Name: Solution

General Instructions:

- You have 50 minutes to complete the exam.
- Write your name on every page of the exam.
- Please do not write on the backs of pages.
- The exam is closed book.
- You are allowed both sides of two 8.5 by 11 inch sheets of paper for your personal notes in addition to the instructor supplied formula sheet.
- Calculators are allowed.
- Your work must be explained to receive full credit. All plots must be carefully drawn with axes labeled.
- Point values for each problem are as indicated. The exam totals 100 points.
- Please do not leave early as it is disruptive to those working around you.

Do not open the exam until you are told to begin.

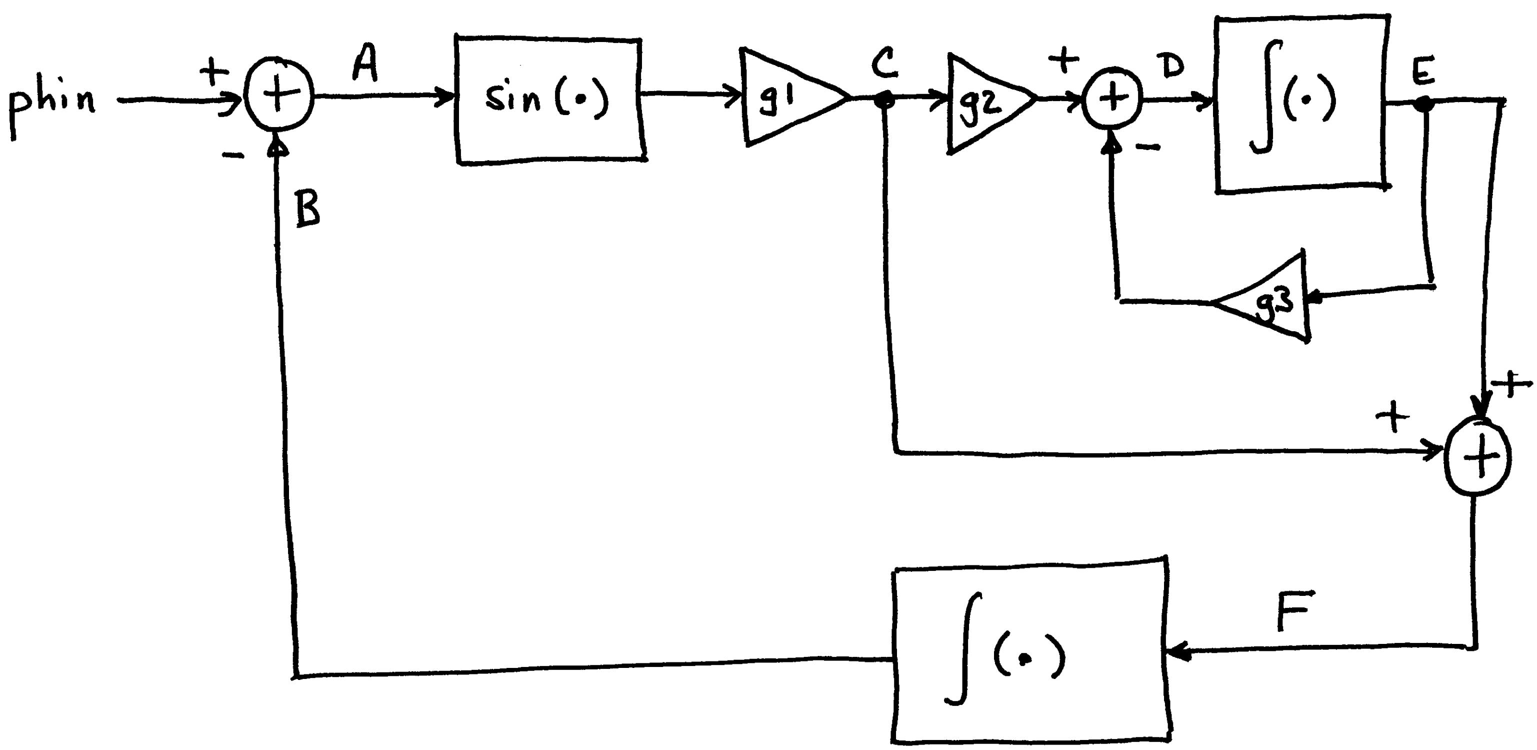
Name: ____________________

Problem 1. *Matlab Implementation of Phase Locked Loop.* [25 pts. total] The code below implements the main body of a phase locked loop you simulated in homework. Variables have been renamed to: A, B, C, D, E, and F (see the code).

```
%% Beginning of Simulation Loop
for i = 1:npts
    if i < nsettle
        fin(i) = 0;
        phin = 0;
    else
        fin(i) = fdel;
        phin = 2*pi*fdel*T*(i-nsettle);
    end

    A = phin - B;
    C = g1*sin(A);

    D = g2*C - g3*E;
    E = E_last + (T/2)*(D + D_last);
    D_last = D;
    E_last = E;

    F = C + E;
    B = B_last + (T/2)*(F + F_last);
    F_last = F;
    B_last = B;

    phierror(i) = A;
    fvco(i) = F/(2*pi);
    freqerror(i) = fin(i) - fvco(i);
end
% End of Simulation Loop
```

(a) [10 pts.] Draw a block diagram for this phase locked loop, labeling the blocks with the parameters of the code and showing the variables A, B, C, D, E, and F.

(a) (cont'd.)



(b) [5 pts.] Answer the following questions with explanation for your answers ...

(b1) What kind of loop filter has been implemented in this code?

This is the imperfect 2nd order loop with loop filter $\frac{s+a}{s+\lambda a}$

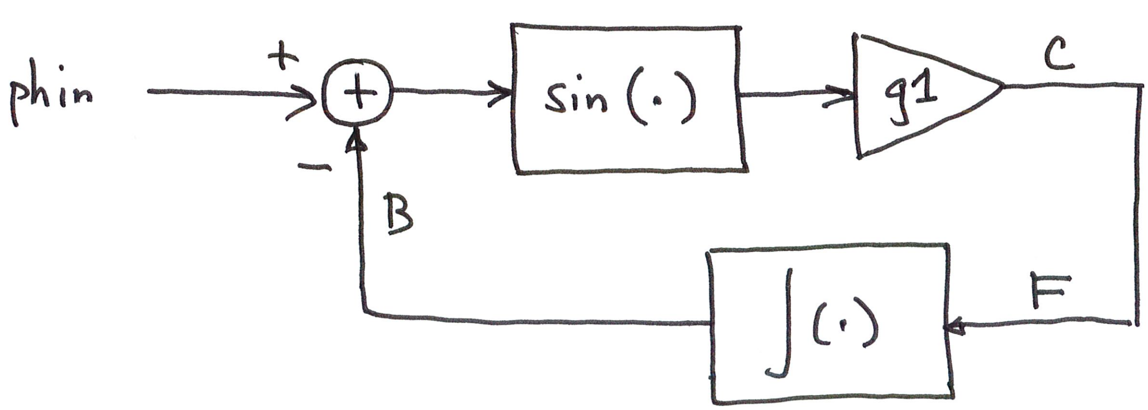
(b2) What is the **type** and **order** of the PLL implemented in this code?

2nd order

Type 1

(c) [10 pts.] Modify the code and the block diagram from part (a) to remove the loop filter creating a first order PLL. Put the new code and the new block diagram on this page.

```
%% Beginning of Simulation Loop
for i = 1:npts
    if i < nsettle
        fin(i) = 0;
        phin = 0;
    else
        fin(i) = fdel;
        phin = 2*pi*fdel*T*(i-nsettle);
    end

    A = phin - B;
    C = g1*sin(A);

    D = g2*C - g3*E;
    E = E_last + (T/2)*(D + D_last);
    D_last = D;
    E_last = E;

    F = C + E;
    B = B_last + (T/2)*(F + F_last);
    F_last = F;
    B_last = B;

    phierror(i) = A;
    fvco(i) = F/(2*pi);
    freqerror(i) = fin(i) - fvco(i);
end
% End of Simulation Loop
```



Name: ____________________

Problem 2. *Power in AM-LC System.* [25 pts. total] An AM-LC signal is given by

$$x_{am-lc}(t) = 10[1 + 0.4 \cdot m(t)] \cos[(1.7 \times 10^5)\pi t] \text{ Volts.}$$

The message power is known to be $P_m = 5$ W. Find:

(a) [15 pts.] The power of $x_{am-lc}(t)$ assuming it is developed across a 1 Ω resistance.

$$\text{Power} = 0.5\,(10^2) + 0.5\,(4^2)\,P_m$$
$$= 50 + 8.5 = 90\text{W}$$

(b) [10 pts.] The efficiency η (i.e., the ratio of power in $x_{am-lc}(t)$ due to the message to the total power in $x_{am-lc}(t)$).

Power due to message $= 40\text{W}$

$$\eta = \frac{40}{90} = \frac{4}{9}$$

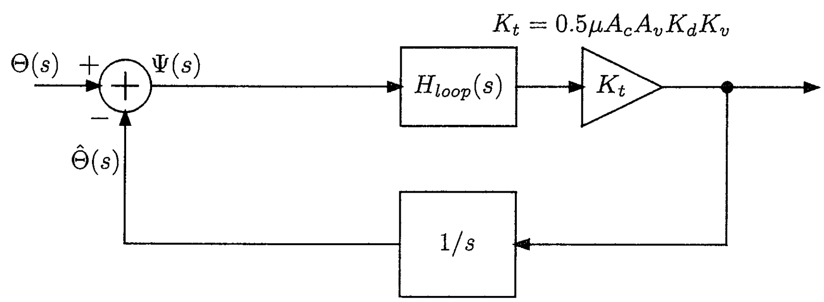
Name: ____________________

Problem 3. *Steady State Response of the Linearized PLL.* [15 pts. total] The standard linearized baseband model for a PLL is shown below in the Laplace Domain. Suppose that the original time domain input to the PLL is of the form

$$x(t) = A_c \cos[10^5 \pi t + \theta(t)]$$

where $\theta(t) = 3t^2 u(t)$ and $u(t)$ denotes a unit step.

What is the requirement on the loop filter $H_{loop}(s)$ in order that the steady state phase error be equal to zero? Explain.



$\theta(t) = 3t^2 u(t)$

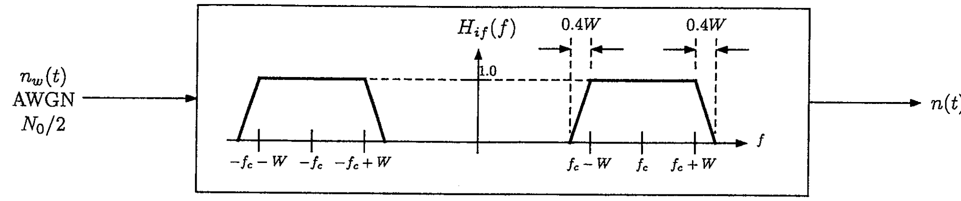
This phase response represents a frequency ramp or chirp. It has a unilateral Laplace transform of the form

$$\frac{K}{s^3}$$

From the "general tracking result" in class the loop filter needs to have at least 2 poles at $s = 0$ in order that the steady state phase error is zero.

Name:

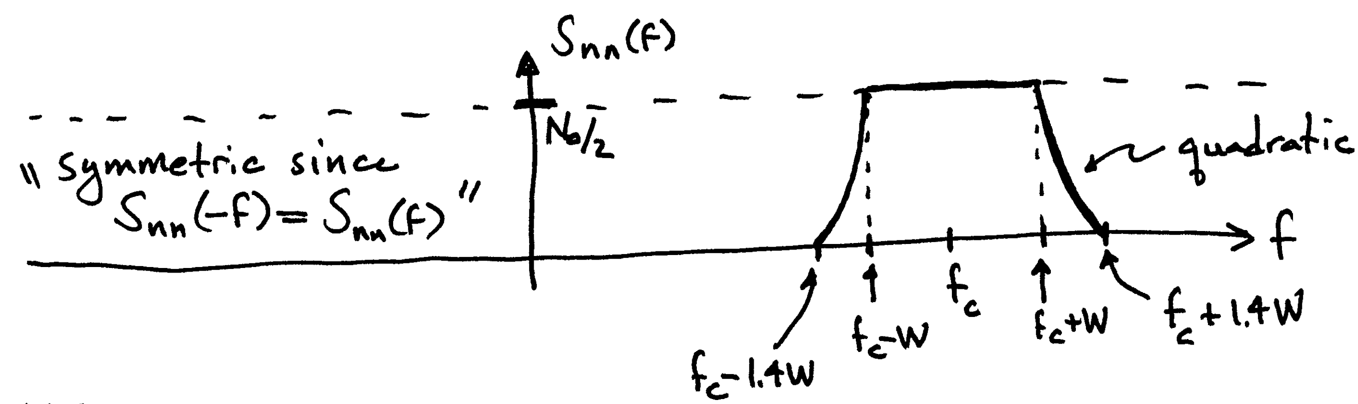
Problem 4. *Noise power in an IF filter output.* [35 points]



The IF filter shown above has input $n_w(t)$, a white Gaussian noise, and output $n(t)$, a non-white Gaussian noise.

(a) [10 pts.] Carefully sketch the power spectral density $S_{nn}(f)$ of the filter output.

$S_{nn}(f) = |H_{if}(f)|^2 \cdot \frac{N_0}{2} \quad \forall f \Rightarrow$ has the shape of $|H_{if}|^2$ which will look like ...



(b) [15 pts.] Find the power of the output noise $n(t)$ as a function of W, $0 < W \ll f_c$ and plot it.

$$\text{Power} = \int_{-\infty}^{\infty} \frac{N_0}{2} |H_{if}(f)|^2 \, df$$

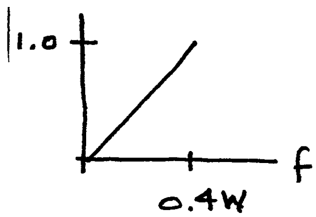
$$= 2 \frac{N_0}{2} (2W) + 4 \frac{N_0}{2} \int \left(\underset{0.4W}{\triangle} \, 1.0 \right)^2 df$$

$$= 2WN_0 + 2N_0 I$$

where I is the integral in question.

(b) (cont'd.)

By a change of variables the integral "I" is the integral of the square of ...



$\longrightarrow$ equation of this line is

$$\frac{f}{0.4W} \qquad 0 \le f \le 0.4W$$

$$I = \int_0^{0.4W} \left(f/0.4W\right)^2 df = \frac{1}{(0.4W)^2}\,\frac{1}{3} f^3 \Big|_0^{0.4W}$$

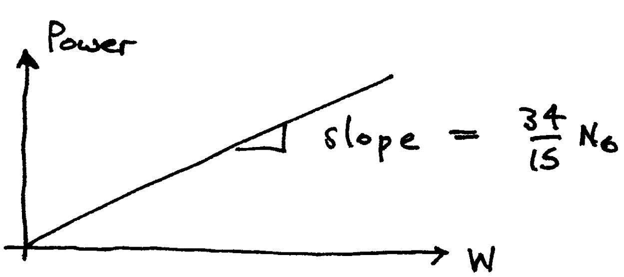
$$= \frac{1}{(0.4W)^2}\,\frac{1}{3}\,(0.4W)^3 = \frac{0.4W}{3}$$

$$= \frac{4W}{30} = \frac{2W}{15}$$

$$\therefore \text{Power} = 2N_0 W + 2N_0 \frac{2W}{15} = \left(2 + \frac{4}{15}\right) N_0 W$$

$$= 2\left(1 + \frac{2}{15}\right) N_0 W = 2\,\frac{17}{15}\,N_0 W$$

$$= \frac{34}{15} W N_0$$



(c) [10 pts.] With $N_0 = kT$, where $k = 1.38 \times 10^{-23}$ J/K and T is the temperature in Kelvin find the power in the output noise for the following cases. Express your answers in both mW and dBm.

(c1) $W = 1$ MHz and $T = 300$ K, roughly the average temperature of the earth as seen by a satellite antenna pointing down.

$$P = \frac{34}{15} W N_0 = \frac{34}{15} \cdot 10^6 \cdot 1.38 \times 10^{-23} \cdot 3 \times 10^2$$

$$= \frac{34}{15} \cdot 1.38 \cdot 3 \times 10^{-15} \text{ W} = 9.38 \times 10^{-15} \text{ W}$$

$$= 9.38 \times 10^{-12} \text{ mW}$$

$$P_{dBm} = 10 \log_{10} (9.38 \times 10^{-12}) = -120 + 10 \log_{10} (9.38)$$

$$= -110.3 \text{ dBm}$$

(c2) $W = 100$ MHz and $T = 1000$ K, corresponding to a fairly noisy receiver.

$$P = \frac{34}{15} \cdot 1.38 \cdot 10^8 \cdot 10^3 \cdot 10^{-23}$$

$$= 3.13 \times 10^{-12} \text{ W} = 3.13 \times 10^{-9} \text{ mW}$$

$$P_{dBm} = -90 + 10 \log_{10} 3.13$$

$$= -85.0 \text{ dBm}$$