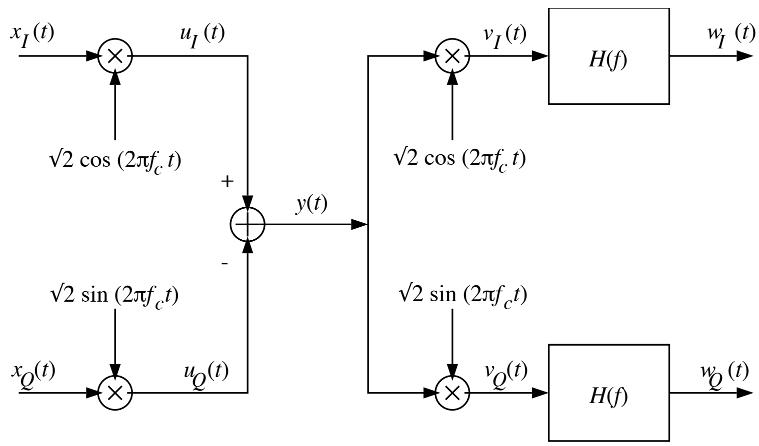
ECE 44000 (Fall 2022)
HW 1 (Probs. 1–4)
Due: 8/29/2022

Problem 1: Consider the following block diagram:



where

- The signals $x_I(t)$ and $x_Q(t)$ have continuous-time Fourier transforms $X_I(f)$ and $X_Q(f)$ which are both bandlimited: $X_I(f) = X_Q(f) = 0$ for $|f| > f_m$.
- $f_c \gg f_m$.
- The filter $H(f)$ is a low-pass filter:

$$H(f) = \begin{cases} 1 & |f| \leq f_m \\ 0 & |f| > f_m \end{cases} .$$

Plot the Fourier transforms of $x_I(t)$, $\sqrt{2}\cos(2\pi f_c t)$, $u_I(t)$, $x_Q(t)$, $\sqrt{2}\sin(2\pi f_c t)$, $u_Q(t)$, $y(t)$, $v_I(t)$, $w_I(t)$, $v_Q(t)$, and $w_Q(t)$ when

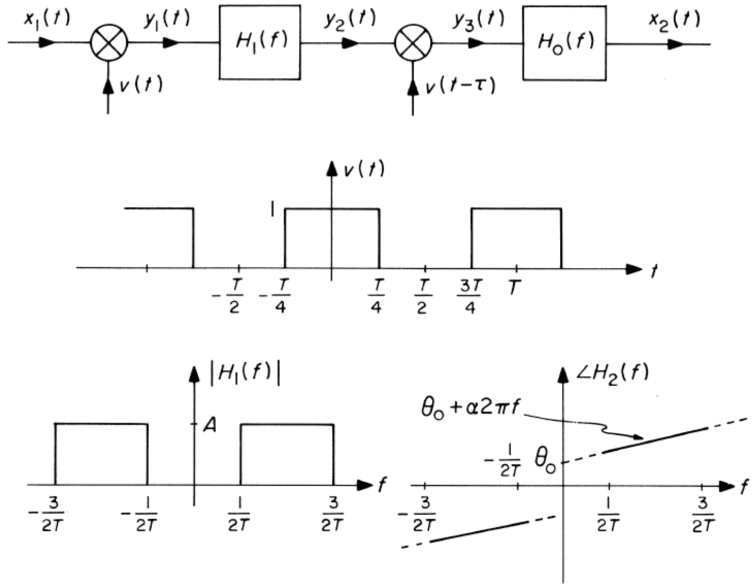
$$\begin{aligned} X_I(f) &= \begin{cases} \chi_I & |f| < f_m \\ 0 & \text{otherwise} \end{cases} \\ X_Q(f) &= \begin{cases} \chi_Q(1 - |f|/f_m) & |f| < f_m \\ 0 & \text{otherwise} \end{cases} . \end{aligned}$$

Problem 2: [From Siebert] The design of high-gain dc amplifiers (i.e., amplifiers whose pass band includes $f = 0$) frequently presents difficulties because small slow changes (due to aging, temperature fluctuations, supply voltage changes, etc.) in the quiescent operating points of the active elements can produce responses indistinguishable from those due to small desired signals. One way to circumvent these difficulties is to use a *chopper* to modulate the signal onto a carrier so that an ac coupled amplifier can be used instead. Another chopper is then used as a synchronous detector to restore the signal to its original frequency range. The scheme is illustrated in the figure below. Assume that the spectrum of the input signal $x_1(t)$ is restricted to $|f| < f_1$. The periodic time functions $v(t)$ and $v(t - \tau)$ describe the chopping action. $H_0(f)$ is an ideal lowpass filter with gain 1 over the pass band $|f| < f_1$; and $H_1(f)$ is a bandpass high-gain amplifier with characteristics shown.

(a) Find the first few terms of the Fourier series for $v(t)$ and $v(t - \tau)$.

(b) Sketch the spectra of $y_1(t)$ and $y_2(t)$ and label them accurately in terms of an assumed spectral shape for $X_1(f)$.

(c) Find an expression for $x_2(t)$ in terms of $x_1(t)$ and the system parameters.



Problem 3: Z&T Computer Exercise 2.3 (page 111).

2.3. Write a computer program to evaluate the coefficients of the complex exponential Fourier series of a

signal by using the FFT. Check it by evaluating the Fourier series coefficients of a square-wave and comparing your results with Computer Exercise 2.2.

Problem 4: Z&T Computer Exercise 2.4 (page 111).

2.4. How would you use the same approach as in Computer Exercise 2.3 to evaluate the Fourier transform of a pulse-type signal. How do the two outputs differ? Compute an approximation to the Fourier transform of a square pulse signal 1 unit wide and compare with the theoretical result.