the peak phase deviation in radians (i.e., the parameter β).

(b) Find the power in the FM wave at the input to the bandpass filter and at the output of the bandpass filter. (A table of Bessel function values in appended to the end of this problem.)

(c) What should be the Carson's rule bandwidth for the FM wave? Does the bandpass filter's bandwidth suffice to pass most of the signal power from input to output?

(d) An FM discriminator is given by the circuit shown for which we have also plotted the magnitude response $|E(f)/X_r(f)|$. Assume the envelope detector is ideal and does not load the tuned circuit. If this discriminator is used to demodulate the FM wave at the output of the bandpass filter mentioned in Part (b) what would be its gain (or loss) relative to an ideal FM discriminator made up of an ideal differentiator followed by an ideal envelope detector.





x	Bessel-function order, n																
m_f	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}	J_{15}	J_{16}
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—	—	—
2.41	0	0.52	0.43	0.20	0.06	0.02	—	—	—	—	—	—	—	—	—	—	—
2.5	–.05	0.50	0.45	0.22	0.07	0.02	0.01	—	—	—	—	—	—	—	—	—	—
3.0	–.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—	—	—
4.0	–.40	–.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—	—	—
5.0	–.18	–.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—	—	—
5.53	0	–.34	–.13	0.25	0.40	0.32	0.19	0.09	0.03	0.01	—	—	—	—	—	—	—
6.0	0.15	–.28	–.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—	—	—
7.0	0.30	0.00	–.30	–.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—	—	—
8.0	0.17	0.23	–.11	–.29	–.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	—	—	—	—	—
8.65	0	0.27	0.06	–.24	–.23	0.03	0.26	0.34	0.28	0.18	0.10	0.05	0.02	—	—	—	—
9.0	–.09	0.25	0.14	–.18	–.27	–.06	0.20	0.33	0.31	0.21	0.12	0.06	0.03	0.01	—	—	—
10.0	–.25	0.04	0.25	0.06	–.22	–.23	–.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01	—	—
12.0	0.05	–.22	–.08	0.20	0.18	–.07	–.24	–.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01

(Table of Bessel Function Values)

Problem 29: For this problem you will need to read the Section 8.3 of Z&T (Seventh Edition) on discriminator detection of FM. The key result is:

> Suppose $x(t)$ is the message corresponding to an FM wave $z(t)$, which has AWGN added to it and then is input to an FM discriminator. Under the assumption that the SNR at IF is high the output of the discriminator is
>
> $$\tilde{x}(t) = x(t) + \tilde{n}(t)$$
>
> where the noise is WSS of zero mean and psd of the form $S_{\tilde{n},\tilde{n}}(f) = \gamma f^2$.

Z+T 6th Edition

7.23. An FDM communication system uses DSB modulation to form the baseband and FM modulation for transmission of the baseband. Assume that there are eight channels and that all eight message signals have equal power P_0 and equal bandwidth W. One channel does *not* use subcarrier modulation. The other channels use subcarriers of the form

$$A_k \cos(2\pi k f_1 t), \quad 1 \leq k \leq 7$$

The width of the guardbands is $3W$. Sketch the power spectrum of the received *baseband* signal showing both the signal and noise components. Calculate the relationship between the values of A_k if the channels are to have equal SNRs.

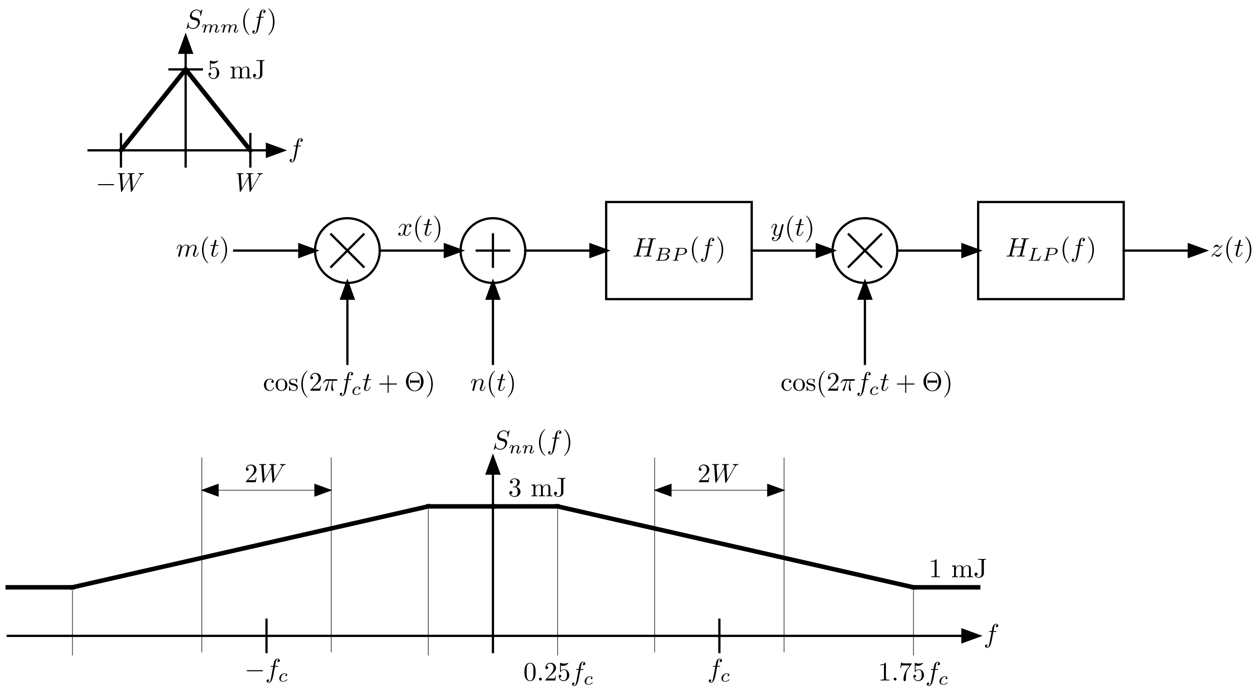
Problem 30: [Exam 2, Fall 2011] Consider the AM-DSB transmitter, channel, coherent receiver block diagram shown below. The message $m(t)$ is modeled as a zero mean WSS random process with bandwidth $W = 5$ kHz and power spectral density shown. The carrier phase is modeled as a random variable Θ uniformly distributed over $[0, 2\pi)$ and independent of the message. The bandpass filter H_{BP} is an ideal brickwall filter with gain equal to one and passband centered at $f_c = 100$ kHz with bandwidth set to just pass the message component without attenuation. The low pass filter H_{LP} is also an ideal brickwall filter of gain equal to one and bandwidth chosen to just pass the message.

(a) Find the power in the message component and the power in the noise component at the output of the bandpass filter.

(b) Take the ratio to find the signal-to-noise ratio at IF.

(c) Find the power in the message component and the power in the noise component at the output of the lowpass filter.

(d) Take the ratio to find the signal-to-noise ratio at the demodulator output.


$S_{mm}(f)$
5 mJ
$-W$
W
f
$m(t)$
$x(t)$
$H_{BP}(f)$
$y(t)$
$H_{LP}(f)$
$z(t)$
$\cos(2\pi f_c t + \Theta)$
$n(t)$
$\cos(2\pi f_c t + \Theta)$
$S_{nn}(f)$
$2W$
3 mJ
$2W$
1 mJ
$-f_c$
$0.25 f_c$
f_c
$1.75 f_c$
f

AM_dsb_ideal.m

```
% AM_dsb_ideal.m: Simulation of DSB with perfect carrier synchronization.

time = .3;                                 % length of simulation (sec)
Ts = 1/10000;                              % sampling interval
t = Ts:Ts:time; lent=length(t);            % define a "time" vector
fc = 1000;                                 % carrier freq (Hz)
c = cos(2*pi*fc*t);                        % define the carrier at freq fc

% Create "message". It will be a sum of linear and sinusoidal terms
fm = 20;                                   % freq of sinuoidal term
w = 0.0007*(1:lent) + cos(2*pi*fm*t);      % create "message"

v = c .* w;                                % modulate with carrier

c2 = cos(2*pi*fc*t);                       % create cosine for demod

x = v .* c2;                               % demod received signal

fbe=[0 0.1 0.2 1]; damps=[1 1 0 0]; fl=100; % low pass filter design
b=remez(fl,fbe,damps);                     % impulse response of LPF
m=2*filter(b,1,x);                         % LPF the demodulated signal

figure(1)
% used to plot figure
subplot(4,1,1), plot(t,w)
ylabel('amplitude'); title('(a) message signal');
subplot(4,1,2), plot(t,v)
ylabel('amplitude'); title('(b) message after modulation');
subplot(4,1,3), plot(t,x)
ylabel('amplitude');
title('(c) demodulated signal');
subplot(4,1,4), plot(t,m)
ylabel('amplitude'); title('(d) recovered message is a LPF applied to (c)');

[ssf,wfxs] = JVK2_plotspec(w,Ts);
[ssf,vfxs] = JVK2_plotspec(v,Ts);
[ssf,xfxs] = JVK2_plotspec(x,Ts);
[ssf,mfxs] = JVK2_plotspec(m,Ts);

% used to plot freq domain picture
figure(2)
subplot(4,1,1), plot(ssf,abs(wfxs))
ylabel('amp. spec.'); title('(a) message signal');
subplot(4,1,2), plot(ssf,abs(vfxs))
ylabel('amp. spec.'); title('(b) message after modulation');
subplot(4,1,3), plot(ssf,abs(xfxs))
ylabel('amp. spec.');
title('(c) demodulated signal before LPF');
subplot(4,1,4), plot(ssf,abs(mfxs))
ylabel('amp. spec.'); title('(d) recovered message is a LPF applied to (c)');
```

JVK2_plotspec.m

```
% plotspec(x,Ts) plots the spectrum of the signal x
% Ts = time (in seconds) between adjacent samples in x
% Modified by JVK to make the spectrum amplitude correspond
% to that of the CT Fourier Transform.

function [ssf,fxs] = JVK2_plotspec(x,Ts)

N=length(x);                          % length of the signal x
t=Ts*(1:N);                           % define a time vector
ssf=(-N/2:N/2-1)/(Ts*N);              % frequency vector
fx=Ts*fft(x(1:N));                    % do DFT/FFT
fxs=fftshift(fx);                     % shift it for plotting
```

Test_fm_and_pm.m

```
fm = 1;
fs = 50*fm;
time = 10;
t = (0:1/fs:time);

m = cos(2*pi*fm*t);
%m = sawtooth(2*pi*fm*t);
%m = square(2*pi*fm*t);

fc = 50;
deltaf = 20;
%deltap = 0.2*deltaf;
%deltap = deltaf/fm;
deltap = 1;

[x_fm, tout_fm] = fmmod_jvk1(m, t, fc, deltaf);
[x_pm, tout_pm] = pmmod_jvk1(m, t, fc, deltap);

figure(1)
subplot(3,1,1)
plot(t,m)
xlabel('Time (sec)')
title('Message')
subplot(3,1,2)
plot(tout_fm,x_fm)
xlabel('Time (sec)')
title('FM')
subplot(3,1,3)
plot(tout_pm,x_pm)
xlabel('Time (sec)')
title('PM')


figure(2)
subplot(3,1,1)
plotspec_v3(m,1/fs)
title('Message Mag Spectrum')
subplot(3,1,2)
plotspec_v3(x_fm,tout_fm(2)-tout_fm(1))
title('FM Mag Spectrum')
subplot(3,1,3)
plotspec_v3(x_pm,tout_pm(2)-tout_pm(1))
title('PM Mag Spectrum')
```

fmmod_jvk1.m

```
function [x, tout] = fmmod_jvk1(m, tin, fc, deltaf)
%fmmod_jvk1: Function to FM modulate a message.
%
%   Inputs: m = vector containing message samples.
%           tin = vector of time samples corresponding to m.
%           fc = desired center frequency of FM output
%           deltaf = peak frequency deviation
%   Outputs:    x = vector containing FM modulated wave.
%               tout = vector of time samples corresponding to x
%
%   The function also performs an increase in the sampling rate between
%   the message and the modulated wave. The upsampling factor is chosen
%   based on the ratio of the desired carrier frequency and the bandwidth
%   of the baseband message.


gamma = deltaf/max(abs(m));         %compute desired freq sensitivity
                                    %of the modulator based on allowed
                                    %peak frequency deviation

y = zeros(size(m));                 %y is the integral of the message
Tin = tin(2) - tin(1);              %Tin is the sampling interval of the
                                    %baseband message

%Integrator
y(1) = (Tin/2)*m(1);

for i = 2:length(m)
    y(i) = y(i-1) + (Tin/2)*(m(i)+m(i-1));
end

%CarsonBW = 2*(deltaf + 1/(2*Tin));
%Nup = 8*ceil((fc + 0.5*CarsonBW)*2*Tin);
Nup = 8*ceil(fc*2*Tin);                 %Upsampling factor

yup = interp(y,Nup);                    %Interpolate
tout = (0:(length(yup) - 1))*(Tin/Nup); %Create output time vector

%FM modulation
theta = 2*pi*(fc*tout + gamma*yup);
x = cos(theta);

end
```

plotspec_v3.m

```
% plotspec_v3(x,Ts) plots the spectrum of the signal x
% Ts = time (in seconds) between adjacent samples in x
% Modified by JVK to make the spectrum amplitude correspond
% to that of the CT Fourier Transform and to properly plot when
% signal length is odd

function plotspec_v3(x,Ts)

N=length(x);                            % length of the signal x
n = 1:N;
ssf=(n - 1 - floor(N/2))/(Ts*N);        % frequency vector
X=Ts*fftshift(fft(x));                  % do DFT/FFT
X_dB = 20*log10(abs(X));
plot(ssf,X_dB)                          % plot magnitude spectrum
xlabel('frequency'); ylabel('magnitude in dB')   % label the axes
```