ECE 44000 (Fall 2022)
HW 5 (Probs. 18–24)
Due: 10/12/2022

Problem 18: Z&T (7th Edition) Problem 6.16.

4.16. Given the Gaussian random variable with the pdf

$$f_X(x) = \frac{e^{-x^2/2\sigma^2}}{\sqrt{2\pi}\sigma}$$

let $Y = X^2$. Find the pdf of Y. *Hint*: Note that $Y = X^2$ is symmetrical about $X = 0$ and that it is impossible for Y to be less than zero.

Problem 19: Z&T (5th Edition) Problem 4.28[1].

4.28. A random variable X is defined by

$$f_X(x) = 4e^{-8|x|}$$

The random variable Y is related to X by $Y = 2 + 3X$.

(a) Determine $E[X]$, $E[X^2]$, and σ_X^2.

(b) Determine $f_Y(y)$.

(c) Determine $E[Y]$, $E[Y^2]$, and σ_Y^2.

(d) If you used $f_Y(y)$ in part (c), repeat that part using only $f_X(x)$.

Problem 20:
A DSB-SC modulated signal is transmitted over an additive noise channel where the psd of the noise is given by

$$S_n(f) = \begin{cases} 1 - \frac{|f|}{400} & |f| < 400 \\ 0 & |f| \geq 400 \end{cases}$$

where the units of f are kHz and the units of $S_n(f)$ are μJ. The message bandwidth is 4 kHz and the carrier frequency is 200 kHz. Assuming that the average power of the modulated wave is 10 W, determine the output signal-to-noise ratio of the receiver.

[1] A minor variation of Problem 6.28 in the 7th edition.

Problem 21: Z&T (7th Edition) Problems 7.3 and 7.5.

5.3. A random process is composed of sample functions that are square waves, each with constant amplitude A, period T_0, and random delay τ as sketched in Figure 5.15. The pdf of τ is

$$f(\tau) = \begin{cases} 1/T_0, & |\tau| \le T_0/2 \\ 0, & \text{otherwise} \end{cases}$$

(a) Sketch several typical sample functions.

(b) Write the first-order pdf for this random process at some arbitrary time t_0. (*Hint*: Because of the random delay τ, the pdf is independent of t_0. Also, it might be easier to deduce the cdf and differentiate it to get the pdf.)

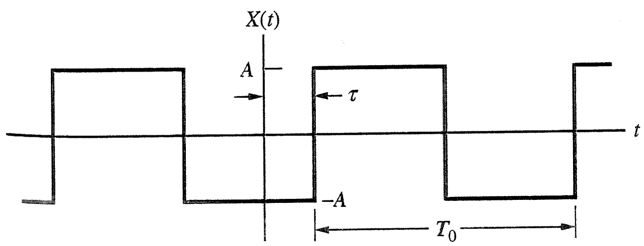


FIGURE 5.15

5.5. Consider the random process of Problem 5.3.

(a) Find the time-average mean and the autocorrelation function.

(b) Find the ensemble-average mean and the autocorrelation function.

(c) Is this process wide-sense stationary? Why?

Problem 22: Z&T (7th Edition) Problem 7.19.

6.19. An ideal finite-time integrator is characterized by the input-output relationship

$$Y(t) = \frac{1}{T}\int_{t-T}^{t} X(\alpha)\, d\alpha$$

a. Justify that its impulse response is $h(t) = \frac{1}{T}[u(t) - u(t-T)]$.

b. Obtain its frequency response function. Sketch it.

c. The input is white noise with two-sided power spectral density $N_0/2$. Find the power spectral density of the output of the filter.

d. Show that the autocorrelation function of the output is

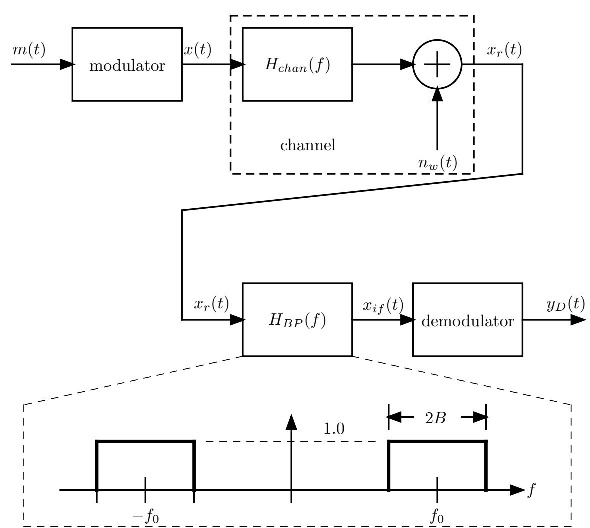
$$R_0(\tau) = \frac{N_0}{2T}\Lambda(\tau/T)$$

where $\Lambda(x)$ is the unit-area triangular function defined in Chapter 2.

e. What is the equivalent noise bandwidth of the integrator?

f. Show that the result for the output noise power obtained using the equivalent noise bandwidth found in part (e) coincides with the result found from the autocorrelation function of the output found in part (d).

Problem 23: A general model for passband communications is based on the block diagram shown below:



The message $m(t)$ will be modeled as a WSS random process with autocorrelation/psd $R_m(\tau) \leftrightarrow S_m(f)$ and bandlimited

$$S_m(f) = 0 \text{ for } f > W.$$

The noise $n_w(t)$ is also modeled as a WSS random process which is Gaussian, white, i.e.,

$$R_{n_w}(\tau) = \frac{N_0}{2}\delta(\tau)$$

$$\updownarrow$$

$$S_{n_w}(f) = \frac{N_0}{2} \text{ for } -\infty < f < \infty,$$

and statistically independent of $m(t)$ and any other random parameters of the communications model. For the purposes of this problem we will also assume that $H_{chan}(f) = 1$.

The purpose of this problem is to compute the signal-to-noise (SNR) power ratio at IF (i.e., associated with $x_{if}(t)$ in the block diagram). Since $m(t)$ is low-pass, the modulated signal $x(t)$ will be bandpass. In addition, we will assume that the bandpass filter $H_{BP}(f)$ in the block diagram passes the signal part without distortion, i.e., $x * h_{BP}(t) = x(t)$. Therefore,

$$\begin{aligned} x_{if}(t) &= x * h_{BP}(t) + n_w * h_{BP}(t) \\ &= x(t) + n(t) \end{aligned}$$

where $n(t)$ is a bandpass Gaussian noise with zero mean and psd of height $N_0/2$ and shape identical to that of $H_{BP}(f)$.

Then the SNR at IF is defined to be

$$\begin{aligned} \text{SNR}_{if} &\stackrel{\text{def}}{=} \frac{\text{power}[x(t)]}{\text{power}[n(t)]} \\ &= \frac{R_x(0)}{2N_0 B} \end{aligned}$$

assuming $x(t)$ is WSS and we can find its autocorrelation and/or psd.

(a) For AM DSB-SC assume

$$x(t) = A_c m(t) \cos(2\pi f_c t + \Theta)$$

where Θ is uniform on $[0,\, 2\pi)$ and statistically independent of message and noise.

- Find the auto-correlation and psd of $x(t)$. For a representative shape for $S_m(f)$ sketch a plot of $S_x(f)$.
- Specify the BPF so that $x * h_{BP}(t) = x(t)$ and the noise $n(t)$ has minimum power.
- Compute SNR_{if}.

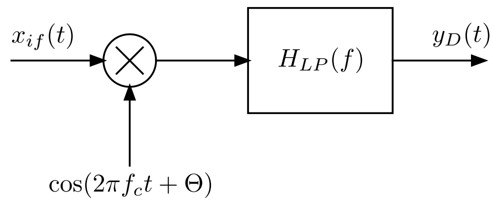
(b) Repeat (a) for AM LC

$$x(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t + \Theta).$$

(c) Repeat (a) for SSB

$$x(t) = 0.5 A_c m(t) \cos(2\pi f_c t + \Theta) \pm\ 0.5 A_c \hat{m}(t) \sin(2\pi f_c t + \Theta).$$

Problems 24: The three linear modulations AM DSB-SC, AM LC, and SSB have the identical coherent demodulator architecture shown in the figure below:



Use the standard assumptions on message, noise, and phase angle and compute the SNR at the demodulator output for the three linear modulations above.