Bilateral Z-Transform

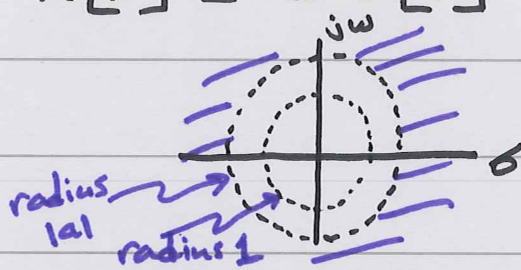
$x[n]$ a D.T. signal

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

for complex $z = re^{j\omega}$ where sum converges absolutely (the R.o.C.)

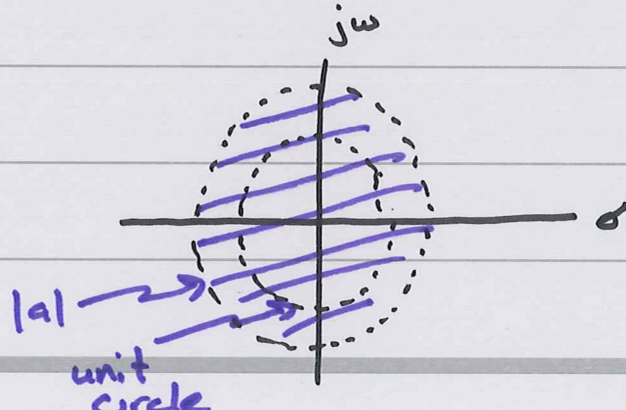
Ex 1 $\quad x[n] = a^n u[n] \longleftrightarrow X(z) = \dfrac{1}{1 - az^{-1}}$

$$\text{Roc} = \{ z \in \mathbb{C} : |z| > |a| \}$$



Ex 2 $\quad x[n] = -a^n u[-n-1] \longleftrightarrow X(z) = \dfrac{1}{1 - az^{-1}}$

$$\text{Roc} = \{ z \in \mathbb{C} : |z| < |a| \}.$$



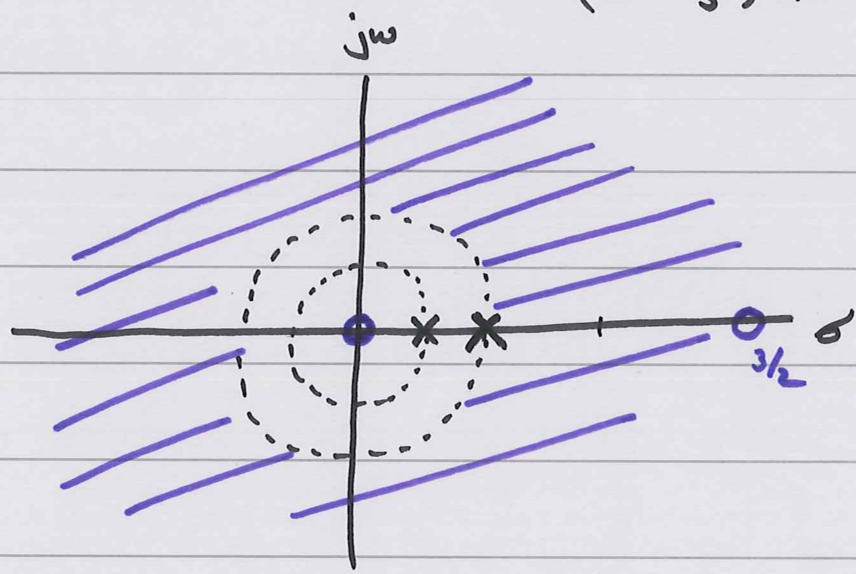
Ex 10.3 $\quad x[n] = 7\left(\frac{1}{3}\right)^n u[n] - 6\left(\frac{1}{2}\right)^n u[n]$

$$\updownarrow$$

$$X(z) = \frac{7}{1-\frac{1}{3}z^{-1}} - \frac{6}{1-\frac{1}{2}z^{-1}}$$

$$= \frac{7\left(1-\frac{1}{2}z^{-1}\right) - 6\left(1-\frac{1}{3}z^{-1}\right)}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)} = \frac{1-\frac{3}{2}z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)}$$

$$= \frac{z\left(z-\frac{3}{2}\right)}{\left(z-\frac{1}{3}\right)\left(z-\frac{1}{2}\right)}$$



$$ROC = \left\{|z| > \tfrac{1}{3}\right\} \cap \left\{|z| > \tfrac{1}{2}\right\}$$

$$= \left\{|z| > \tfrac{1}{2}\right\}$$

Ex 10.4

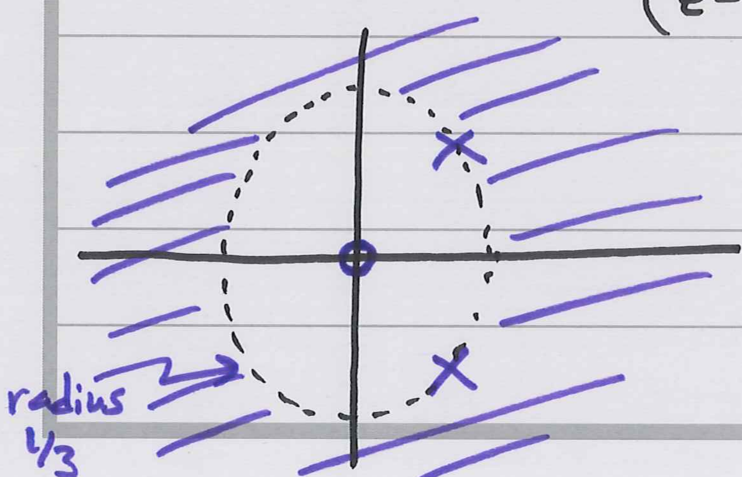
$$x[n] = \left(\frac{1}{3}\right)^n \sin\left(\frac{\pi}{4}n\right) u[n]$$

$$\frac{e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}}{j2}$$

$$= \left\{ \frac{1}{j2}\left(\frac{1}{3}e^{j\pi/4}\right)^n - \frac{1}{j2}\left(\frac{1}{3}e^{-j\frac{\pi}{4}}\right)^n \right\} u[n]$$

$$= \frac{1/j2}{1 - \frac{1}{3}e^{j\frac{\pi}{4}}z^{-1}} - \frac{1/j2}{1 - \frac{1}{3}e^{-j\pi/4}z^{-1}}$$

$$= \frac{\frac{1}{3\sqrt{2}}\ z}{\left(z - \frac{1}{3}e^{j\pi/4}\right)\left(z - \frac{1}{3}e^{-j\pi/4}\right)}$$



Properties of ROCs

	Laplace Transform	Z-Transform
1	ROCs are strips parallel to $j\omega$ axis.	ROCs are annular regions centered at origin
2	ROC does not contain any poles.	Ditto.
3	$x(t)$ of finite duration and abs. integrable $\Rightarrow$ ROC $\supseteq \mathbb{C}$	$x[n]$ of finite duration $\Rightarrow$ ROC $\supseteq \mathbb{C}\setminus\{\text{origin}\}$
4	$x(t)$ is right-sided then if $s_0 \in$ ROC all $s \in \mathbb{C}$ st $Re(s) > Re(s_0)$ are also in ROC (if ROC is not empty then it is a right half plane).	$x[n]$ is right sided and if $z_0 \in$ ROC then all finite $z \in \mathbb{C}$ st $\lvert z \rvert > \lvert z_0 \rvert$ are also in ROC. (if ROC is not empty it is exterior of a disk).
5	"left-sided signals have ROCs which are left half planes."	"left-sided signals have ROCs interior to disk ... with poss. exception of $z=0$".
6	two-sided signals have strip ROCs. (could be ϕ)	two-sided signals have annular ROCs. (could be ϕ).
7	$x(t) \longleftrightarrow X(s)$ a rational funct. then ROC is bounded by poles or extends to infinity.	$x[n] \longleftrightarrow X(z)$ a rat. funct. Ditto.
8a	$x(t) \longleftrightarrow X(s)$ a rat., $x(t)$ right sided ROC = region to right of right most pole	$x[n] \longleftrightarrow X(z)$ a rat., $x[n]$ right sided ROC = region outside circle of rad. equal to pole of largest mag. (if causal then $+\infty$ also in ROC).
8b	same but $x(t)$ left sided ROC = region to left of left most pole.	same but $x[n]$ left sided. ROC = region inside circle of rad. equal to mag. of smallest nonzero pole, except poss. $z=0$. (if anti causal then includes $z=0$).