ECE 44000 (Fall 2022)
HW 3 (Probs. 10–13)
Due: 9/21/2022

Problem 10: Z&T 6th Ed., Problem 3.13 (7th Ed., Problem 3.14).

3.13. Consider the system shown in Figure 3.72. Assume that the average value of $m(t)$ is zero and that the maximum value of $|m(t)|$ is M. Also assume that the square-law device is defined by $y(t) = 4x(t) + 2x^2(t)$.

a. Write the equation for $y(t)$.

b. Describe the filter that yields an AM signal for $g(t)$. Give the necessary filter type and the frequencies of interest.

c. What value of M yields a modulation index of 0.1?

d. What is an advantage of this method of modulation?

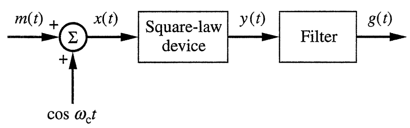


Figure 3.72

Problem 11: Z&T 7th Ed., Computer Exercises 3.2 (page 155).

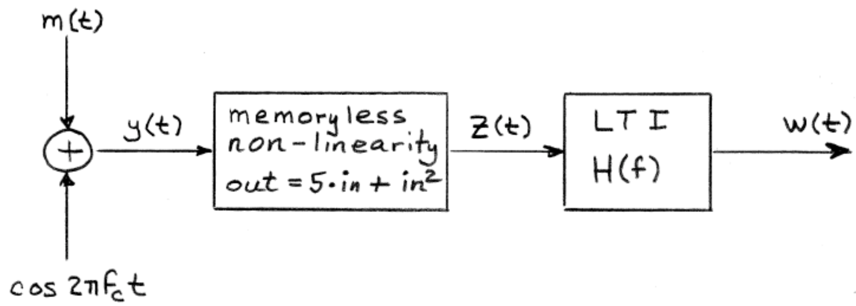
3.2. The purpose of the exercise is to demonstrate the properties of SSB modulation. Develop a computer program to generate both upper-sideband and lower-sideband SSB signals and display both the time-domain signals and the amplitude spectra of these signals. Assume the message signal

$$m(t) = 2\cos(2\pi f_m t) + \cos(4\pi f_m t)$$

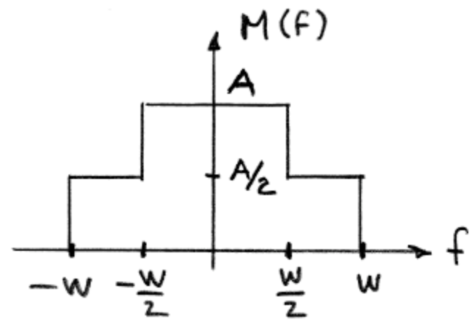
Select both f_m and f_c so that both the time and frequency axes can be easily calibrated. Plot the envelope of the SSB signals, and show that both the upper-sideband and the lower-sideband SSB signals have the same envelope. Use the FFT algorithm to generate the amplitude spectrum for both the upper-sideband and the lower sideband SSB signal.

Problem 12: A voice signal occupying the frequency band 0.3 – 3.4 kHz is to be SSB modulated onto a carrier wave of frequency 11.6 MHz. Assume the availability of bandpass filters which provide an attenuation of 50 dB in a transition band that is one percent of the mid-band frequency. Design a system to generate this SSB wave using the frequency discrimination method.

Problem 13: [Fall 2007 Exam 1] In the modulator shown below, the message waveform $m(t)$ and a sinusoid at the intended carrier frequency f_c are added to produce $y(t) = m(t) + \cos(2\pi f_c t)$, which is then passed through a memoryless non-linearity described by $z(t) = 5y(t) + y^2(t)$.



(a) Assume that $m(t)$ is real and even with the Fourier transform $M(f)$ shown. Find and carefully sketch $Z(f)$. Assume that $W \ll f_c$.



(b) Choose (and sketch) a filter frequency response $H(f)$ such that $w(t)$ is an AM large carrier wave at carrier frequency f_c. Write down the resulting time-domain waveform $w(t)$.

(c) Consider the AM-LC wave $w(t)$ found in (b). Find a value of K such that if

$$\max_t |m(t)| < K$$

then $w(t)$ will not be overmodulated (i.e., such that $m(t)$ could be recovered from $w(t)$ using an envelope detector).

(d) For the spectrum $M(f)$ given in part (a) find the inverse transform $m(t)$ and sketch it. This part can be solved independently of the rest of the problem.

(e) From the sketch of (d) and the solution to (c) what bound should we place on the product AW to be certain there is no overmodulation?

Z+T Problem 3.13 p 204

3.13. Consider the system shown in Figure 3.72. Assume that the average value of $m(t)$ is zero and that the maximum value of $|m(t)|$ is M. Also assume that the square-law device is defined by $y(t) = 4x(t) + 2x^2(t)$.

a. Write the equation for $y(t)$.

b. Describe the filter that yields an AM signal for $g(t)$. Give the necessary filter type and the frequencies of interest.

c. What value of M yields a modulation index of 0.1?

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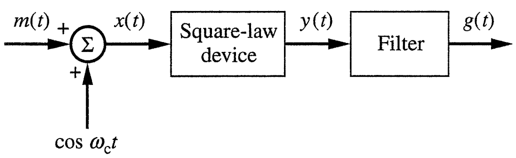


Figure 3.72

(a) $y(t) = 4x(t) + 2x^2(t)$

$$= 4\left[m(t) + \cos\omega_c t\right] + 2\left[m(t) + \cos\omega_c t\right]^2$$

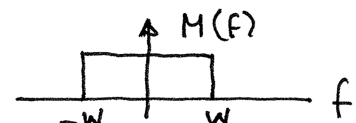
$$= 4m(t) + 4\cos\omega_c t + 2m^2(t) + 4m(t)\cos\omega_c t + 2\cos^2\omega_c t$$

$$= 4m(t) + 4\cos\omega_c t + 2m^2(t) + 4m(t)\cos\omega_c t + 1 + \cos 2\omega_c t$$

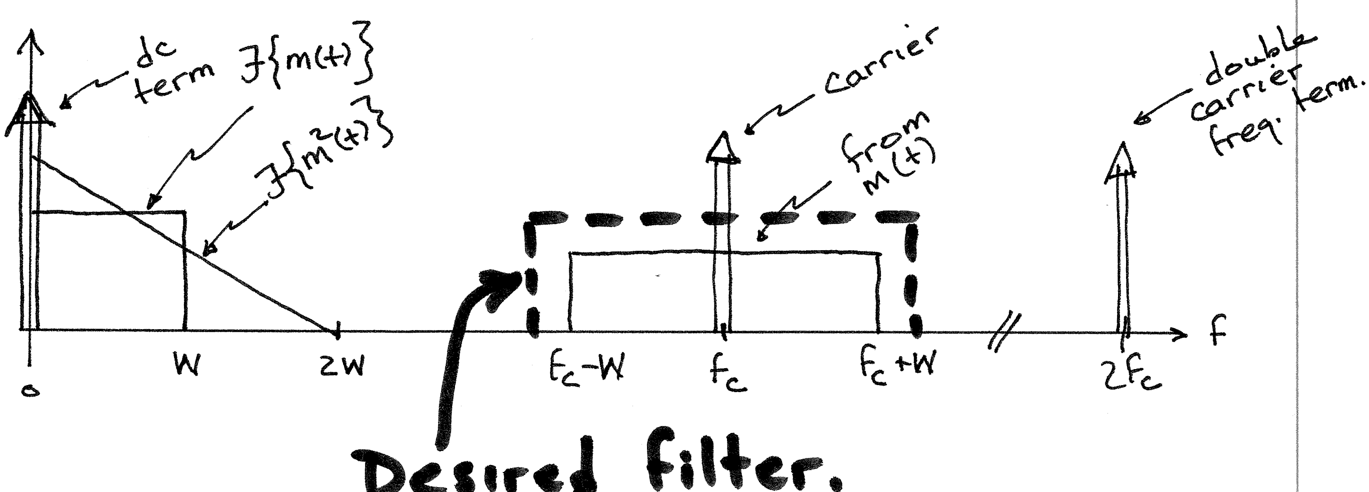
$$= \underbrace{1 + 4m(t) + 2m^2(t)}_{\text{Terms near BB}} + \underbrace{\cos 2\omega_c t}_{\text{Double carrier freq. term}} + \underbrace{4\big(1 + m(t)\big)\cos\omega_c t}_{\text{An AM-LC term.}}$$

(b) Want an LTI filter which will pass the AM-LC terms and reject the others. To see what we will need it is helpful to make a rough spectrum sketch of $Y(f)$.

Suppose $M(f)$ is rectangular, say



Then $\mathcal{F}\{m^2(t)\}$ will be triangular, and extending out to 2W.



Therefore, as long as

$$\left.\begin{array}{l} f_c - W > 2W \\ \text{and} \\ f_c + W < 2f_c \end{array}\right\} \Longrightarrow \begin{array}{c}\text{both satisfied if} \\ f_c > 3W. \end{array}$$

we can separate the desired AM–LC term in $y(t)$ from the undesired terms.

(c) $g(t) = 4\left(1 + m(t)\right) \cos \omega_c t$

$$= 4\left(1 + M m_n(t)\right) \cos \omega_c t$$

$\Longrightarrow$ Therefore, for a modulation index of $a = 0.1$ we have

$$M = a = 0.1$$

(d) The method avoids the need for an analog multiplier.

Solution to Z&T Computer Exercise 3.2 page 208

We use the phase shift modulator approach where

$x_{SSB}(t) = 0.5A_c m(t)\cos(2\pi F_c t) \pm 0.5A_c \hat{m}(t)\sin(2\pi F_c t)$

and

- $\hat{m}(t)$ is the Hilbert transform of $m(t)$
- the plus sign is used for lower sideband SSB
- the minus sign is used for upper sideband SSB

Contents

Design an FIR Hilbert transform filter

Use Matlab command `firpm`, which implements the Parks-McClellan algorithm for creating linear phase equi-ripple FIR filters. Note that linear phase filters of the Hilbert type have the following symmetry in the filter taps (here using standard, i.e., non-Matlab indexing

h(k) = -h(N-k) for k = 0, 1, 2, ..., N

For the case N even it will turn out that h(N/2) = 0 and the DTFT will be of the form

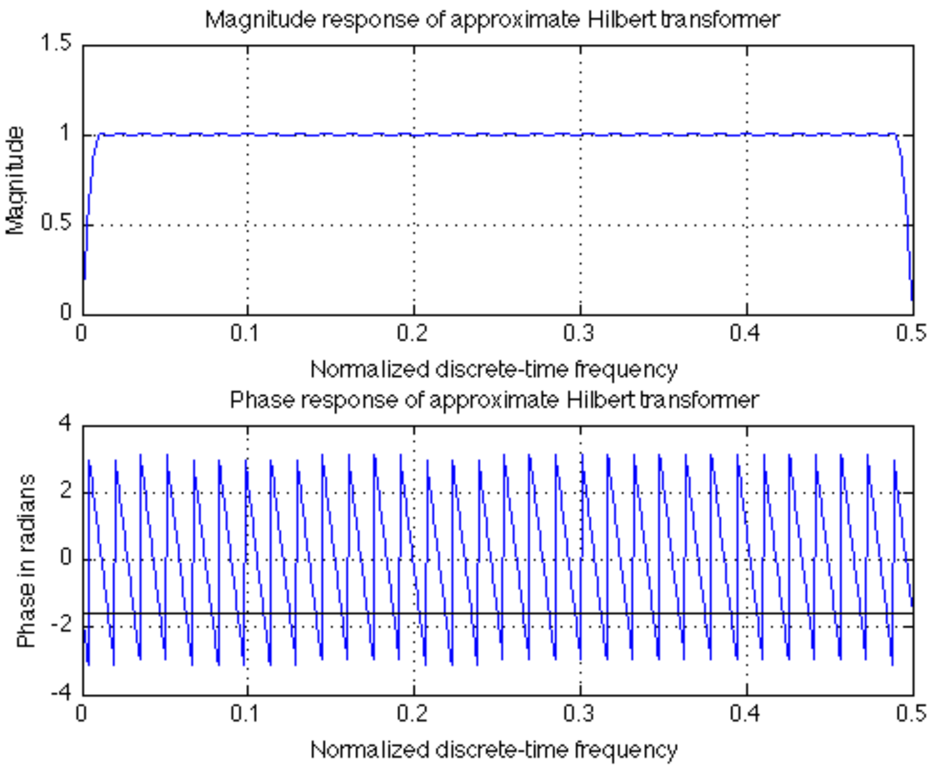
$H(f) = -je^{-j2\pi f N/2} 2\sum_{n=0}^{(N/2)-1} h(n)\sin[2\pi f(n - N/2)]$

This explains the step taken below to undo the linear part of the phase variation.

```
f = [0.02 0.98];               %Vector of frequencies to spec mag resp.
a = [1 1];                     %Vector of amplitude constraints.
N = 128;                       %Order of FIR filter. Length will be $N+1$
h = firpm(N,f,a,'hilbert');

[H,w] = freqz(h,1,1024);       %Evaluates frequency response of filter.

figure(1)
subplot(2,1,1)
plot(w/(2*pi),abs(H));
xlabel('Normalized discrete-time frequency')
ylabel('Magnitude')
title('Magnitude response of approximate Hilbert transformer')
grid

subplot(2,1,2)
plot(w/(2*pi),angle(H));
xlabel('Normalized discrete-time frequency')
ylabel('Phase in radians')
title('Phase response of approximate Hilbert transformer')
```

```
grid

hold
plot(w/(2*pi),angle(H .* exp(j*(N/2)*w)),'k');  %Remove linear part of
                                                 %phase so we can see
                                                 %desired phase of the
                                                 %Hilbert transformer,
                                                 %which should equal -90
                                                 %degrees.
```

```
Current plot held
```

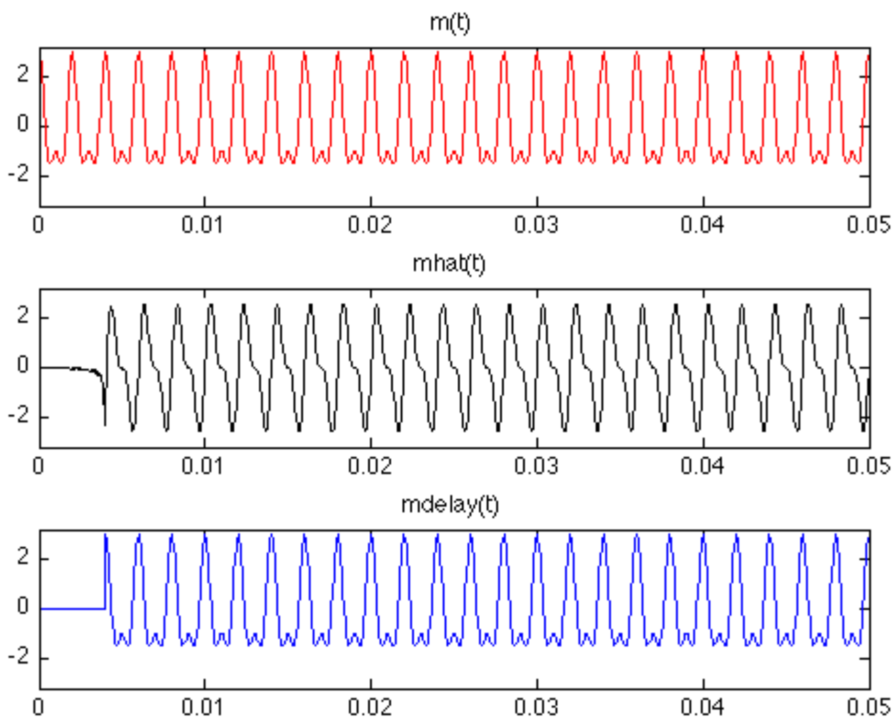


Create the message waveform and filter it to get its Hilbert transform

Different message waveforms could be substituted for that given here. Since the Hilbert transformer output contains a delay of N/2 samples, we must delay m(t) in order to properly line it up with mhat(t).

```
Fm = 500;
Fs = 32*Fm;                              %Oversampling factor is larger
                                         %than it needs to be in order
                                         %to make smoother looking
                                         %plots

T = 1;
t = 0:1/Fs:T-1/Fs;
m = 2*cos(2*pi*Fm*t) + cos(2*pi*2*Fm*t);  %The message specified by the
                                         %problem statement

figure(2)
subplot(3,1,1); plot(t,m,'r')
```

```
set(gca,'xlim',[0 0.05],'ylim',[-3.2 3.2])
title('m(t)')


mhat = filter(h,1,m);                          %Hilbert transform of message
subplot(3,1,2); plot(t,mhat,'k')
set(gca,'xlim',[0 0.05],'ylim',[-3.2 3.2])
title('mhat(t)')

hdelay = [zeros(1,N/2) 1];

mdelay = filter(hdelay,1,m);                   %Delayed message
subplot(3,1,3); plot(t,mdelay)
set(gca,'xlim',[0 0.05],'ylim',[-3.2 3.2])
title('mdelay(t)')
```



Create the SSB waves

"-" sign for upper sideband SSB, "+" sign for lower sideband SSB. Here we upsample the message and message Hilbert transform in order to fit with higher sampling rate needed by carrier

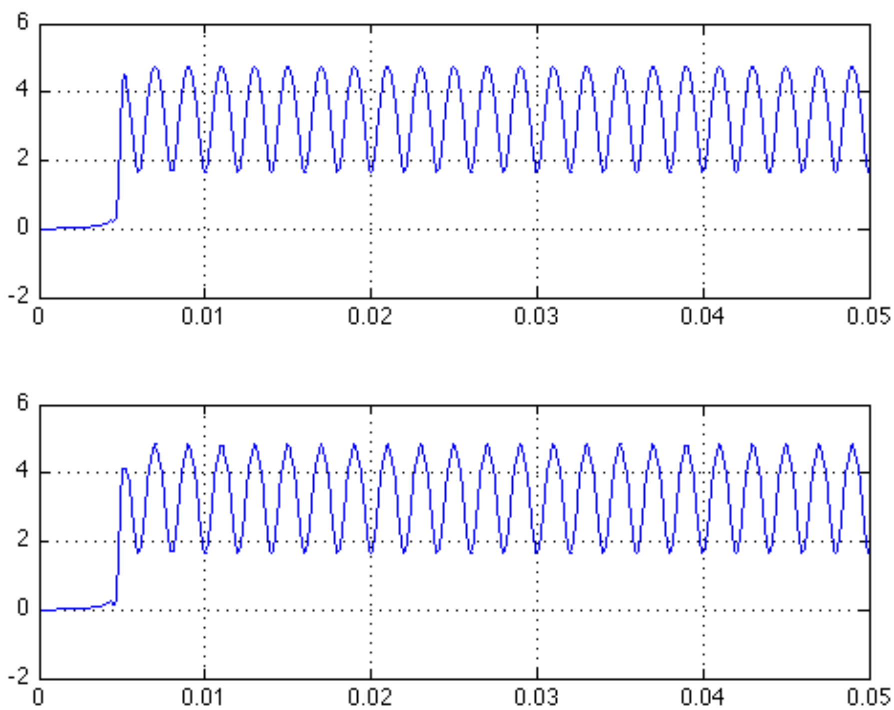
```
Fc = 2000;                                     %Carrier freq
Nup = floor(Fc/Fm);                            %Upsampling factor

%Signal processing toolbox command "interp" approximates ideal bandlimited
%interpolation
mdelay_up = interp(mdelay,Nup);
mhat_up = interp(mhat,Nup);

L = length(mhat_up);
t_up = 0:1/(Fs*Nup):T+1;
```

```
t_up = t_up(1:L);                              %Create new time sample vector

Ac = 5;
xssb_upper = 0.5*Ac*(mdelay_up .* cos(2*pi*Fc*t_up)) - ...
    0.5*Ac*(mhat_up .* sin(2*pi*Fc*t_up));
xssb_lower = 0.5*Ac*(mdelay_up .* cos(2*pi*Fc*t_up)) + ...
    0.5*Ac*(mhat_up .* sin(2*pi*Fc*t_up));
```

Compute the envelopes

We do this by rectifying and then low pass filtering

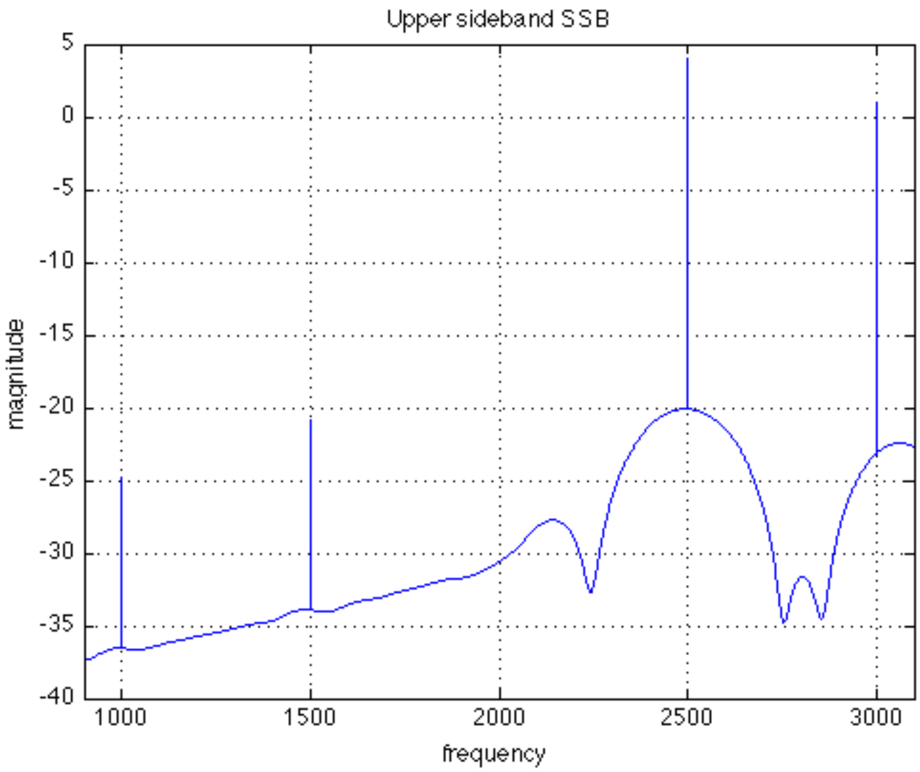
```
f = [0 .008 .1 1];                             %Specification of LP filter
a = [1 1 0 0];
hlp = firpm(128,f,a);

xssb_upper_rect = abs(xssb_upper);             %Rectify
env_upper = filter(hlp,1,xssb_upper_rect);     %Low pass filter

xssb_lower_rect = abs(xssb_lower);             %Rectify
env_lower = filter(hlp,1,xssb_lower_rect);     %Low pass filter

figure(3)
subplot(2,1,1); plot(t_up,env_upper); set(gca,'xlim',[0 0.05]); grid
subplot(2,1,2); plot(t_up,env_lower); set(gca,'xlim',[0 0.05]); grid
```

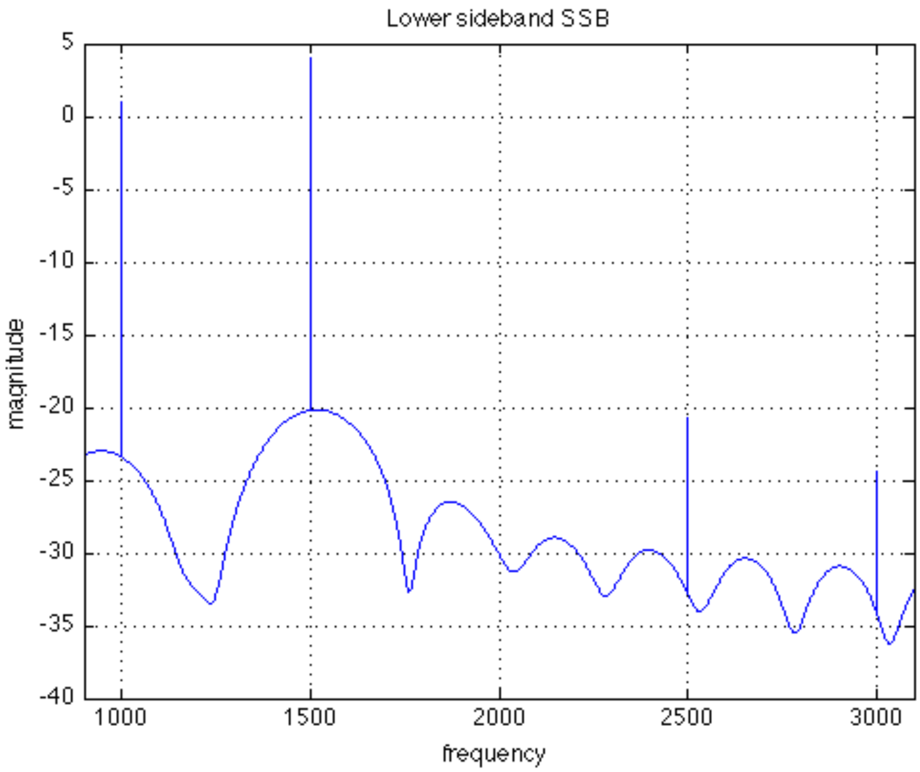


Plot the spectra using function plotspec_dB

```
figure(4)
plotspec_dB(xssb_upper,1/(Nup*Fs))
title('Upper sideband SSB')
```

```
set(gca,'xlim',[900 3100])
grid

figure(5)
plotspec_dB(xssb_lower,1/(Nup*Fs))
title('Lower sideband SSB')
set(gca,'xlim',[900 3100])
grid
```





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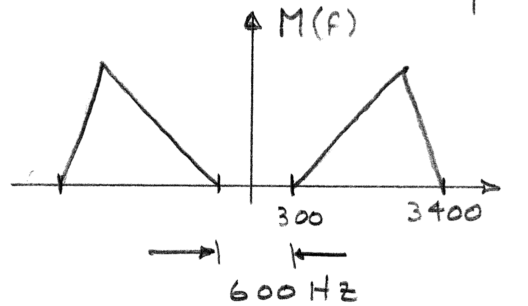
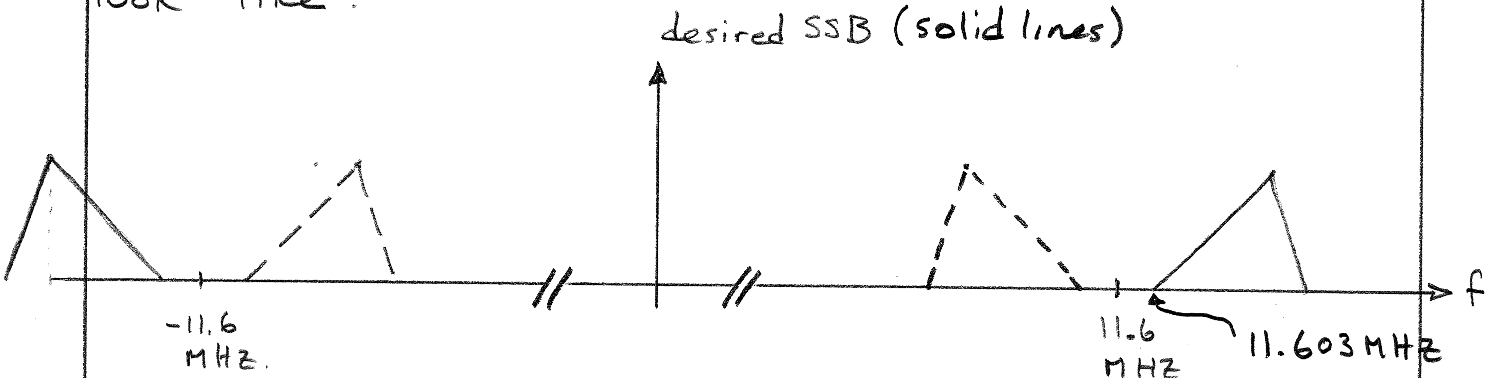
A voice signal occupying the frequency band 0.3 – 3.4 kHz is to be modulated onto a carrier wave of frequency 11.6 MHz. Assume the availability of bandpass filters which provide an attenuation of 50 dB in a transition band that is one percent of the mid-band frequency. Design a system to generate this SSB wave using the frequency discrimination method.

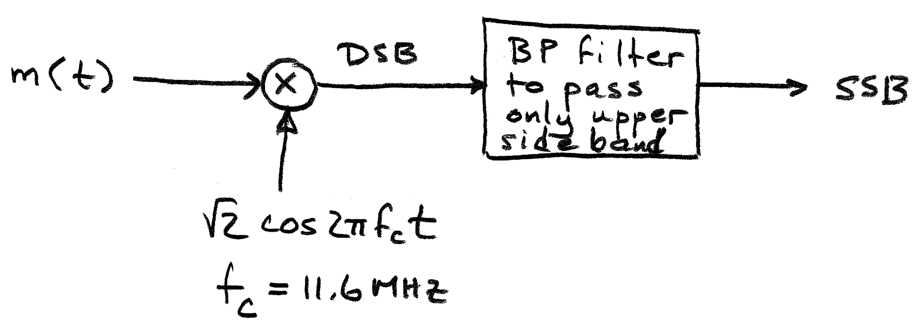
For purposes of illustration suppose the baseband message looks like (in the frequency domain)



We wish to create an SSB (say upper sideband) wave on an 11.6 MHz carrier. The resulting spectrum should look like:

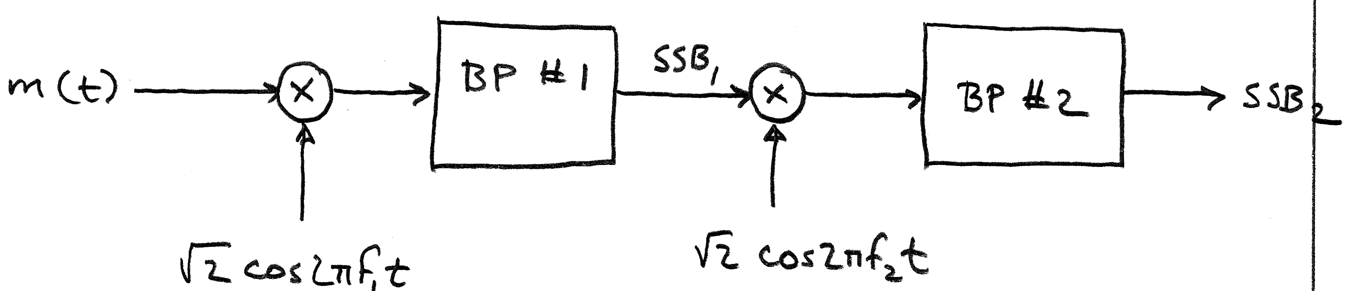


The following transmitter block diagram could be used to create the desired SSB wave shown as a solid line in the figure



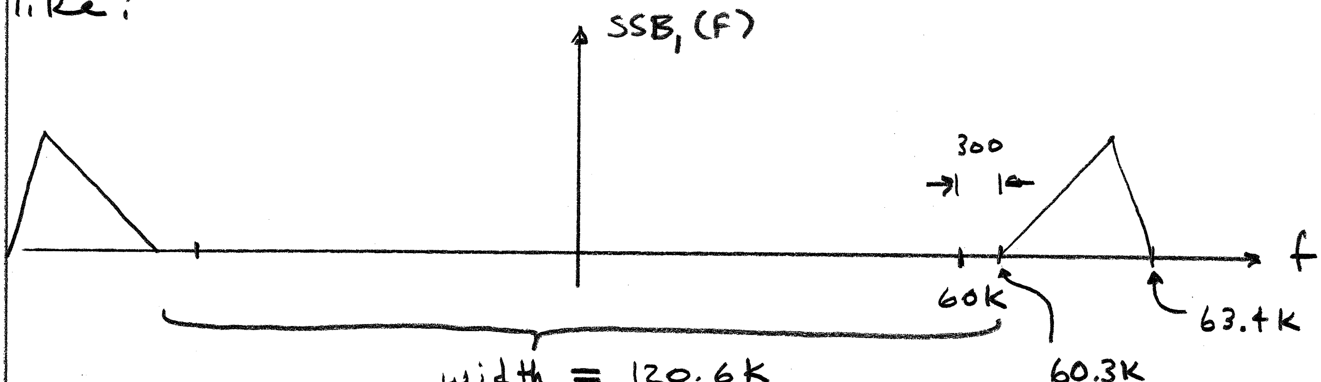
For simplicity suppose that 50 dB of attenuation is essentially infinite attenuation. Then to create the required BP filter, we would need to transition from 0 dB attenuation to 50 dB attenuation in only 600 Hz, which is about 0.005% of 11.6 MHz. According to the problem statement, we need about 1% of 11.6 MHz or 116 kHz to make the transition.

The solution is to use multiple stages of up conversion and filtering such as



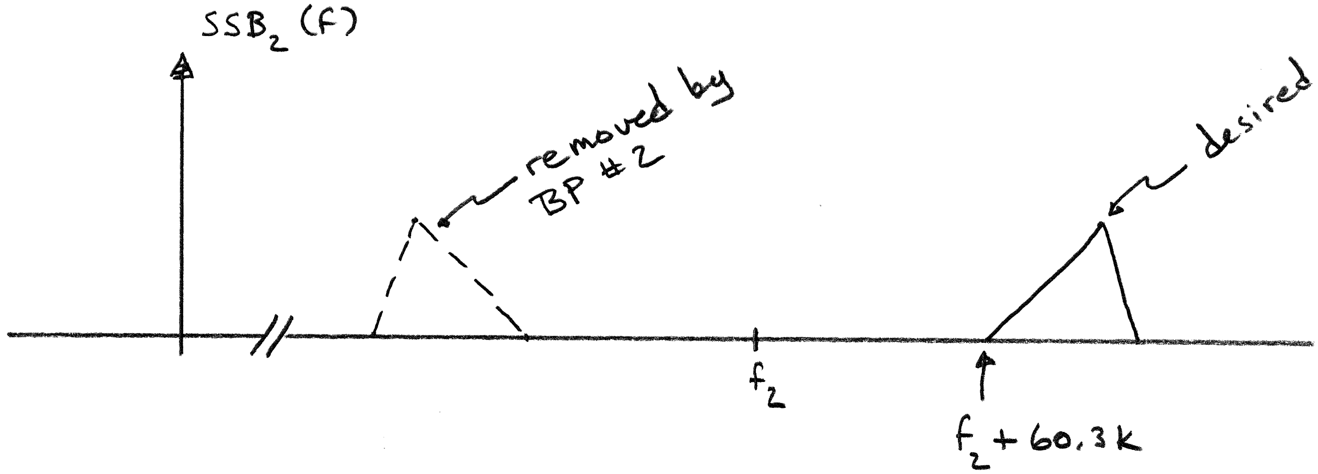
It remains to be seen if 2 stages will suffice.

With 600 Hz for transition, the 1st carrier can be at $f_1 = 60\text{kHz}$. The spectrum of SSB_1 will look like:



In a second stage we could reach a max. center frequency of 12.06 MHz and have just enough room to rolloff BP #2 $\Longrightarrow$ Therefore, 2 stages will work.

See picture on following page.



We want $f_2 + 60.3k = 11{,}603\text{ kHz}$

$$\Longrightarrow f_2 = 11\text{ MHz}.$$

Problem 3

(a) $y(t) = m(t) + \cos 2\pi f_c t$

$$y^2(t) = m^2(t) + 2m(t)\cos 2\pi f_c t + \cos^2 2\pi f_c t$$

$$= m^2(t) + 2m(t)\cos 2\pi f_c t + \frac{1}{2} + \frac{1}{2}\cos(2\pi(2f_c)t)$$

$$z(t) = 5y(t) + y^2(t)$$

$$= 5m(t) + 5\cos 2\pi f_c t + \{y^2 \text{ above}\}$$

$$= \underbrace{\left\{\frac{1}{2} + 5m(t) + m^2(t)\right\}}_{\text{baseband terms}} + \underbrace{\{5 + 2m(t)\}\cos 2\pi f_c t}_{f_c \text{ terms}} + \underbrace{\frac{1}{2}\cos(2\pi(2f_c)t)}_{2f_c \text{ term}}.$$

To find $Z(f)$ we need the F.T.s of all signals above

$$\frac{1}{2} \longleftrightarrow \frac{1}{2}\delta(f)$$

$$5m(t) \longleftrightarrow 5M(f)$$

$$m^2(t) \longleftrightarrow M * M(f) \longrightarrow$$ we will need to calculate this later.

$$5\cos 2\pi f_c t \longleftrightarrow \frac{5}{2}\delta(f-f_c) + \frac{5}{2}\delta(f+f_c)$$

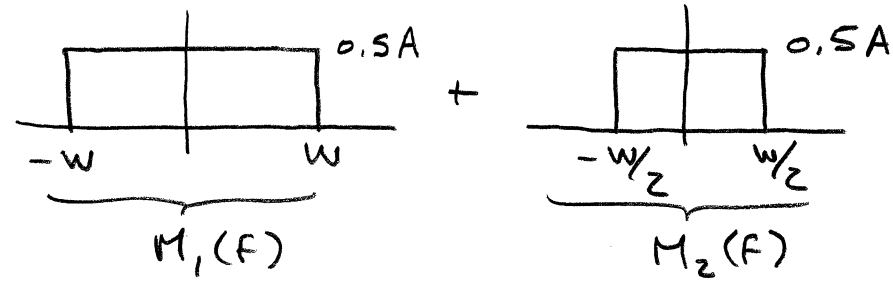
$$2m(t)\cos 2\pi f_c t \longleftrightarrow M(f-f_c) + M(f+f_c)$$

$$\frac{1}{2}\cos(2\pi(2f_c)t) \longleftrightarrow \frac{1}{4}\delta(f-2f_c) + \frac{1}{4}\delta(f+2f_c)$$

Calculation of $M * M(f)$

Transform will be symmetric wrt $f=0$ and will consist of linear segments with varying slopes. Could also decompose $M(f)$ as

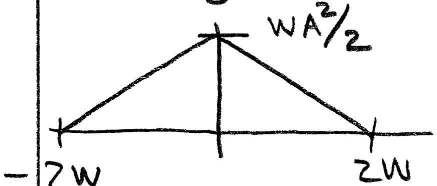
$M(f) =$



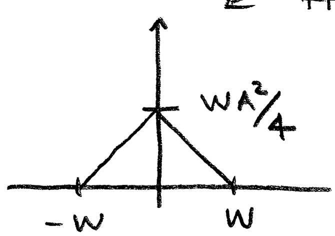
Then

$$M * M(f) = \underbrace{M_1 * M_1(f)}_{\text{triangular}} + \underbrace{M_2 * M_2(f)}_{\text{triangular}} + \underbrace{2M_1 * M_2(f)}_{\text{triangular with flat top}}$$

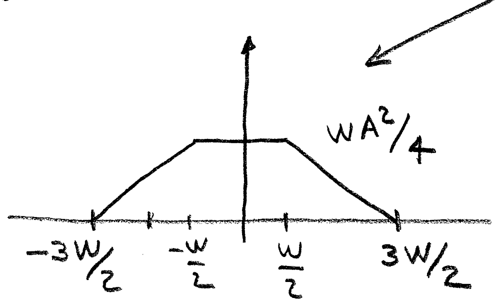
triangular



triangular



triangular with flat top



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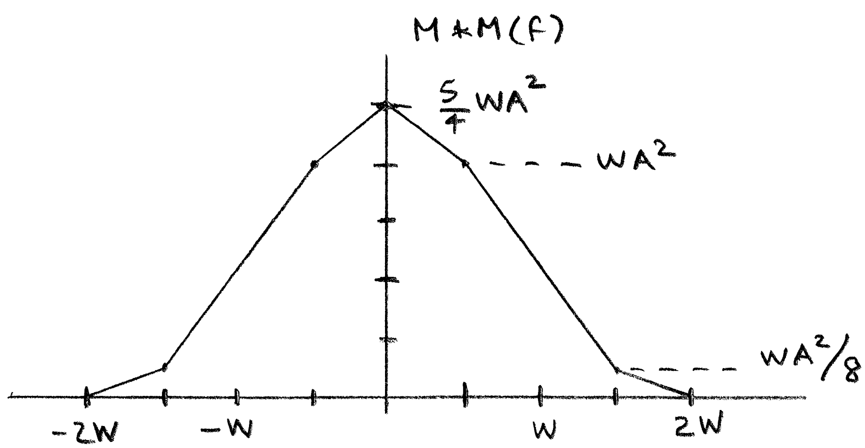
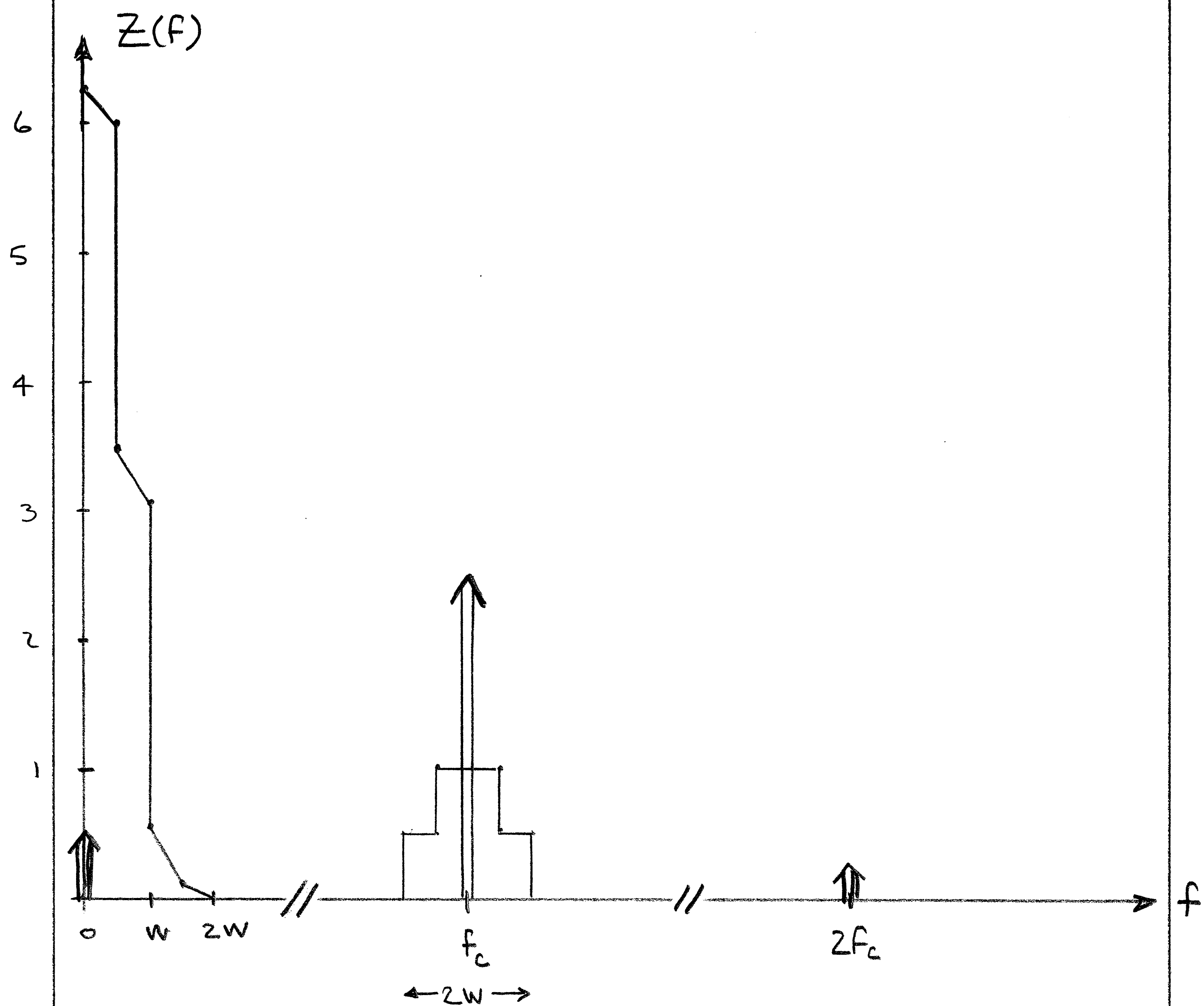
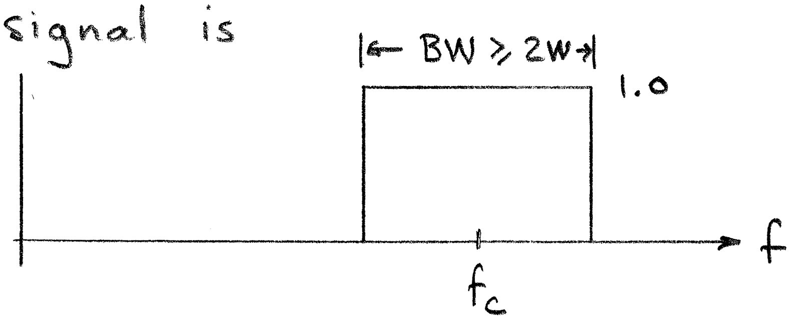
Adding up and carefully sketching:



For purposes of plotting the complete spectrum of $Z(f)$ set $A=1$, $W=1$ when adding up the individual terms. Also, the spectrum is symm. wrt. $f=0$ so only plot the $f \geq 0$ part.



ⓑ A filter $H(f)$ to extract the desired AM-LC signal is



The desired AM-LC signal is

$$w(t) = [5 + 2m(t)] \cdot \cos 2\pi f_c t$$

(c) A condition on $m(t)$ that will ensure that overmodulation does not occur is

$$5 + 2m(t) \geq 5 - 2\max_t |m(t)| > 0$$

ie

$$\frac{5}{2} > \max_t |m(t)|.$$

(d) To find the inverse transform of $M(f)$ we write it as

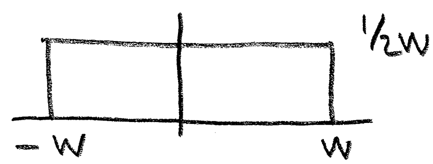
$$M(f) = M_1(f) + M_2(f)$$

where M_1 and M_2 are as previously defined. Then

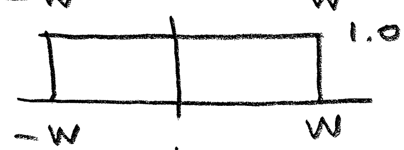
$$m(t) = m_1(t) + m_2(t).$$

Now go to the F.T. table

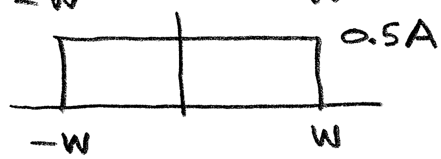
$\text{sinc}\, 2Wt \longleftrightarrow$



$2W \,\text{sinc}\, 2Wt \longleftrightarrow$



$AW \,\text{sinc}\, 2Wt \longleftrightarrow$



$\| $

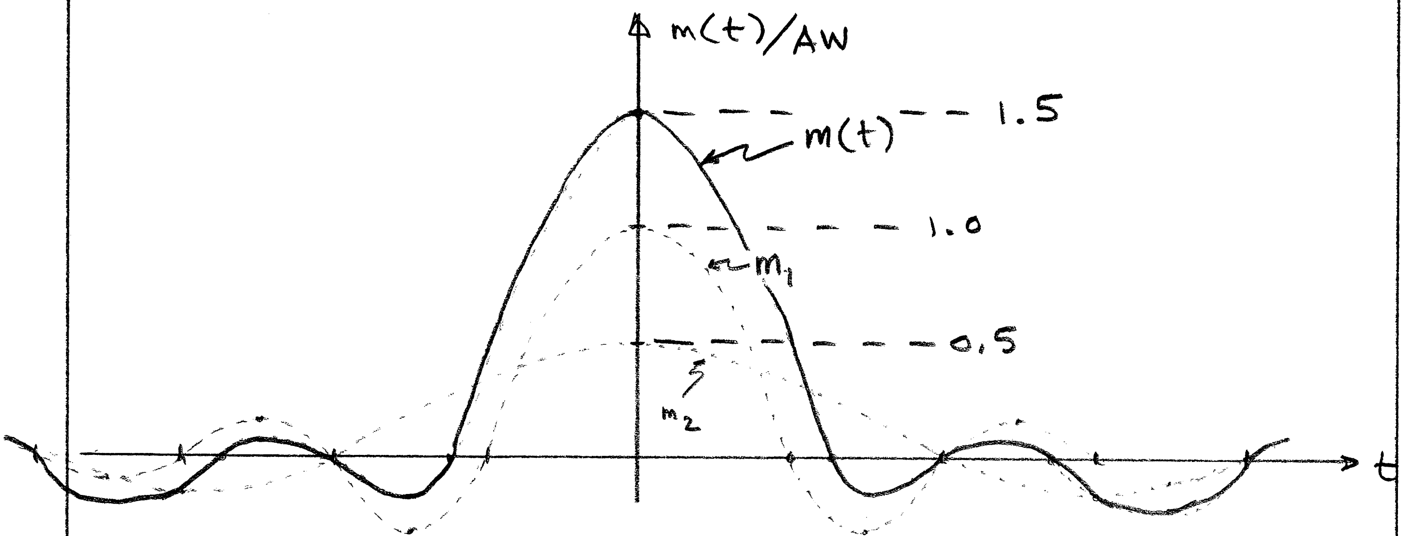
$m_1(t)$

Replace W by $W/2$ in the above to get

$$m_2(t) = \frac{AW}{2}\,\text{sinc}\, Wt$$

$$\therefore \quad m(t) = m_1(t) + m_2(t)$$
$$= AW\left[\frac{1}{2}\operatorname{sinc} Wt + \operatorname{sinc} 2Wt\right]$$

To plot $m(t)$ note that the zero crossings of $\operatorname{sinc} Wt$ are at $t = k/W$ $k = \pm 1, \pm 2 \ldots$ and those of $\operatorname{sinc} 2Wt$ are at $t = \ell/2W$ $\ell = \pm 1, \pm 2 \ldots$



ⓔ The rough sketch of $m(t)$ shows that

$$\max_t |m(t)| = m(0) = \frac{3}{2}AW$$

$\therefore$ a conservative design to ensure no overmodulation is

$$\frac{3}{2}AW < \frac{5}{2}$$
$$AW < 5/3.$$