(MONDAY) Summary of Week 5

Linear + Time Invariant

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(DT) $x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k] \implies y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$

impulse; impulse response

(CT) $x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau \implies y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$

Recall the conv. notation

$x * y$

Computational Examples

Properties of LTI systems

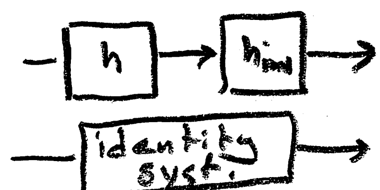
$x * h = h * x$

$x * (h_1 + h_2) = (x * h_1) + (x * h_2)$

$x * y * z = (x * y) * z = x * (y * z)$

LTI memoryless $\iff$ $h[n] = K\delta[n]$; $h(t) = K\delta(t)$

LTI h invertible $\iff$ $\exists\, h_{inv}$ st $h * h_{inv} = \delta$

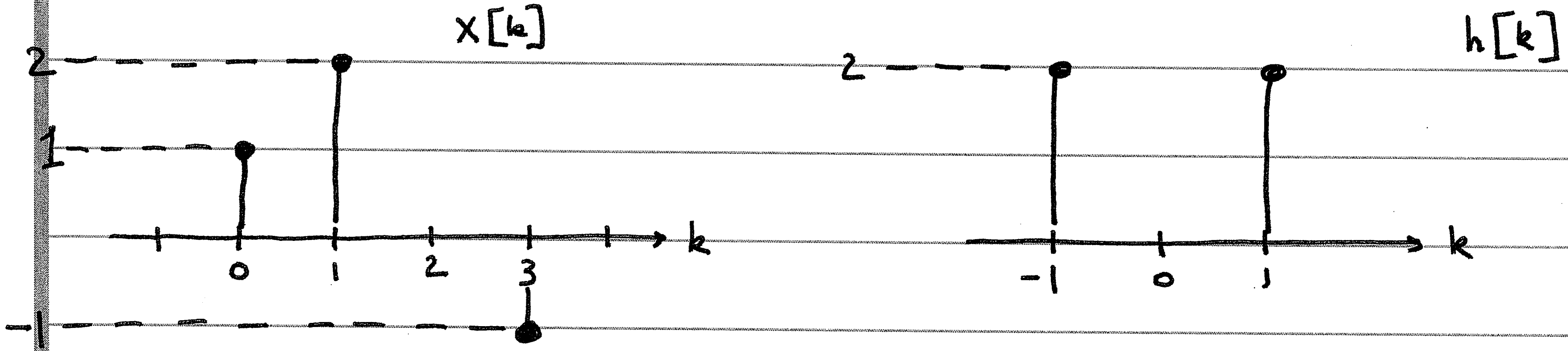


LTI causal $\iff$ $h[n] = 0$ for $n < 0$; $h(t) = 0$ for $t < 0$

Stability means: Bounded inputs result in bounded outputs

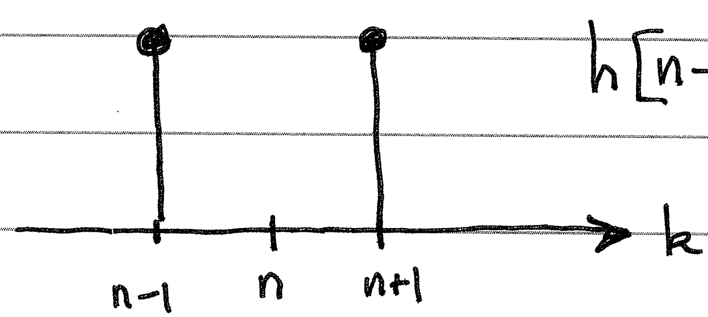
BIBO $\iff$ $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$; $\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$

$x * h$ for $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$
$h[n] = 2\delta[n+1] + 2\delta[n-1]$.



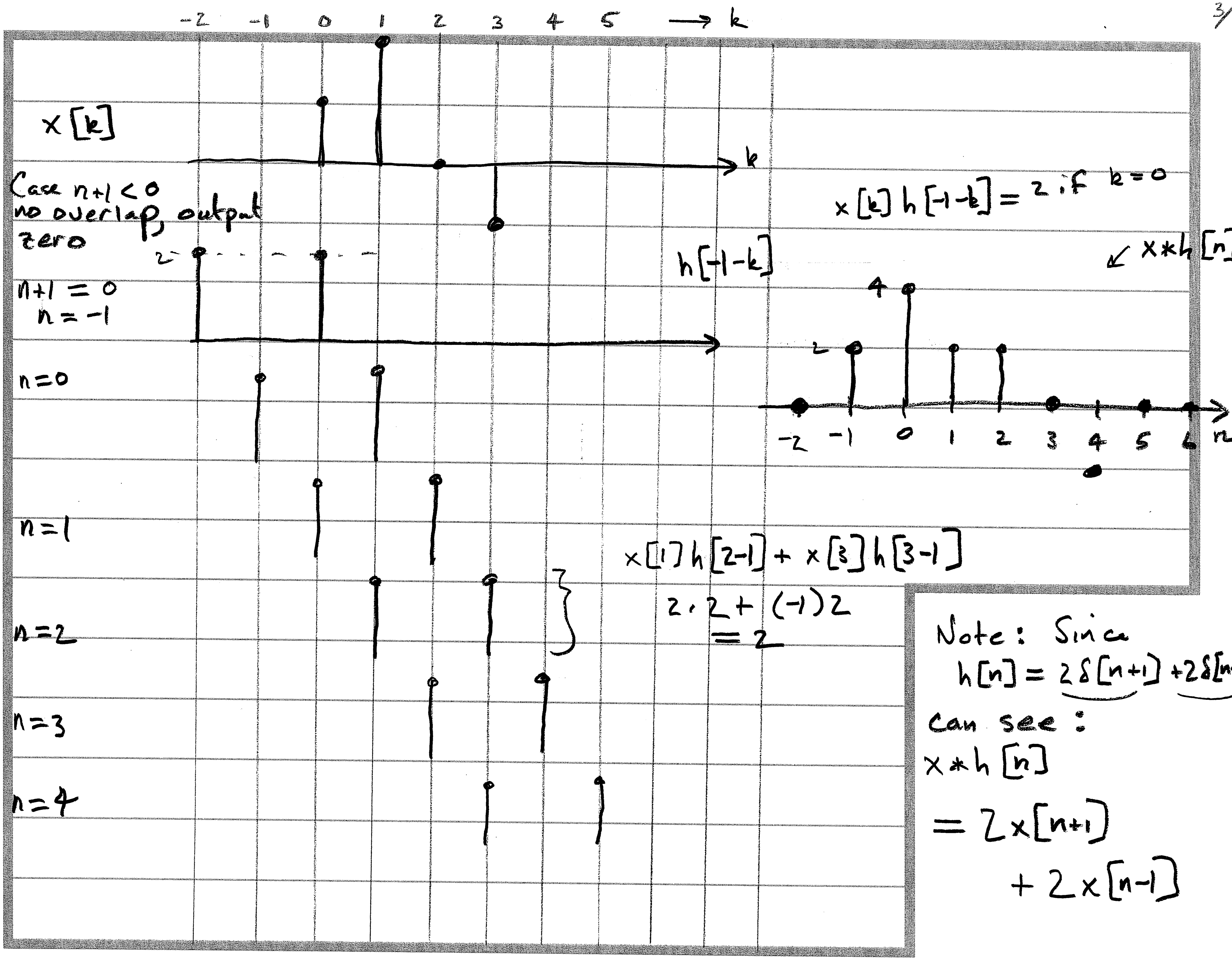
$$y[n] = x*h[n] = \sum_k x[k]\,h[n-k]$$

Look at the shifted and time-reversed (wrt k) impulse response



$h[n-k]$ considered as function of k

Must consider several cases.



Note: Since

$h[n] = 2\delta[n+1] + 2\delta[n-1]$

can see:

$x*h[n]$

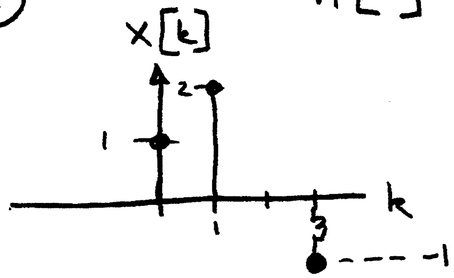
$= 2x[n+1]$

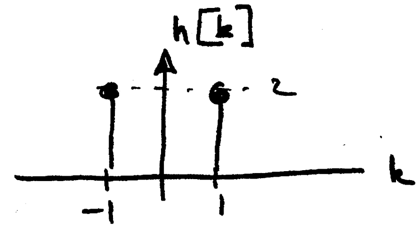
$+ 2x[n-1]$

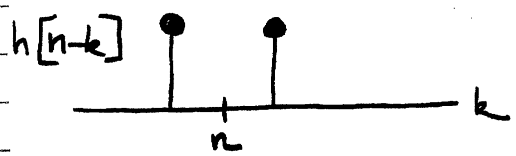
(Neater Version of Prev. Example)

O+W 2.1 $\quad x[n] = \delta[n] + 2\,\delta[n-1] - \delta[n-3]$

(a) $\quad h[n] = 2\,\delta[n+1] + 2\,\delta[n-1]$





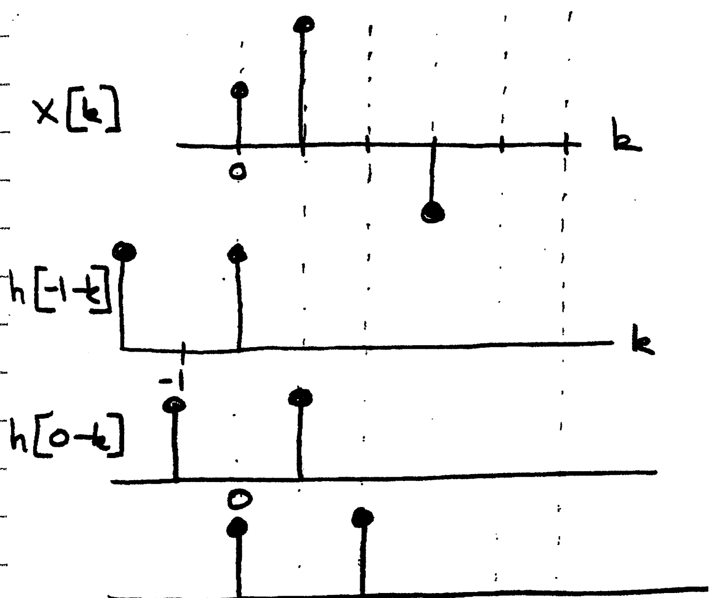


$\Longrightarrow$ several cases to consider

Case $\quad n+1 < 0 \iff n < -1 \implies$ no overlap so $x * h[n] = 0$

Case $\quad n-1 > 3 \iff n > 4 \implies$ " " " "

Case $\quad -1 \le n \le 4 \quad$ overlap can happen



$h * x[-1] = 2 \cdot 1 = 2$

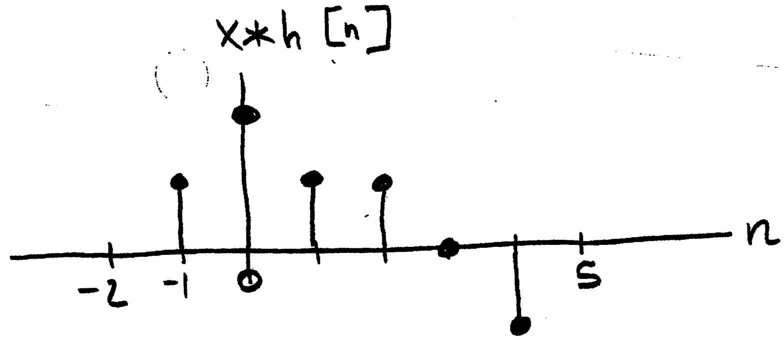
$h * x[0] = 2 \cdot 2 = 4$

$h * x[1] = 2 \cdot 1 = 2$

$h * x[2] = 2 \cdot 2 - 2 \cdot 1 = 2$

$h * x[3] = 0$

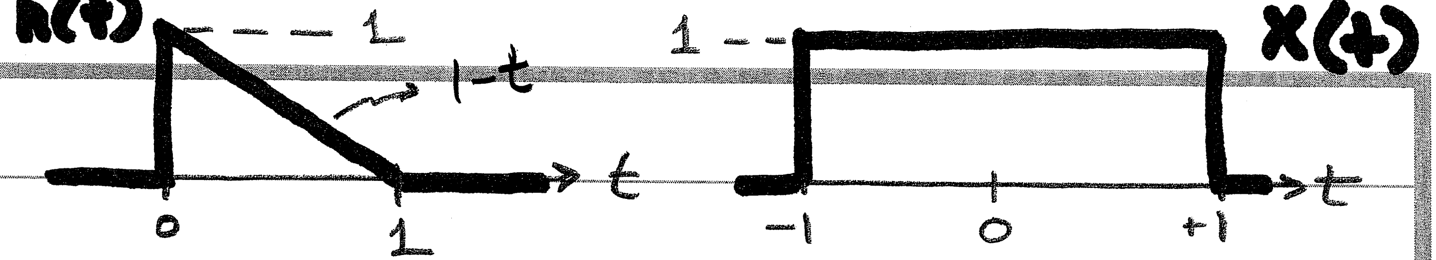
$h * x[4] = 2 \cdot (-1) = -2$

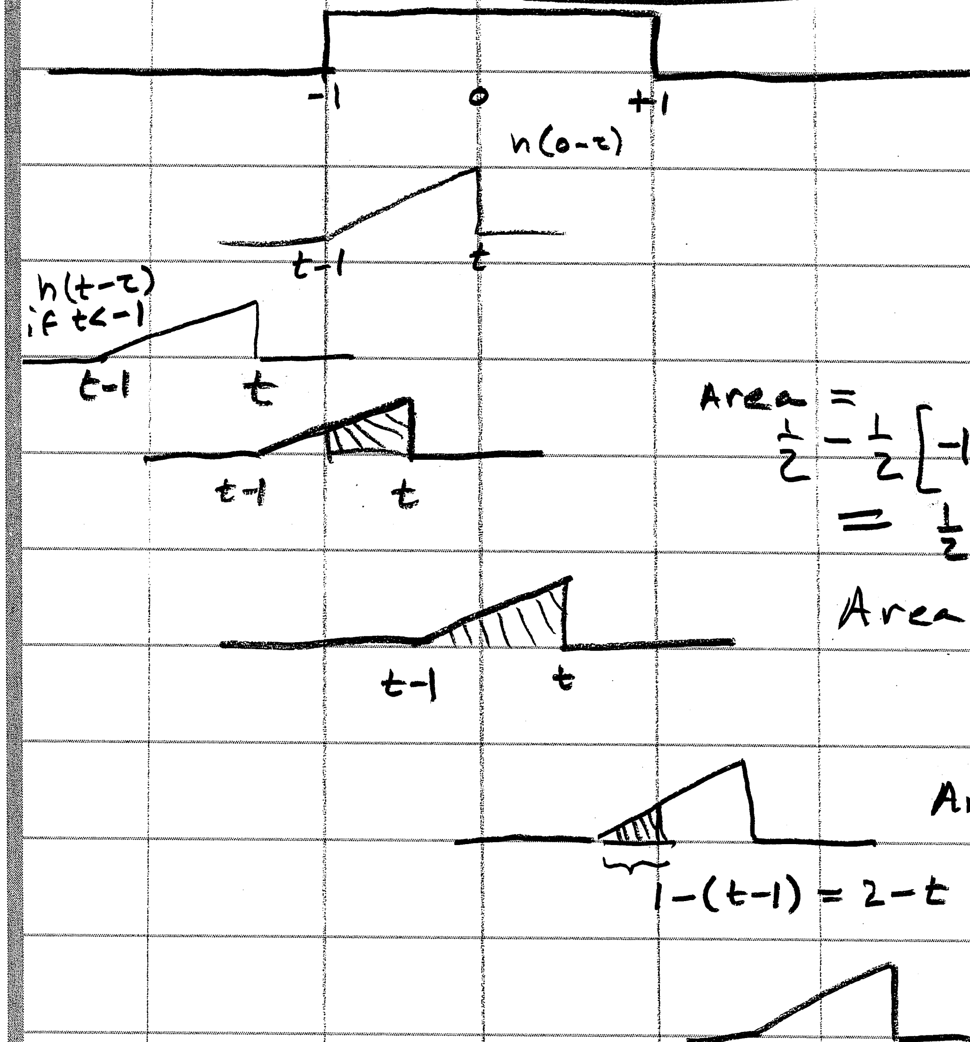


<u>Notice</u>

$$x*h[n] = 2x[n+1] + 2x[n-1]$$

x*h for





$\rightarrow\ h*x(t) = 0$ if $t<-1$

Area $= \frac{1}{2} - \frac{1}{2}\left[-1-(t-1)\right]^2$

$= \frac{1}{2}(1-t^2)$

$-1<t<0 \quad \int_{-1}^{t}(1-t+\tau)\,d\tau = \frac{1}{2}(1-t^2)$

Area $= 1/2$

Area $= \frac{1}{2}(2-t)^2$

$\underbrace{\quad}_{1-(t-1)\,=\,2-t}$

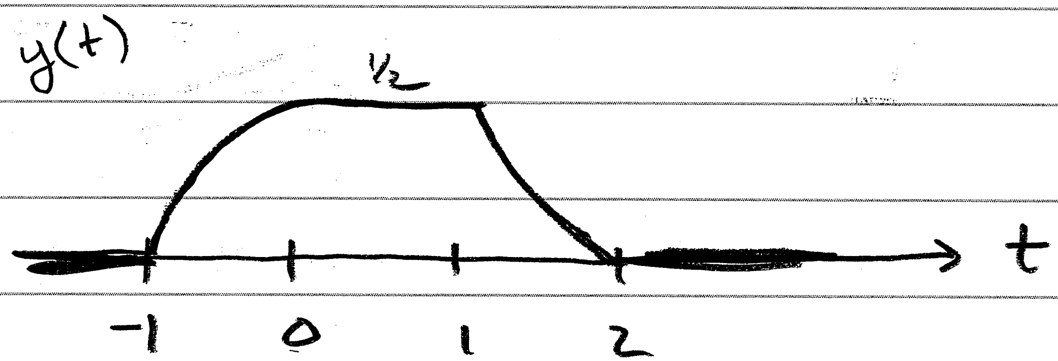
$h*x(t) = \int x(\tau)\,h(t-\tau)\,d\tau$

$h(t) = 1-t \quad 0<t<1$

$h(t-\tau) = 1-(t-\tau)$

$= 1-t+\tau$

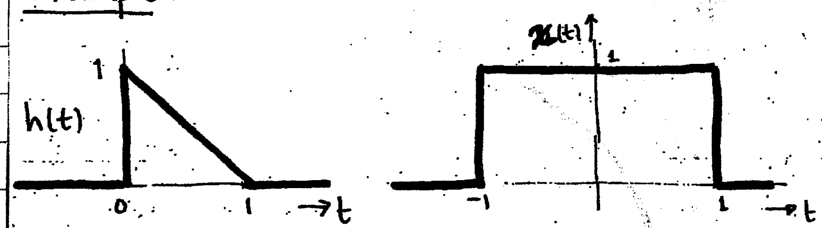
$$y(t) = x * h(t) = \begin{cases} 0 & t < -1 \\ \frac{1}{2}(1-t^2) & -1 < t < 0 \\ \frac{1}{2} & 0 < t < 1 \\ \frac{1}{2}(2-t)^2 & 1 < t < 2 \\ 0 & t > 2 \end{cases}$$



(Neater Version of Prev.)

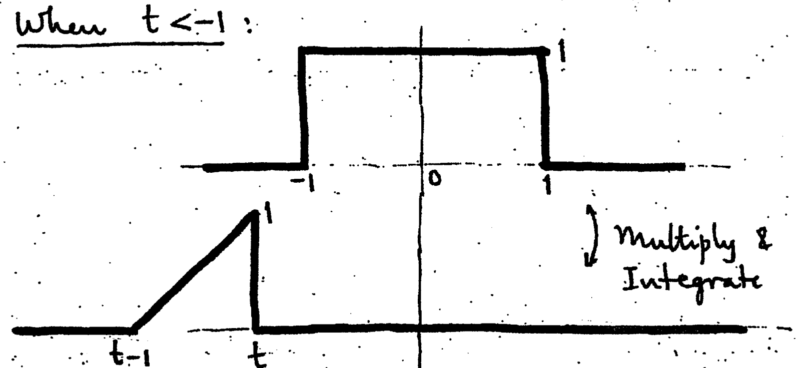
$\sim \quad y(t) = x * h(t)$

Example:



$\boxed{\text{Find } y(t)}$

When $t < -1$:



Multiply & Integrate

$y(t) = 0$

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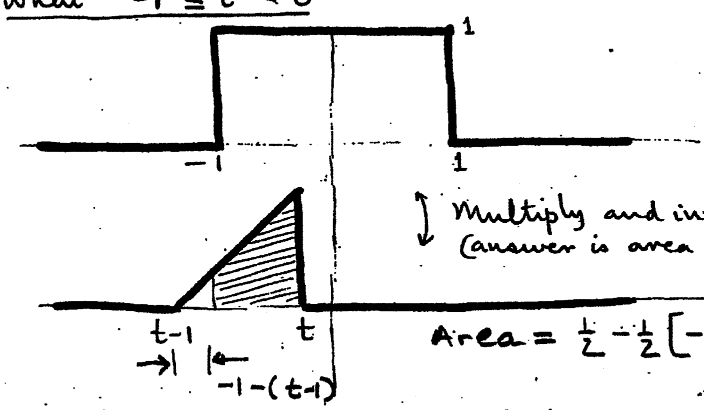
When $-1 \le t < 0$

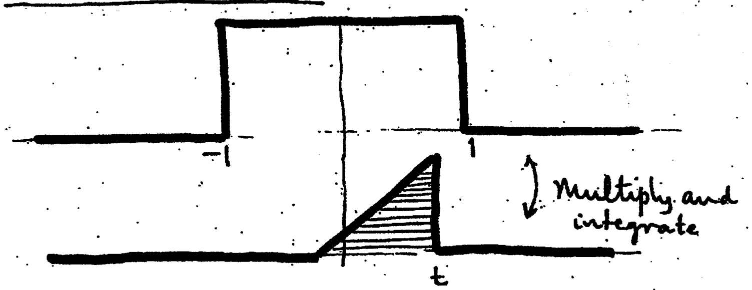


Multiply and integrate (answer is area shaded)

$\text{Area} = \frac{1}{2} - \frac{1}{2}\left[-1-(t-1)\right]^2$

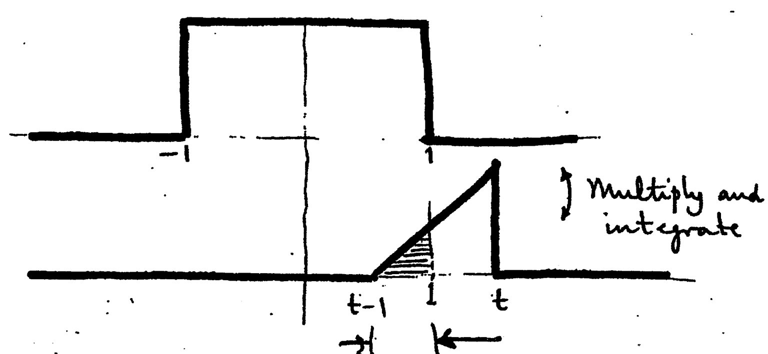
$y(t) = \frac{1}{2}(1-t^2)$

When $0 \le t < 1$



$y(t) = \frac{1}{2}$

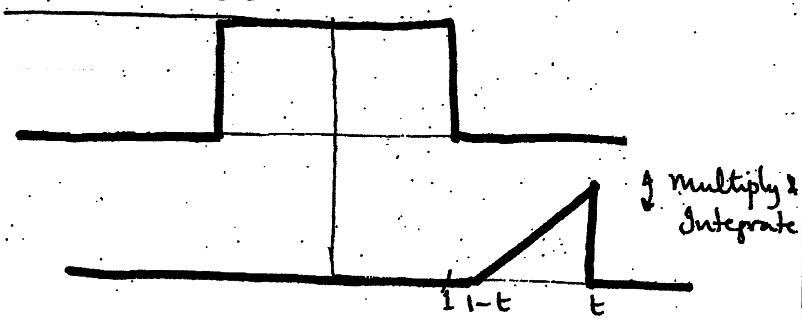
When $1 \le t < 2$



$y(t) = \frac{1}{2}(2-t)^2$

$1-(t-1) = 2-t$

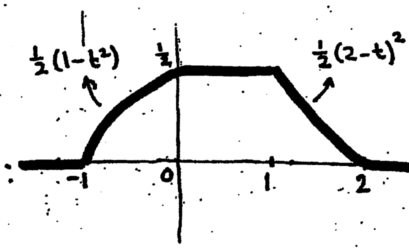
When $2 \le t$



$\underline{y(t) = 0}$

Thus,

$y(t)$ looks like this:



$$y(t) = \begin{cases} 0 & t < -1 \\ \frac{1}{2}(1-t^2) & -1 < t < 0 \\ 1/2 & 0 < t < 1 \\ \frac{1}{2}(2-t)^2 & 1 \le t < 2 \end{cases}$$