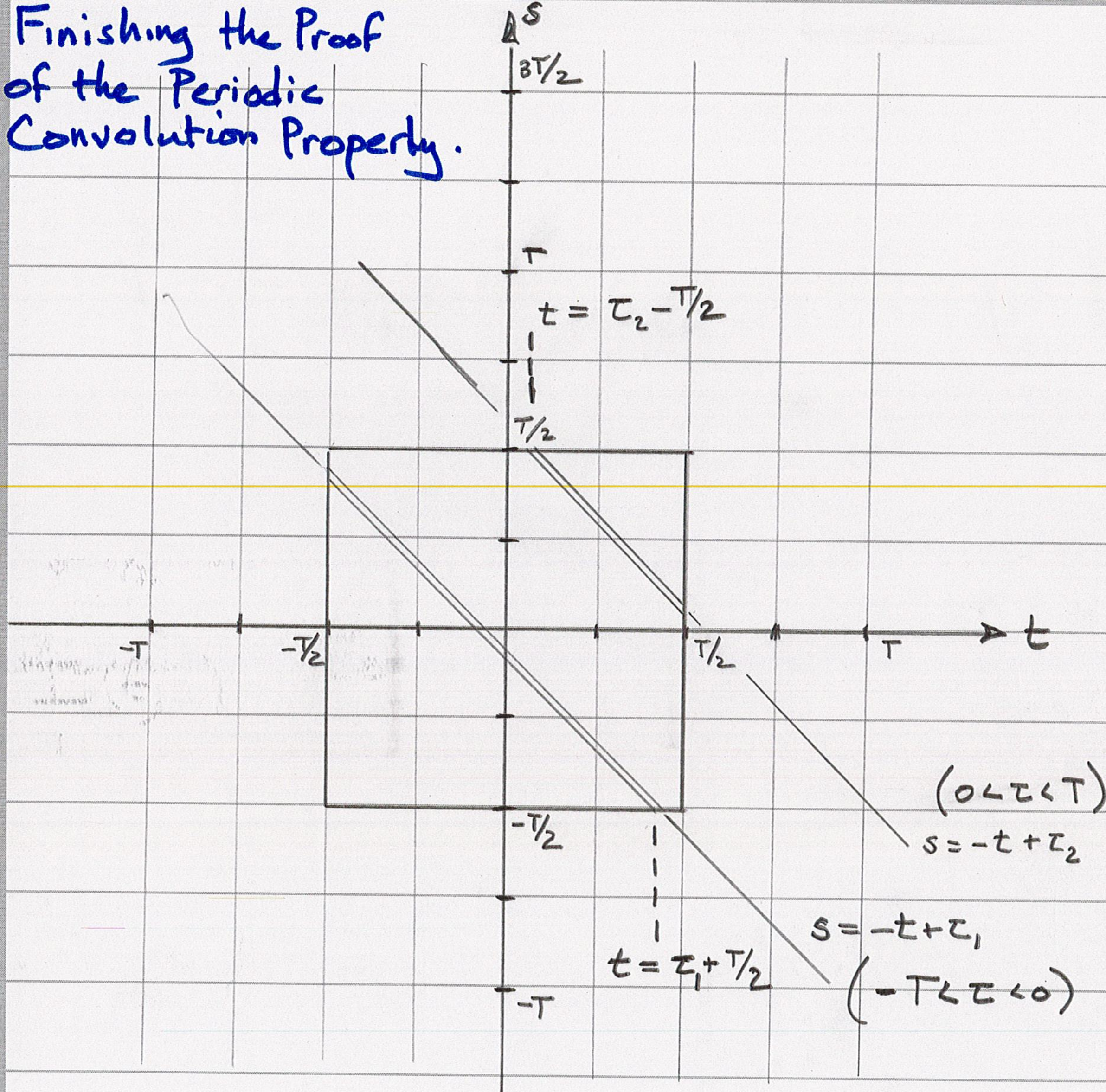
(Week 7 - Monday)

Finishing the Proof of the Periodic Convolution Property.



- Must scan τ from $-T$ to $+T$ in order that lines sweep over the whole rectangle
- The t limits will depend on τ

$$-T < \tau < 0 \rightarrow -\frac{T}{2} < t < \tau + \frac{T}{2}$$

$$0 < \tau < T \rightarrow \tau - \frac{T}{2} < t < \frac{T}{2}$$

$$T X_k Y_k = \frac{1}{T}\iint_{\text{rectangle}} I(t,s)\,dt\,ds = \underbrace{\frac{1}{T}\int_{-T}^{0}\int_{-T/2}^{\tau+T/2} I(t,\tau-t)\,dt\,d\tau}_{A_1}$$

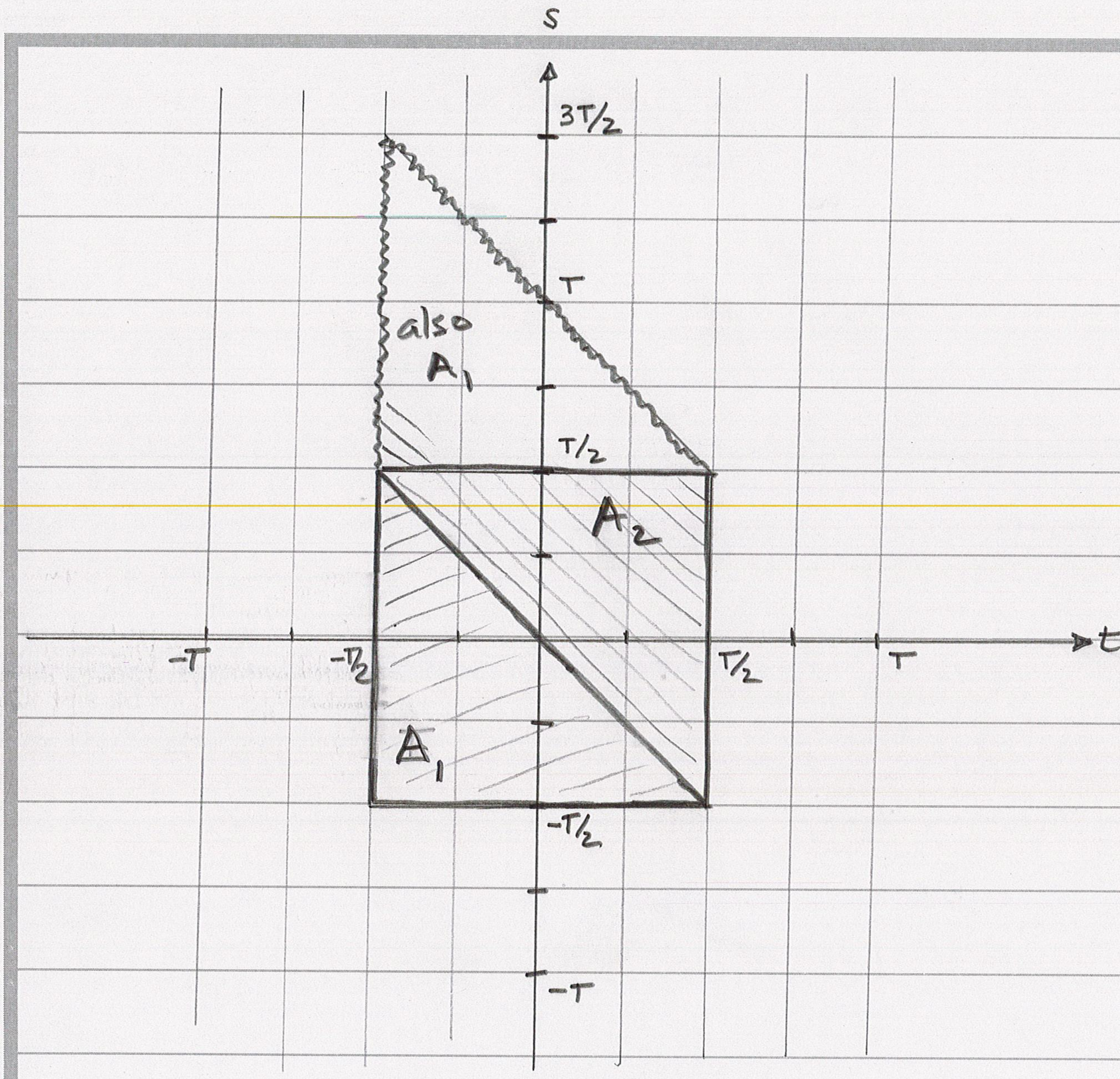
$$+ \underbrace{\frac{1}{T}\int_{0}^{T}\int_{\tau-T/2}^{T/2} I(t,\tau-t)\,dt\,d\tau}_{A_2}$$

(See the following figure and note)

$$= \underbrace{\frac{1}{T}\int_{0}^{T}\int_{-T/2}^{\tau-T/2} I(t,\tau-t)\,dt\,d\tau}_{\text{also } = A_1} + \text{above expression for } A_2$$

$$T X_k Y_k = \frac{1}{T}\int_{0}^{T}\int_{-T/2}^{T/2} I(t,\tau-t)\,dt\,d\tau$$

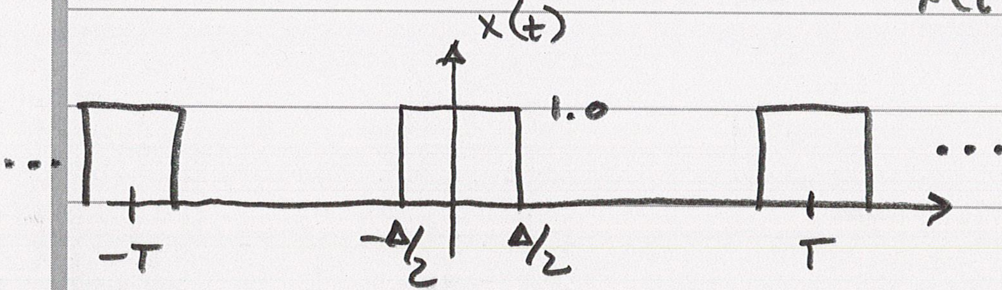
$$= \frac{1}{T}\int_{0}^{T}\left\{\int_{-T/2}^{T/2} x(t)\,y(\tau-t)\,dt\right\} e^{-j\omega_0\tau}\,d\tau$$


s
3T/2
T
T/2
-T/2
-T
t
-T
-T/2
T/2
T
also
A1
A2
A1

Periodic Pulse Trains

$$x(t) \overset{F.S}{\underset{T}{\longleftrightarrow}} X_k$$

$$x(t) = \sum_k X_k e^{jk\omega_0 t} \longleftrightarrow X_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t}\, dt$$



$\Delta \leq T$

$\frac{\Delta}{T} \triangleq$ pulse duty cycle.

$$X_k = \frac{1}{T}\int_{-\Delta/2}^{\Delta/2} 1.0\, e^{-jk\omega_0 t}\, dt \qquad \omega_0 = 2\pi/T$$

$$= \frac{1}{T}\frac{1}{-jk\omega_0} e^{-jk\omega_0 t}\Big|_{t=-\Delta/2}^{\Delta/2} = \frac{1}{T}\frac{1}{-jk\omega_0}\left[e^{-jk\omega_0\Delta/2} - e^{+jk\omega_0\Delta/2}\right]$$

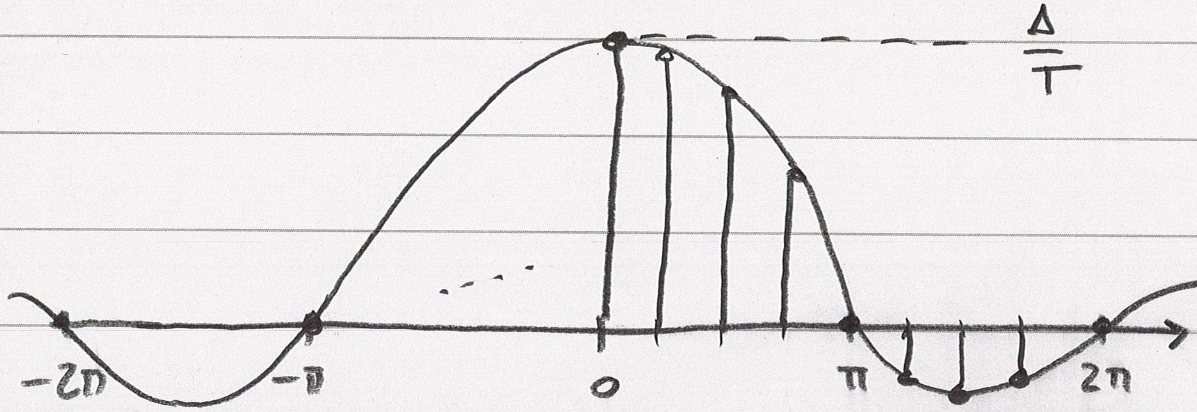
$$= \frac{1}{j2\pi k}\left[e^{jk\omega_0\Delta/2} - e^{-jk\omega_0\Delta/2}\right] = \frac{1}{\pi k}\sin\left(k\omega_0\Delta/2\right)$$

$k\frac{2\pi}{T}\cdot\frac{\Delta}{\cancel{2}}$

$k\pi\frac{\Delta}{T}$

$$= \frac{\Delta}{T}\cdot\frac{1}{\pi k\Delta/T}\sin\left(\pi k\frac{\Delta}{T}\right) = \frac{\Delta}{T}\,\frac{\sin(\pi k\Delta/T)}{(\pi k\Delta/T)}$$

scaled version of sinc.



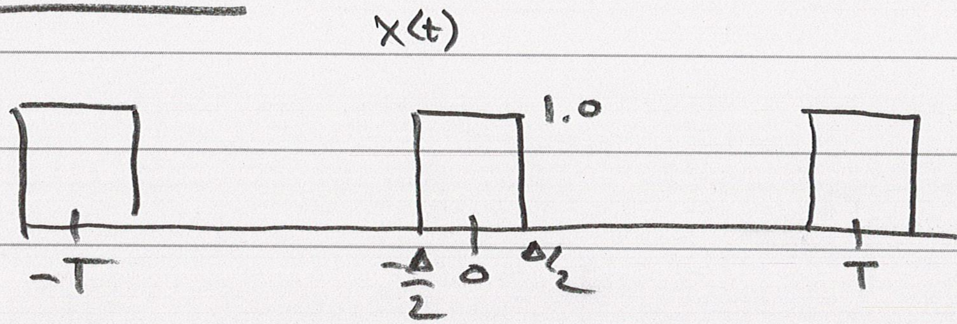
$$\frac{\sin\omega}{\omega}\begin{cases} = 1 & \omega = 0 \\ = 0 & \omega = \pi l \\ & l \neq 0. \end{cases}$$

"stemplot".

$$X_k = \frac{\Delta}{T} \cdot \frac{\sin\left(\pi k \Delta / T\right)}{\left(\pi k \Delta / T\right)} \qquad \frac{\Delta}{T} = 0.25$$

See Pulse Train Example
and the Matlab m file.

Scale amplitude of prev. pulse train s.t. area remains <u>constant</u>



$\tilde{x}(t) = \frac{1}{\Delta} x(t)$ $\longrightarrow$ Look @ behavior as $\Delta \to 0$.