Summary Week 2 Main Lect.

• Transformation of time index

+ time shift $x(t-t_0)$ delay or advance
+ time reversal $x(-t)$

} work for DT sigs.

+ time scaling $x(\alpha t)$
+ gen. time transfs. $x(2-3t)$ etc.

} be careful with DT sigs.

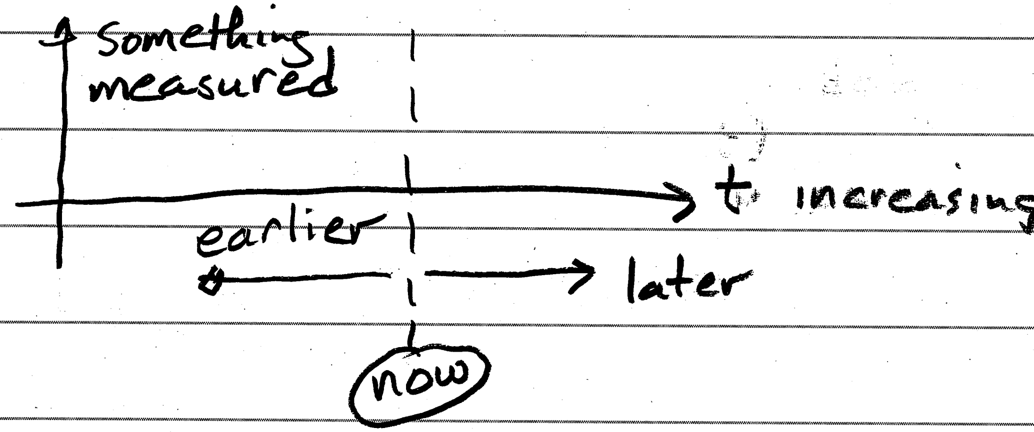
• Periodic Signals

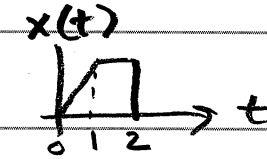
+ $x(t) = x(t+T)$ $\forall t \in \mathbb{R}$
+ $x[n] = x[n+N]$ $\forall n \in \mathbb{Z}$
+ def of the fundamental period

• Even, Odd Signals

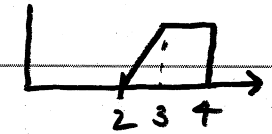
$$x(t) = \underbrace{\left(\frac{x(t)+x(-t)}{2}\right)}_{x_{even}(t)} + \underbrace{\left(\frac{x(t)-x(-t)}{2}\right)}_{x_{odd}(t)}$$

Alt. Illustration: Time Delay / Advance

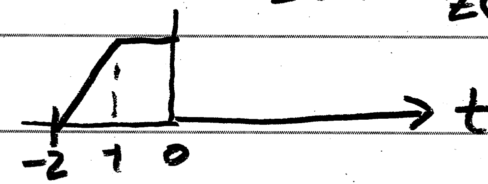




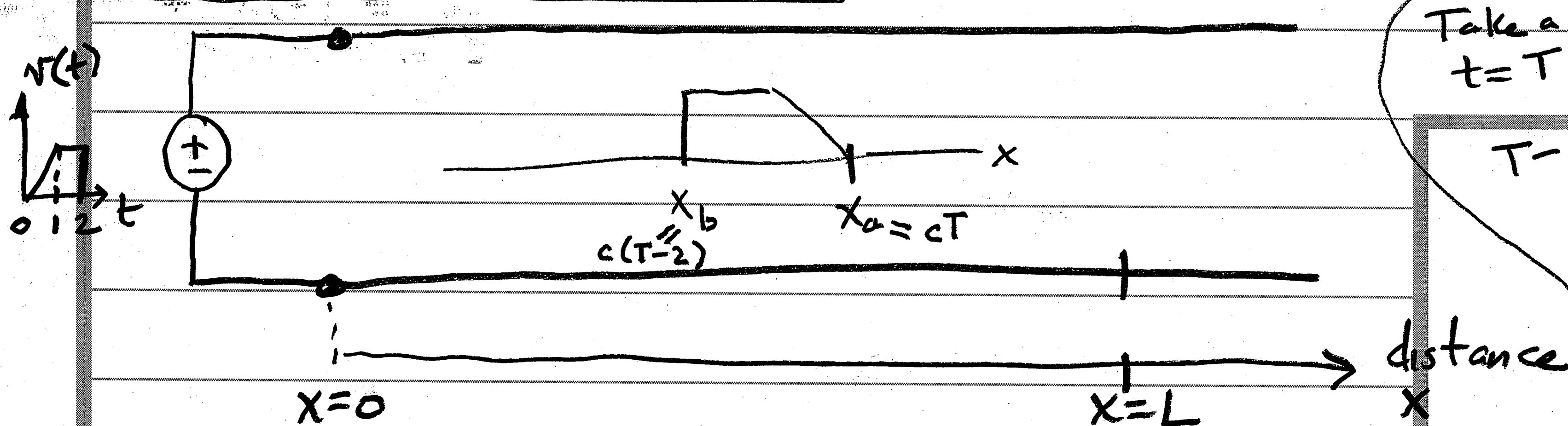
$y(t) = x(t-2)$



$z(t) = x(t+2)$



Transmission Line Example



$$w(t,x) = v(t - x/c)$$

space time wave

c = speed of wave.

Take a snapshot: $t = T$ snap.

$$T - \frac{x_a}{c} = 0$$

$$cT = x_a$$

$$T - \frac{x_b}{c} = 2$$

$$c(T-2) = x_b$$

To ~~compute the~~ wave prop speed on the trans line

$$\underbrace{t - \frac{x}{c}}_{} = \boxed{K}$$

$$\frac{d}{dt}(\downarrow) \Rightarrow 1 - \frac{\dot{x}}{c} = 0$$

$$\dot{x} = c > 0.$$

Exponential/Sinusoidal CT case

CT case general $\quad x(t) = Ce^{at} \quad t \in \mathbb{R} \quad C, a \in \mathbb{C}$

Special cases

- real-exponential signal $\quad C, a \in \mathbb{R}$
- complex sinusoid $\quad C \in \mathbb{C},\ a = j\omega$ (a pure imag. say) $\omega > 0$

$$x(t) = Ce^{j\omega t} = \underbrace{Ae^{j\phi}}_{C} e^{j\omega t} = Ae^{j(\omega t + \phi)}$$

periodic of period $T = \frac{2\pi}{\omega}$

Euler's Identity $\quad e^{j\theta} = \cos\theta + j\sin\theta$

Note

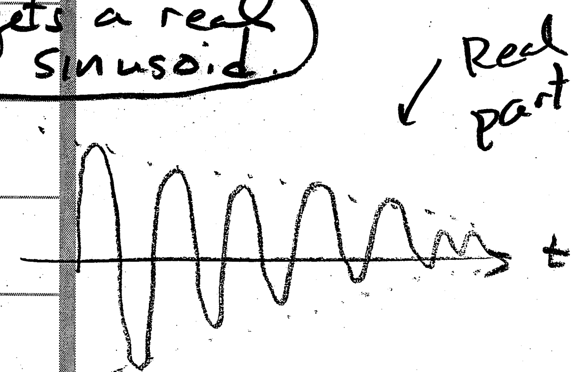
Paul Nahin (Dr. Euler's Fabulous Formula)

$$Ce^{j\omega t} + C^{*}e^{-j\omega t}$$ gets a real sinusoid.

- Gen. case $\quad C = Ae^{j\phi},\ a = r + j\omega$

$$x(t) = Ae^{rt}e^{j(\omega t + \phi)} \qquad t \in \mathbb{R}$$

Signals are form of geneal soln of natural respone of underdamped RLC circuit



Exponential / Sinusoidal DT case

$$x[n] = C\alpha^n \qquad C, \alpha \in \mathbb{C} \qquad n \in \mathbb{Z}$$

- real case $\Longrightarrow$ geometric sequence
- complex sinusoid $\quad C = Ae^{j\phi} \quad \alpha = e^{j\omega_0} \quad x[n] = Ae^{j\phi}e^{j\omega_0 n}$
- gen. case $\quad C = Ae^{j\phi} \quad \alpha = |\alpha|e^{j\omega_0}$

$$x[n] = Ae^{j\phi}|\alpha|^n e^{j\omega_0 n}$$

$$= A|\alpha|^n e^{j(\omega_0 n + \phi)}$$

$$= A|\alpha|^n \cos(\omega_0 n + \phi) + jA|\alpha|^n \sin(\omega_0 n + \phi)$$

Periodicity Props of $x[n] = e^{j\omega_0 n}$ $n \in \mathbb{Z}$

Two "periodicities"

① Alway periodic in "freq" ω_0

$$e^{j\omega_0 n} = e^{j(\omega_0 + 2\pi)n} \quad \forall n \in \mathbb{Z}$$

"low freqs"

$\omega_0 \approx 0$ or $\approx 2\pi$

"high freqs"

$\omega_0 \approx \pi$

② Periodic in Time $n \in \mathbb{Z}$

$$e^{j\omega_0 n} = e^{j\omega_0 (n+N)} \quad \forall n \in \mathbb{Z}$$

$$= e^{j\omega_0 n} \underbrace{e^{j\omega_0 N}}_{1}$$

is there a solution N ?

Solve

$$1 = e^{j\omega_0 N}$$

$$\omega_0 N = 2\pi k \quad \text{some } k$$

$$\frac{\omega_0}{2\pi} = \frac{k}{N}$$

$$\omega_0 = \frac{k}{N} \cdot 2\pi \qquad k = 0, 1, 2, \ldots N-1$$