Z-transform : $X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$ $\quad z = re^{j\omega}$

$$= \sum_{n=-\infty}^{\infty} x[n]\, r^{-n} e^{-j\omega n}$$

ie the Z transform can be viewed as the DTFT of

$x[n]\, r^{-n}$

evaluated at ω.

So the inversion formula for Z transform can be gotten from inverse DTFT

$$x[n]\, r^{-n} = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(re^{j\omega})\, e^{+j\omega n}\, d\omega$$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(re^{j\omega}) (re^{j\omega})^{n}\, d\omega$$

Since we want to write this as a contour integral

$z = re^{j\omega}$

$dz = jre^{j\omega}\, d\omega$

$\Rightarrow d\omega = \frac{1}{j} (re^{j\omega})^{-1} dz$

Then

$$x[n] = \frac{1}{j2\pi} \int_{\substack{z:\, z = re^{j\omega} \\ r \in \text{ROC}}} X(z)\, z^{n-1}\, dz = \frac{1}{j2\pi} \oint_C X(z)\, z^{n-1}\, dz$$

As with L.T. inversion, above leads a complex analysis & Cauchy's Theorem.

↳ outside of 301 scope so we resort to partial fraction expansion + tables.

$$X(z) = \sum_{i=1}^{m} \frac{A_i}{1 - a_i z^{-1}}$$

(simple case where poles are simple)

Take the given ROC for $X(z)$ and use to infer ROCs assoc. with each term...

For ith term $|z| > |a_i|$ $\quad A_i a_i^n u[n]$

or

$|z| < |a_i|$ $\quad -A_i a_i^n u[-n-1]$.

$\boxed{\text{O+W Ex 10.9—11}}$

$$X(z) = \frac{3 - \frac{5}{6}z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)} = \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{2}{1-\frac{1}{3}z^{-1}}$$

1 ROC : $|z| > 1/3$ $\quad x[n] = \left(\frac{1}{4}\right)^n u[n] + 2\left(\frac{1}{3}\right)^n u[n]$

2 ROC : $\frac{1}{4} < |z| < \frac{1}{3}$ $\quad x[n] = \left(\frac{1}{4}\right)^n u[n] - 2\left(\frac{1}{3}\right)^n u[-n-1]$

3 ROC : $|z| < \frac{1}{4}$ $\quad x[n] = -\left(\frac{1}{4}\right)^n u[-n-1] - 2\left(\frac{1}{3}\right)^n u[-n-1]$