ECE 44000 (Fall 2022)
HW 2 (Probs. 5–9)
Due: 9/12/2022

Problem 5: The Matlab code below concerns errors in approximating the CTFT using a DFT. The CTFT pair of interest is non-time-limited and non-bandlimited so there are both aliasing and windowing errors to consider. The CTFT pair used in the code is:

$$x(t) = e^{-2\pi t}u(t) \leftrightarrow X(f) = \frac{1}{2\pi(1 + jf)}.$$

The code is:

```
K = [2 3 4 5 6];                %Vector of TD window length parameters to
                                %try

M = 100;                        %Number of sampling rates to try
B = 10;                         %Bandwidth over which we approximate
f_s = linspace(2*B,200*B,M);    %Sampling rates to try
T = 1 ./ f_s;                   %Corresponding vector of sampling intervals

alpha = 2*pi;                   %Exponential decay parameter. Choice of
                                %2*pi gives a 3 dB BW of 1 Hz, equivalently
                                %time constant equals 1/(2*pi) sec.

for k = 1:length(K)

    T_0 = K(k)/alpha;           %Length of time domain window

    N = floor(T_0 ./ T);        %Vector containing the number of time
                                %domain samples corresponding to each
                                %choice of sampling interval

    nrmse_dB = zeros(1,M);

    for i = 1:M
        n = 1:N(i);
        f = (n - 1 - floor(N(i)/2))/(N(i)*T(i));
        index = (abs(f) <= B);
        t = (0:(N(i)-1))*T(i);
        x = exp(-alpha*t);
        X = T(i)*fftshift(fft(x));
        X_c = 1 ./ (alpha + 1j*2*pi*f);
        X_short = X(index);
        X_c_short = X_c(index);
        Error = X_c_short - X_short;
        nrmse_dB(i) = 20*log10(norm(Error)/norm(X_c_short));
    end

    figure(1)
    semilogx(f_s,nrmse_dB,'LineWidth',1.5)
    grid on
    hold on

end
```

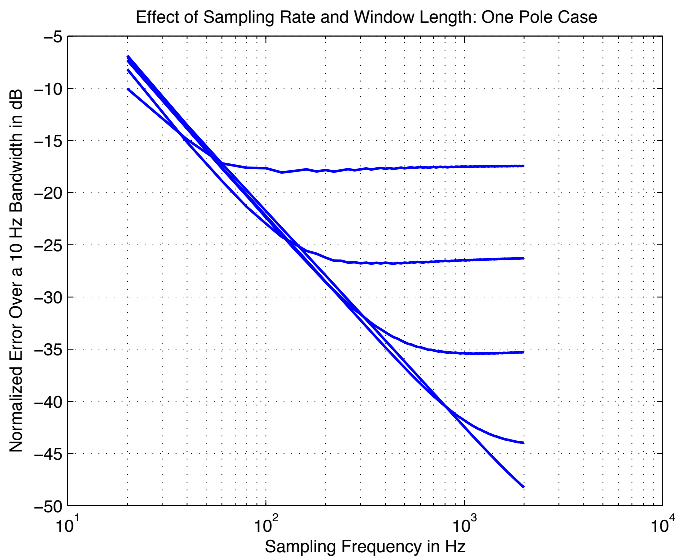


Figure 1: Aliasing Error as a Function of Sampling Frequency and Parameterized by Window Length.

```
figure(1)
xlabel('Sampling Frequency in Hz')
ylabel('Normalized Error Over a 10 Hz Bandwidth in dB')
title('Effect of Sampling Rate and Window Length: One Pole Case')
hold off
```

(a) Explain the code in detail and run it to generate the curves of Figure 1.

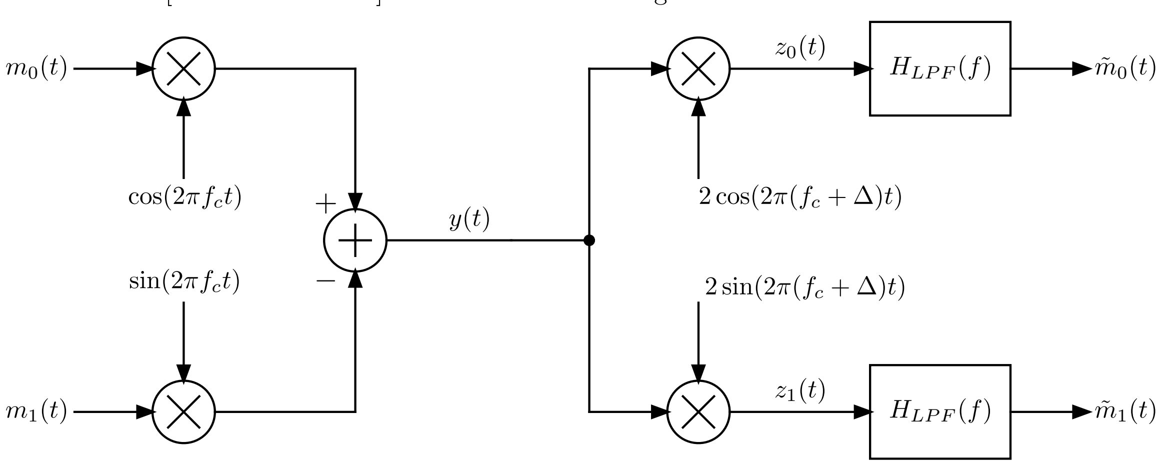
(b) Explain the curves.

(c) Modify the code to handle the CTFT pair

$$y(t) = te^{-\alpha t}u(t) \leftrightarrow Y(f) = \frac{1}{(\alpha + j2\pi f)^2}$$

where you should pick the decay parameter $\alpha > 0$ so that the 3 dB bandwidths of $Y(f)$ and $X(f)$ are approximately the same. Run the new code and produce new curves. Compare the new curves to the old curves and explain.

Problem 6: [Fall 2011 Exam 1] Consider the block diagram below:



where

- The signals $m_0(t)$ and $m_1(t)$ have the real-valued, bandlimited Fourier Transforms shown below:

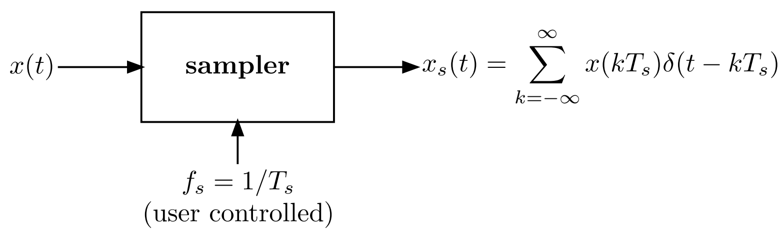
- $f_c \gg W$, Δ and $W = 2\Delta$.
- H_{LPF} is the ideal low pass filter:

$$H_{LPF}(f) = \begin{cases} 1 & |f| \leq W + \Delta \\ 0 & |f| > W + \Delta \end{cases} .$$

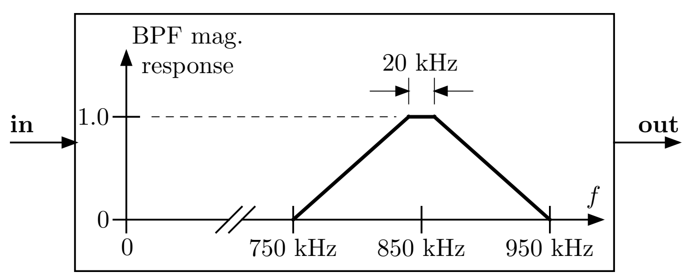
Find time domain expressions for $\tilde{m}_0(t)$ and $\tilde{m}_1(t)$ in terms of $m_0(t)$ and $m_1(t)$. Also, find and plot the spectra $\tilde{M}_0(f)$ and $\tilde{M}_1(f)$.

Problem 7: [Fall 2011 Final Exam] I want to make an AM DSB modulator out of the following parts:

- A sampler with a variable sampling frequency $f_s = 1/T_s$ which takes a continuous time signal as input and produces an amplitude modulated train of delta functions, e.g.,



- A bandpass filter centered at my desired AM DSB carrier frequency f_c = 850 kHz[1] with a transmission bandwidth of 20 kHz and a rolloff to zero as shown:



Assume a baseband message waveform $m(t)$ of bandwidth 10 kHz and show how to connect the blocks to make an AM DSB modulator for carrier frequency f_c = 850 kHz. What is the smallest acceptable sampling frequency f_s?

Problem 8: Transmission line calculations.

(a) Use the equations given in the transmission line notes to compute and fill in a table of parameter values for three different transmission lines at three different frequencies. The rows are:

[1] KOA News Radio in Denver, CO (aka "God's Country").

Name	f (kHz)	R (Ω/km)	L (mH/km)	C (μF/km)
22 AWG	0.1	107	0.6	0.05
22 AWG	10	107	0.6	0.05
22 AWG	100	107	0.6	0.05
19 AWG	0.1	52	0.68	0.04
19 AWG	10	52	0.68	0.04
19 AWG	100	85	0.68	0.04
Telephone	0.1	6.4	2.1	0.0055
Telephone	10	8.0	2.1	0.0055
Telephone	100	22	2.1	0.0055

For all of these the dielectric losses are assumed to be insignificant (i.e., assume $G = 0$). Telephone refers to the old fashioned situation of a pair of wires on top of a pole. The wire diameter in this case is assumed to be 2.6 mm. The wires are also quite far apart, which is why the distributed capacitance is so small. Then compute the following columns for each row:

- Magnitude of the characteristic impedance $|Z_0|$ in Ω/km.
- Angle of the characteristic impedance $\angle Z_0$ in degrees/km.
- Loss in dB/km.
- Phase constant β in radians/km.
- Wavelength λ in m.
- Phase velocity in km/s.

(b) Use the formula for the resistance of a cylindrical wire to compute and compare the resistance per unit distance values of the transmission lines above. Assume the wires are made of copper. Explain any discrepancies you might see. Assume operation at a low frequency.

(c) The resistance values given in the table are functions of frequency, at least for the thicker wires. Why is this? Can you find a way to calculate the resistance as a function of frequency? Do so if you are able.

(d) Using the values given in the table and assuming infinitely long transmission lines (so you don't have to worry about reflections from the load), plot the frequency response $e^{-\gamma z}$ for a particular positive value of z, the distance down the line, at the three frequencies in the table. Compare the lines. Use dB for amplitude response and a log plot for frequency. Sketch a smooth line between your points. Comment.

Problem 9: Look up information on the spark gap transmitter of the early days of radio telegraphy. Find a circuit and translate to a system block diagram. Explain the relationship between the sparking frequency and the center frequency of the radio wave oscillations, if any. One place to look might be in IEEE Xplore in Journals from about 100 years ago.