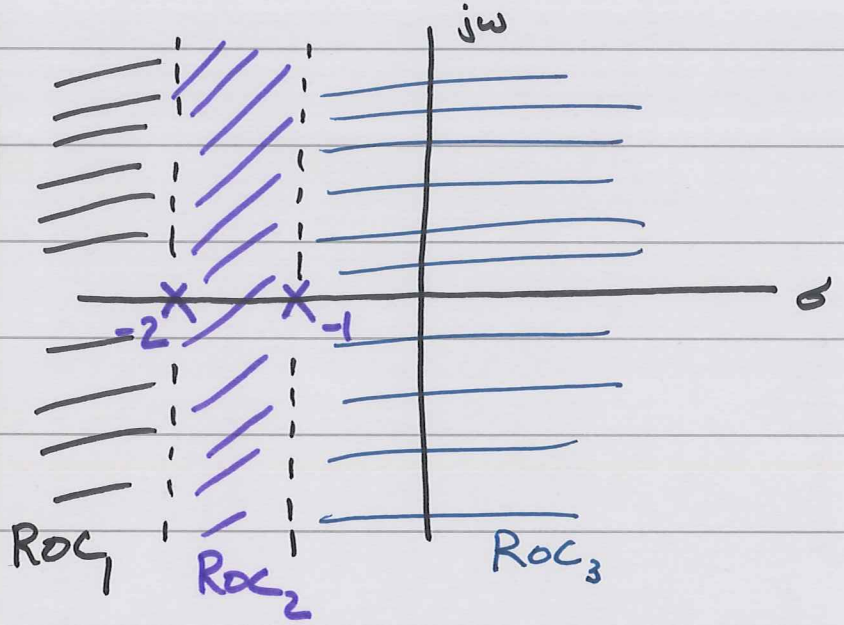
Another Bilateral L.T. Example (Week 15 Session b)

$$X(s) = \frac{1}{(s+1)(s+2)} \rightarrow \text{2 poles: } \begin{matrix} s=-1 \\ s=-2 \end{matrix}$$



There are 3 possible time domain signals ....

$x_1(t)$ corresp to $X(s)$ & ROC_1 (anti-causal)

$x_2(t)$ corresp to $X(s)$ & ROC_2 (two-sided non-causal)

$x_3(t)$ corresp to $X(s)$ & ROC_3 (causal).

First Step : PFE

$$X(s) = \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$\Rightarrow A = \left.\frac{1}{s+2}\right|_{s=-1} = 1; \quad B = \left.\frac{1}{s+1}\right|_{s=-2} = -1$$

$$X(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

ROC_1 both anti-causal

$\mathrm{Re}(s) < -1$ $\quad$ $\mathrm{Re}(s) < -2$

$$-e^{-t}u(-t) - \left(-e^{-2t}u(-t)\right) = x_1(t) = \left(e^{-2t} - e^{-t}\right)u(-t)$$

ROC_2 one causal, one anti-caucal

$\mathrm{Re}(s) < -1$ $\quad$ $\mathrm{Re}(s) > -2$

$$-e^{-t}u(-t) - e^{-2t}u(t) = x_2(t) = \begin{matrix} -e^{-t}u(-t) \\ -e^{-2t}u(t) \end{matrix}$$

ROC_3 both causal

$\mathrm{Re}(s) > -1$ $\quad$ $\mathrm{Re}(s) > -2$

$$e^{-t}u(t) - e^{-2t}u(t) = x_3(t) = \left(e^{-t} - e^{-2t}\right)u(t).$$