<u>Generic F.S.</u> CT sig. $x(t) = x(t+T) \quad \forall t \in \mathbb{R}$

$$x(t) \underset{T}{\overset{FS}{\longleftrightarrow}} X_k \qquad \omega_0 = 2\pi/T$$

$$\sum_k X_k e^{jk\omega_0 t} \qquad \qquad = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$$

(synthesis) (analysis)

Come up with generic variables:

$$t \longmapsto \gamma$$

$$T \longmapsto \Gamma$$

$$x(t) \longmapsto Z(\gamma)$$

$$X_k \longmapsto z[k]$$

Then $Z(\gamma) = Z(\gamma + \Gamma) \quad \forall \gamma \in \mathbb{R}$

$$Z(\gamma) = \sum_{k=-\infty}^{\infty} z[k]\, e^{jk\frac{2\pi}{\Gamma}\gamma}$$

$$\updownarrow$$

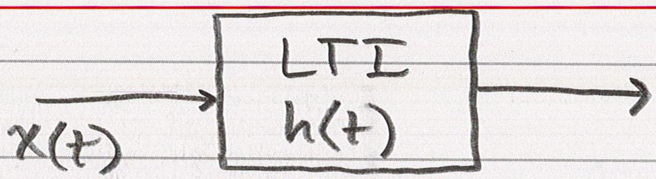
$$z[k] = \frac{1}{\Gamma}\int_{-\Gamma/2}^{\Gamma/2} Z(\gamma)\, e^{-j\frac{2\pi}{\Gamma}\gamma}\, dt.$$

when think of the $z[k]$ as a discrete time signal, then $Z(\gamma)$ can be interp as a spectrum.

This is discrete-time Fourier transform (DTFT)

[actually $\Gamma = 2\pi$].

One More Thing for F.S.



$$y(t) = \int_{-\infty}^{\infty} h(\tau) \overbrace{x(t-\tau)}^{e^{s(t-\tau)} = e^{st} e^{-s\tau}} d\tau$$

Looked $x(t) = e^{st}$ $t \in \mathbb{R}$ ... most interesting is $s = j\omega$.

If specialize for $s = j\omega$:

$$y(t) = e^{j\omega t} \underbrace{\left(\int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau \right)}_{H(\omega)}$$

Now if $x(t) = \sum_k X_k e^{jk\omega_0 t}$ and apply linearity + time invarience:

$$y(t) = \sum_k X_k e^{jk\omega_0 t} \int_{-\infty}^{\infty} h(\tau) e^{-j\omega_0 k \tau} \, d\tau$$

$H(\omega)\Big|_{\omega = k\omega_0}$

$$= \sum_k \underbrace{H(k\omega_0) X_k}_{Y_k} e^{jk\omega_0 t}$$

Leads to a method for designing the filter $h(t)$ ....

Now: Discrete Time Periodic Signals

$$x[n] = x[n+N] \quad \forall n \in \mathbb{Z}$$

(A) We observed that:

$$x[n] = e^{j\omega_0 n} \quad n \in \mathbb{Z}$$ a discrete-time complex sinusoid.

is periodic $\Longleftrightarrow$ $\omega_0 / 2\pi$ = a rational number

(B) Further if fix N and solve for all DT complex sinusoids of period N we found that there were only N such signals ...

$$\phi_\ell[n] = e^{j 2\pi \ell n / N} = \Big(\underbrace{e^{j2\pi\ell/N}}_{\text{Nth "roots of unity"}}\Big)^n$$

$\ell = 0, 1, 2 \ldots N-1$ these N signals have fundamental freqs which are all multiples of $2\pi/N$

ie they are <u>harmonically related</u>

<u>Idea of the DTFS</u> To represent a general DT periodic signal as a linear combination of the $\phi_\ell[n]$.

$$x[\cdot] = \sum_\ell a_\ell \, \phi_\ell[\cdot] \qquad ⊛$$

There are only N different $\phi_\ell[\cdot]$ in $\phi_\ell[\cdot] = \phi_{\ell+rN}[\cdot]$ so could sum over any period in ⊛

$$\sum_{\ell=0}^{N-1} \cdots \quad \text{or} \quad \sum_{\ell=2}^{N+1} \cdots$$

and the book uses

$$\sum_{\ell=\langle N \rangle}$$

to indicate this.

Fact If $x[\cdot]$ is any N-periodic sequence $\exists$ a unique set of DTFS coeffs

$a_0 \quad a_1 \quad \ldots \quad a_{N-1}$

st

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk(2\pi/N)n} \qquad \text{analysis eqN}$$

$$\Updownarrow$$

$$x[n] = \sum_{k=0}^{N-1} a_k e^{+jk(2\pi/N)n} \qquad \text{synthesis eqN}$$

Notes: ① The a_k's can be repeated to define them to also be N-periodic.

② Often write the a_k as X_k or $X[k]$

③ Proof of Fact comes down to $\displaystyle\sum_{n=0}^{N-1} e^{jk(2\pi/N)n} = \begin{cases} N & \text{if } k=0, \pm N, \ldots \\ 0 & \text{else.} \end{cases}$

If wanted to prove fact, then what want to show is

$$x[n] = \sum_{k=0}^{N-1} \left(\frac{1}{N} \sum_{m=0}^{N-1} x[m] e^{-jk(2\pi/N)m} \right) e^{+jk(2\pi/N)n}$$