ECE 440 Exam 1 9/23/22

Name: Solution

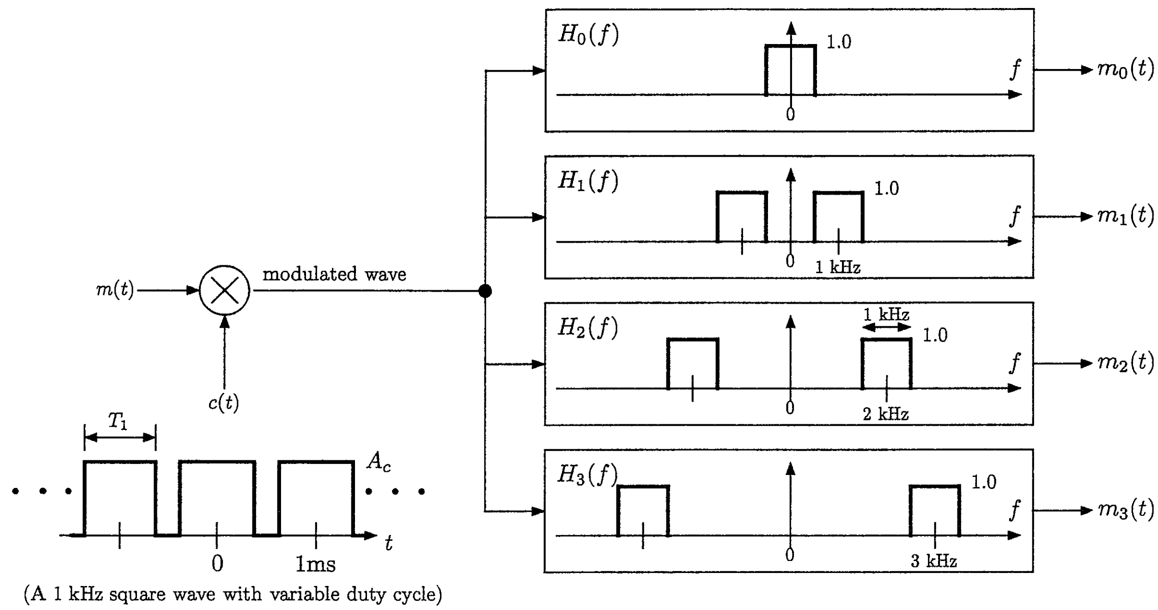
General Instructions:

- **You have 50 minutes to complete the exam.**
- Write your name on every page of the exam.
- Please do not write on the backs of pages.
- The exam is closed book. You are allowed both sides of one 8.5 by 11 inch sheet of paper for your personal notes in addition to the instructor supplied formula sheet.
- Your work must be explained to receive full credit. All plots must be carefully drawn with axes labeled.
- Point values for each problem are as indicated. The exam totals 100 points.
- Please do not leave early as it is disruptive to those working around you.

Do not open the exam until you are told to begin.

Name: ____________________

Problem 1. *Related to Fourier Series and DSB Modulation.* [35 pts. total]



A message $m(t)$ of baseband bandwidth less than 500 Hz modulates the square-wave carrier wave $c(t)$ shown the in the block diagram above. The carrier has a 1 ms period and a variable duty cycle determined by $T_1 \leq 1$ ms.

Then the modulated wave is input to four LTI filters in parallel. The filters are centered at the frequencies indicated and perfectly pass all frequencies within $\pm$ 500 Hz of the center frequency.

(a) [25 pts.] The general problem is to find the output signals $m_0(t)$, $m_1(t)$, $m_2(t)$, and $m_3(t)$ in terms of $m(t)$, A_c, T_1, and the various center frequencies. First, describe the general solution procedure you would use, i.e., outline the steps taken to solve the problem. Then solve it in the general case and write down the expressions for the filter outputs. Show your work and explain completely.

(b) [5 pts.] Find $m_0(t)$, $m_1(t)$, $m_2(t)$, and $m_3(t)$ when $T_1 = 0.5$ ms.

(c) [5 pts.] Find $m_0(t)$, $m_1(t)$, $m_2(t)$, and $m_3(t)$ when $T_1 = 0.75$ ms.

Related to Fourier Series and DSB Modulation

ⓐ The first step is to find the Fourier Series coeffs corresponding to $c(t)$ ie

$$C_k = \frac{1}{T_o}\int_{-T_o/2}^{T_o/2} c(t)\, e^{-j2\pi kt/T_o}\, dt \qquad \text{where } T_o = 1\text{ms}$$

Then

$$c(t) = \sum_{n=-\infty}^{\infty} C_n e^{+j2\pi nt/T_o}$$

and the modulated wave would be

$$x(t) = m(t)c(t) = \sum_n m(t)\, C_n e^{+j2\pi n f_o t}$$

where $f_o = 1/T_o$. Thus there will be spectral copies of $M(f)$ centered at the harmonic frequencies nf_o $n \in \mathbb{Z}$. These are the center frequencies of the BPFs in the figure. Note also that the filter bandwidths are sufficient to pass the shifted spectral copies of $M(f)$.

Clearly

$$m_0(t) = C_0\, m(t)$$

$$m_1(t) = m(t)\left[C_1 e^{+j2\pi f_o t} + C_{-1} e^{-j2\pi f_o t}\right]$$

$$m_2(t) = m(t)\left[C_2 e^{+j2\pi 2f_o t} + C_{-2} e^{-j2\pi 2f_o t}\right]$$

$$m_3(t) = m(t)\left[C_3 e^{+j2\pi 3 f_0 t} + C_{-3} e^{-j2\pi 3 f_0 t}\right]$$

Now since $c(t)$ is real-valued the F.S. coeffs have a conjugate symmetry ...

$$C_k = C_{-k}^{*} \qquad \forall k \in \mathbb{Z}$$

Thus we may write $C_k = |C_k| e^{j\angle C_k}$ and then can rewrite the expressions as e.g.

$$m_3(t) = m(t)\, 2|C_3| \cos(2\pi 3 f_0 t + \angle C_3)$$

To continue need to actually find the C_k ...

$$C_k = \frac{1}{T_0}\int_{-T_1/2}^{T_1/2} A_c e^{-j2\pi k t/T_0}\, dt = \frac{1}{T_0}\frac{A_c}{-j2\pi k/T_0} e^{-j2\pi k t/T_0}\Big|_{-T_1/2}^{T_1/2}$$

$$= \frac{A_c}{\pi k}\,\frac{e^{-j\pi k T_1/T_0} - e^{+j\pi k T_1/T_0}}{-j2}$$

$$= \frac{A_c}{\pi k}\sin \pi k T_1/T_0 = \frac{T_1}{T_0}A_c \frac{\sin \pi k T_1/T_0}{\pi k T_1/T_0}$$

where $T_0 = 1\text{ms}$.

(b) $T_1 = 0.5\,\text{ms} \implies T_1/T_0 = 0.5/1 = 1/2$

$$\implies \frac{\pi k T_1}{T_0} = \pi k/2$$

$$C_0 = \frac{A_c}{2}$$

$$C_1 = \frac{A_c}{2}\,\frac{1}{\pi/2} = \frac{A_c}{\pi} = C_{-1}$$

$$C_2 = \frac{A_2}{2}\cdot 0 = C_{-2}$$

$$C_3 = \frac{A_c}{2}\cdot\frac{-1}{3\pi/2} = -\frac{A_3}{3\pi} = C_{-3}$$

(c) $T_1 = 0.75\,\text{ms} \implies \frac{T_1}{T_0} = \frac{0.75}{1} = \frac{3}{4}$

$$\implies \frac{\pi k T_1}{T_0} = \frac{3\pi k}{4}$$

$$C_0 = \frac{3A_c}{4}$$

$$C_1 = \frac{3A_c}{4}\,\frac{\sqrt{2}/2}{3\pi/4} = \frac{\sqrt{2}\,A_c}{2\pi} = C_{-1}$$

$$C_2 = \frac{3A_c}{4}\,\frac{-1}{3\pi/2} = -\frac{A_c}{2\pi} = C_{-2}$$

$$C_3 = \frac{3A_c}{4}\,\frac{\sqrt{2}/2}{9\pi/4} = \frac{\sqrt{2}\,A_c}{6\pi} = C_{-3}$$

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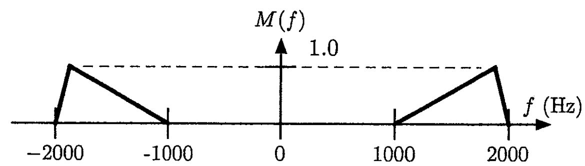
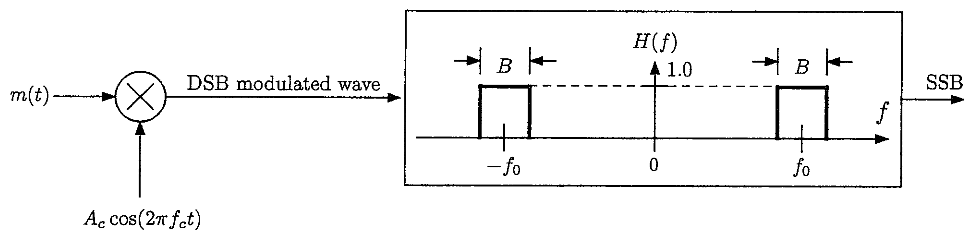
Name: ______________________

Problem 2. *SSB AM Transmitter and Coherent Receiver Using Ideal Filtering.* [30 pts. total]

We desire to create an SSB system for a baseband message $m(t)$ with a spectrum of the form shown below.



The general transmitter architecture uses sideband filtering as shown below ...



(a) [15 pts.] Assuming we want the SSB wave at the transmitter output to be lower sideband SSB, what should be chosen as the minimum value of B and the value picked for f_0. Draw a picture of the SSB spectrum carefully labeling frequency and amplitude axes.

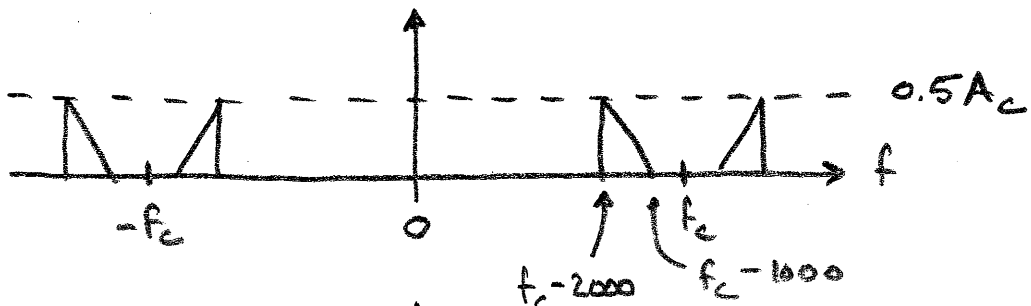
(b) [15 pts.] Draw the block diagram of the ideal coherent receiver for your SSB transmitter of part (a). Explain why it works, the values chosen for important parameters, and illustrate with spectral plots.

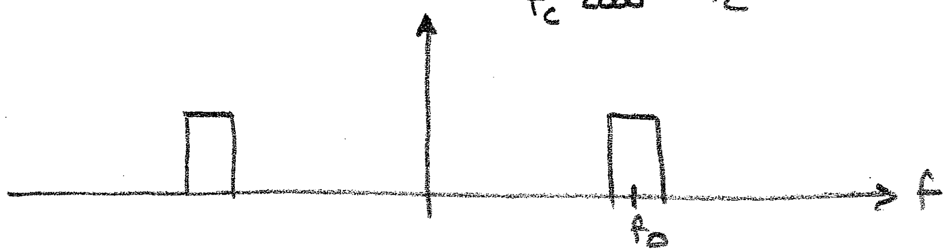
SSB AM Transmitter & Coherent Receiver Using Ideal Filtering

(a) $x_{dsb}(t) = A_c m(t) \cos 2\pi f_c t$

$\updownarrow$

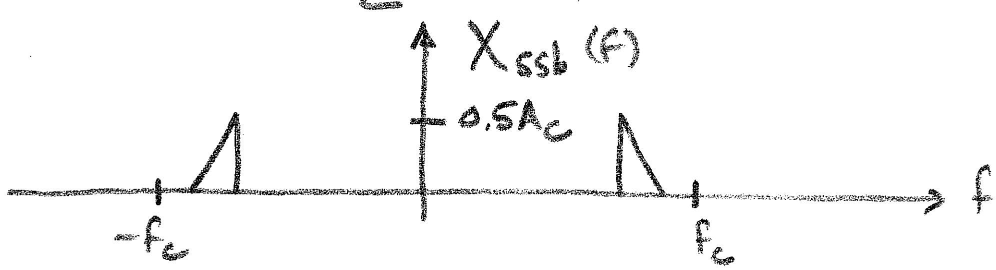
$$X_{dsb}(f) = 0.5 A_c M(f-f_c) + 0.5 A_c M(f+f_c)$$



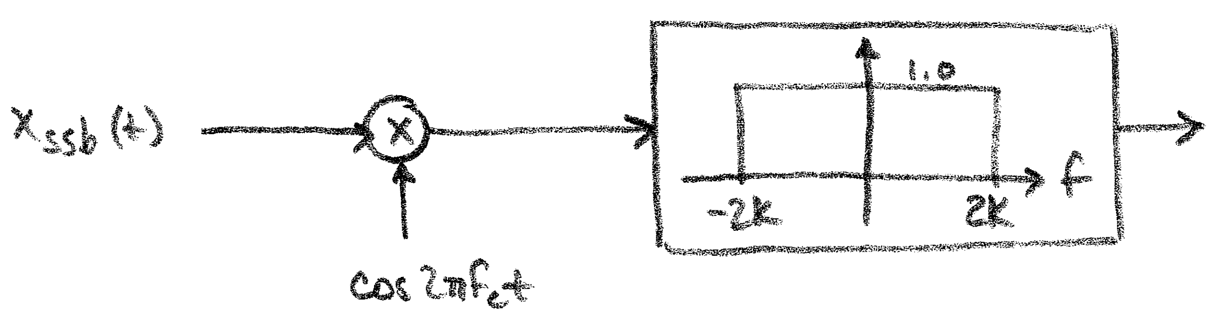


Must set B = 1000 and choose f_o in the center ...

$$f_o = \frac{f_c - 2000 + f_c - 1000}{2} = f_c - 1500 \text{ Hz}$$



(b) The coherent receiver is ...



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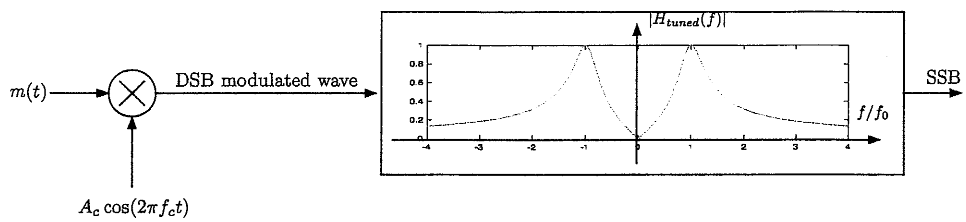
Name: ______________________

Problem 3. *SSB AM Transmitter Using Non-Ideal Filtering.* [35 pts. total]



This is an extension of Problem 2 where we consider only the SSB transmitter and the use of non-ideal lowpass and bandpass filters as illustrated generically in the above figure. The two types of filters available to us are Butterworth lowpass filters with magnitude response transfer functions of the form (n is the filter order and B is the half-power bandwidth) ...

$$|H_{butter,n}(f)| = \frac{1}{\sqrt{1 + (f/B)^{2n}}}$$

and second-order tuned circuits of transfer function $H_{tuned}(f)$ with center frequency f_0 and quality factor Q ...

$$H_{tuned}(f) = \frac{1}{1 + jQ\left(\frac{f}{f_0} - \frac{f_0}{f}\right)}.$$

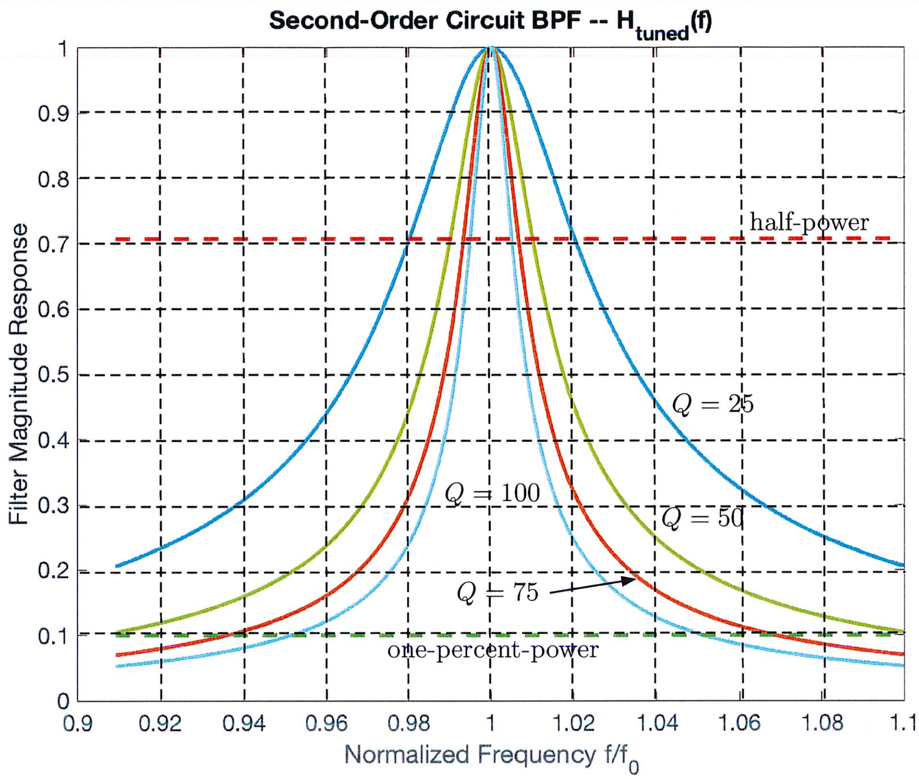
For either lowpass or bandpass we will assume that the passbands are the frequencies where the magnitude response is larger than that at the half-power points and that the rejection bands are the frequencies where the magnitude response is smaller than that at the one-percent power points. All other frequencies are transitions between passbands and rejection bands. Use the zoomed spectral plots on the following pages to answer parts (a), (b), and (c).

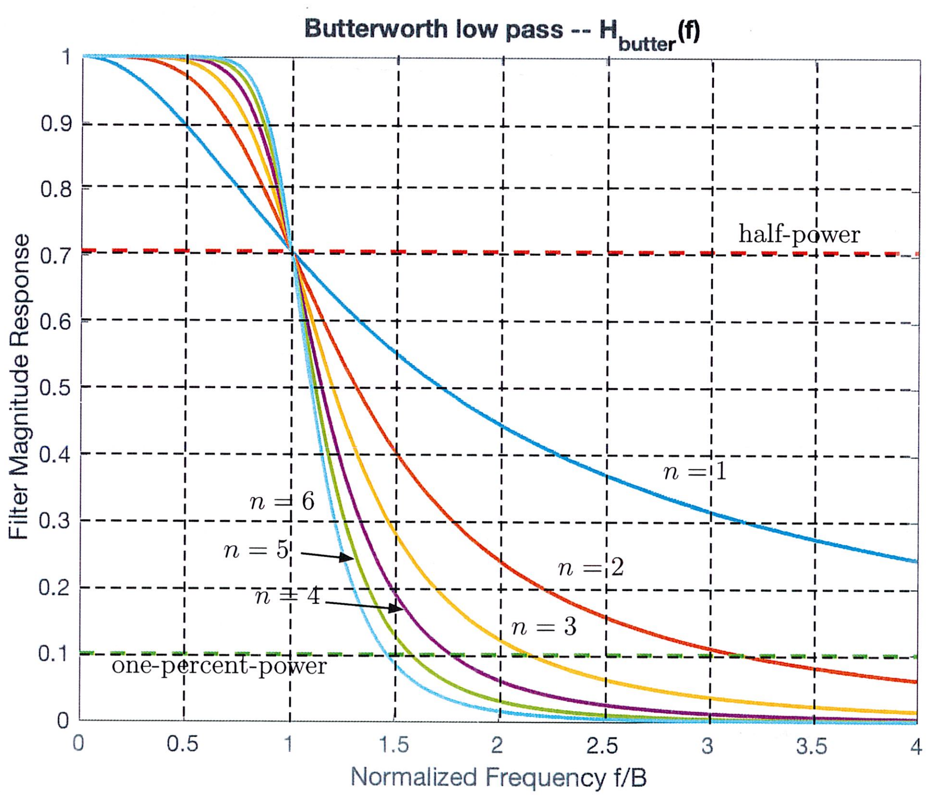
(a) [10 pts.] Is it possible to use only the tuned circuit to make an SSB-lower wave if $Q = 100$ and f_0 is chosen sufficiently small? In otherwords, can we both pass the 1000 Hz bandwidth of the lower sideband of $M(f)$ and reject the upper sideband. Explain and comment on the effect that changing Q would have.

(b) [10 pts.] Now suppose we use the Butterworth lowpass filter to separate the sidebands. What is the smallest filter order for which the lower sideband can be separated from the upper sideband assuming a small enough carrier frequency?

(c) [15 pts.] Propose a scheme to create SSB-lower on a 100 kHz carrier using multiple stages of these filters. How many stages will you need? What types of filters would you be able to use?

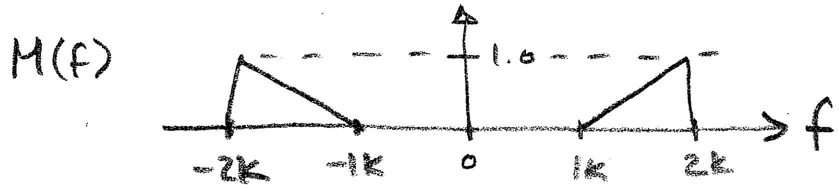
Problem 3. (cont'd.) Name: ____________________





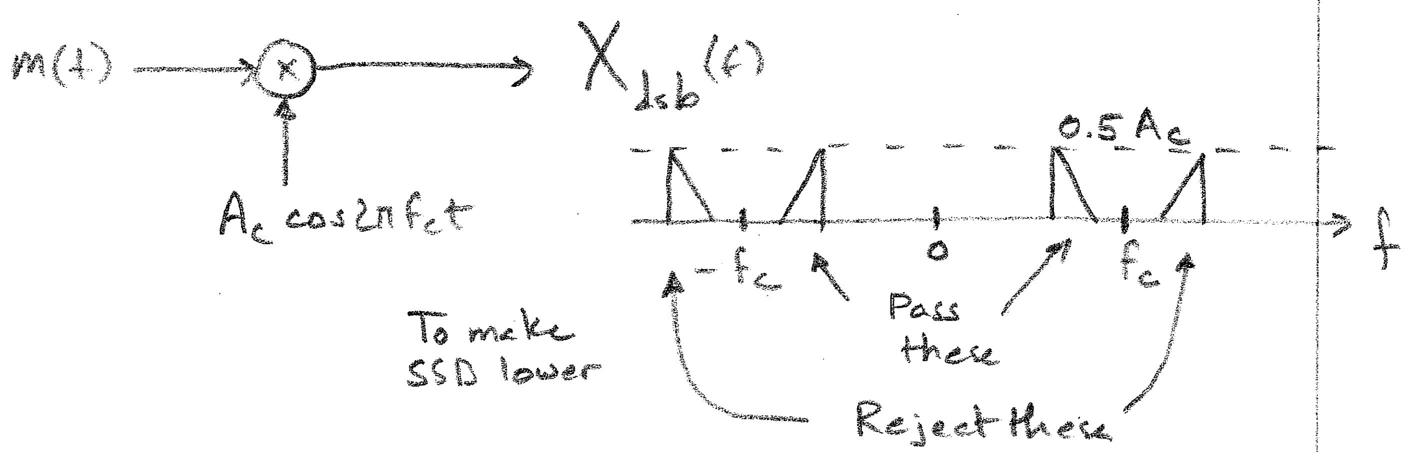
SSB AM Transmitter Using Non-Ideal Filtering

The baseband message $m(t) \longleftrightarrow M(f)$ is the same as in the previous problem...



The minimum bandwidth or width of the frequencies where $M(f) > 0$ and $f > 0$ is 1 KHz. There is also a gap where no message energy is found from dc to 1KHz.

The setup for the sideband filtering approach starts with a dsb wave



ⓐ To accomplish the above with a Q = 100 resonant second-order filter we need to center f_0 at the midpoint

$$f_0 = \frac{f_c - 1000 + f_c - 2000}{2} = f_c - 1500$$

and the passband would need to be 1000 Hz wide. From the passband edge at $f_c - 1500$ the filter must transition to rejection at $f_c + 1500$.

In normalized frequencies

passband: $\frac{f}{f_o} = 0.995$ to $\frac{f}{f_o} = 1.005$

$$BW = (1.005 - 0.995) f_o = 0.01 f_o$$

$$\Longrightarrow BW = 0.01 f_o \geq 1000$$

$$\Longrightarrow f_o \geq 100\,\text{kHz}$$

transition: $\frac{f}{f_o} = 1.005$ to $\frac{f}{f_o} = 1.05$

$$\Longrightarrow TW = (1.05 - 1.005) f_o = 0.045 f_o$$

$$TW = 0.045 f_o \leq 2000$$

$$\Longrightarrow f_o \leq \frac{2000}{0.045} = 44.4\,\text{kHz}$$

Therefore, there is no solution with the $Q = 100$ 2nd order filter.

We can solve for a TW that could work with this filter ...

$$\frac{\text{min TW}}{0.045} = 100\,\text{kHz}$$

$$\Longrightarrow \quad \text{min TW} = (100\,\text{kHz})(0.045) = 4500\,\text{Hz}$$

ie If we can increase the space between the upper and lower sideband using some other stage then 2nd-order tuned circuits could be used.

Note that increasing Q doesn't help because the ratio between passband and transition widths is independent of Q.

(b) Now we consider a Butterworth LPF as a way to pass the lower sideband and reject the upper sideband in the spectral plot on the previous page

We would need $B = f_c - 1000$ and would need to transition to rejection by $f_c + 1000$ (ie 2KHz).

From the Butterworth normalized curve the rejection bands start at

$$\frac{f}{B} = 1.4 \quad \text{if } n = 6$$
$$1.6 \quad \text{if } n = 5$$
$$1.75 \quad \text{if } n = 4$$
$$2.2 \quad \text{if } n = 3$$

call these factors α

Then

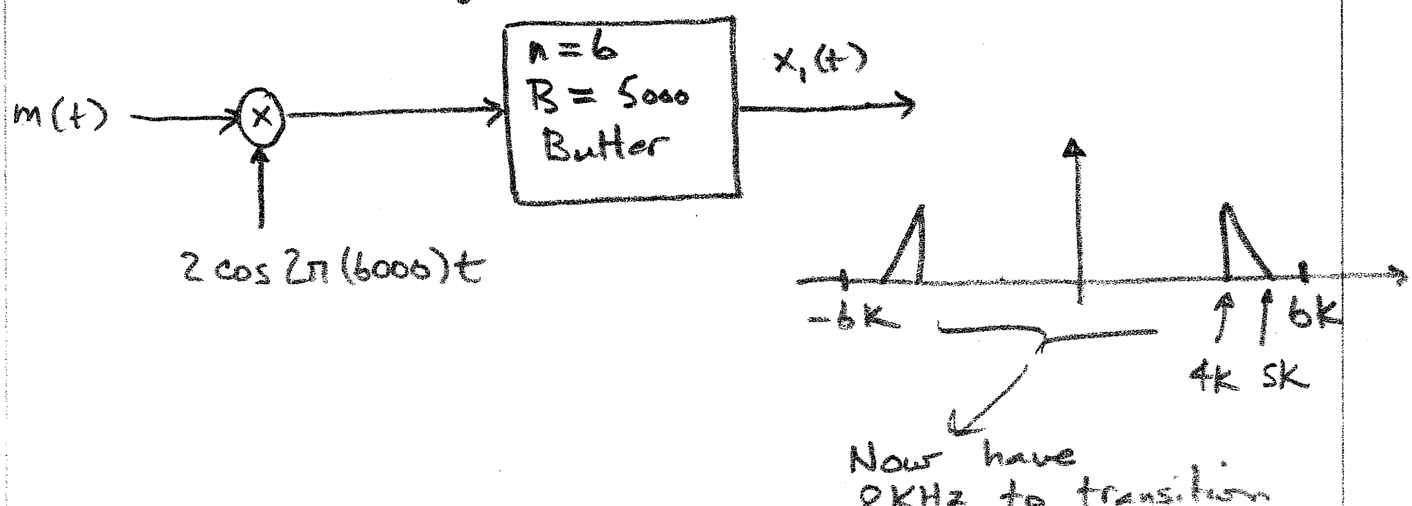
$$\text{rejection band start} = \alpha \cdot \text{bandwith}$$

$$f_c + 1000 = \alpha (f_c - 1000)$$
$$= \alpha f_c - 1000\alpha$$
$$1000(\alpha + 1) = (\alpha - 1) f_c$$
$$1000 \frac{\alpha + 1}{\alpha - 1} = f_c$$

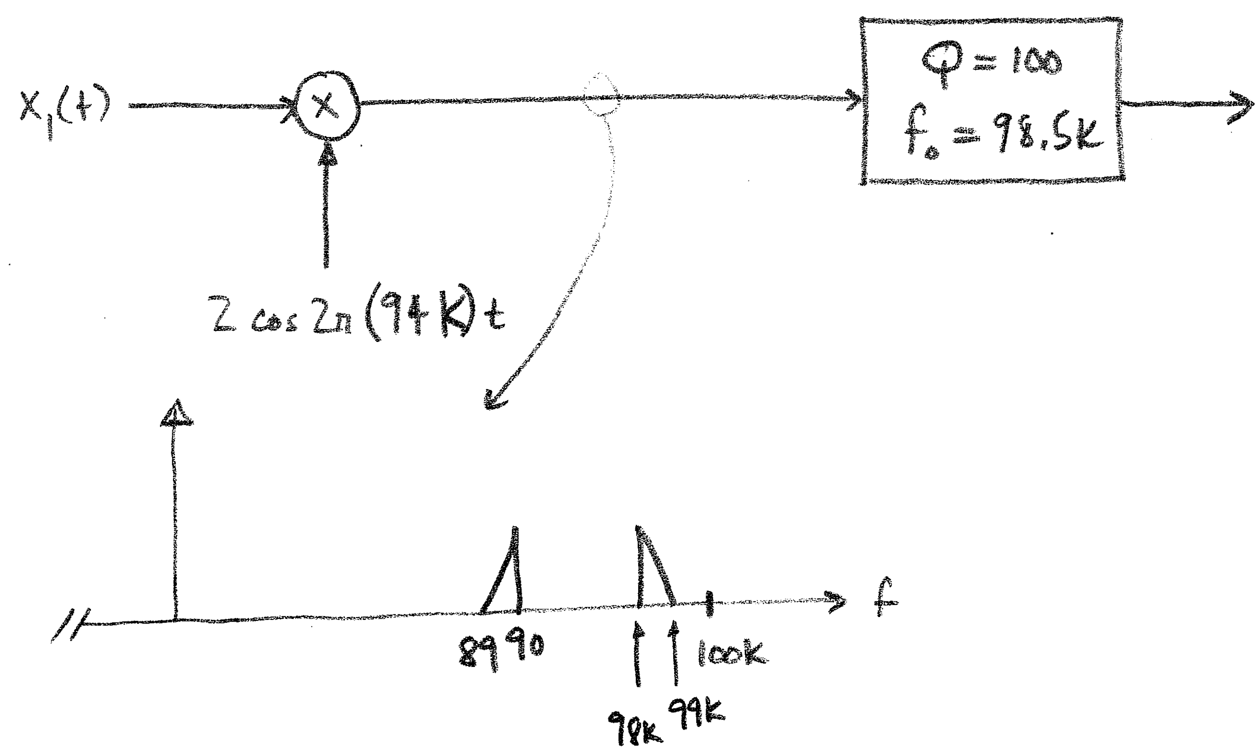
α	$\frac{\alpha+1}{\alpha-1}$	f_c
1.4	6	6000
1.6	4.33	4333
1.75	3.67	3666
2.2	2.67	2666

All of these choices would allow separating the lower and upper sidebands. However only the top 3 result in increasing the allowable transition width for a following stage.

(c) Let's use $n = 6$ Butterworth and $f_c = 6000$ Hz for the 1st stage.



Now it looks like a single additional stage could work even using a tuned circuit ...



Actually this won't quite work because the passband of the tuned is only 985 Hz, so we'd do a bit of filtering on the message

Two possible solutions

1) Lower Q a little
2) Use Butterworth.