Bilateral Laplace Transform (Week 15 – Monday) (Session a)

$$x(t) \longleftrightarrow X(s) \triangleq \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$$

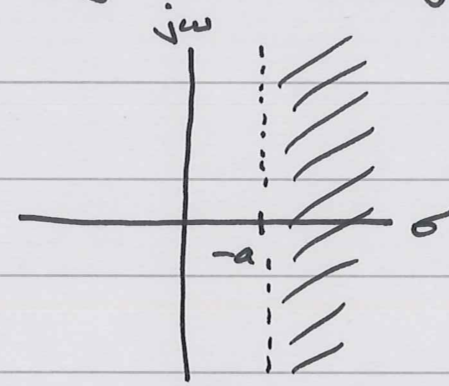
where $s = \sigma + j\omega$

In the region of convergence (ROC) where integral converges absolutely.

Ex 1

$$x(t) = e^{-at}u(t)$$

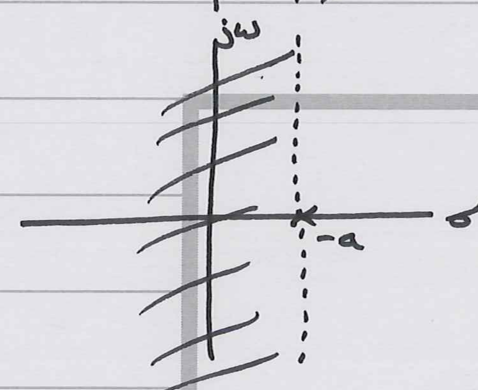
$$X(s) = \frac{1}{s+a} \qquad Re(s) > -a$$



Ex 2

$$x(t) = -e^{-at}u(-t)$$

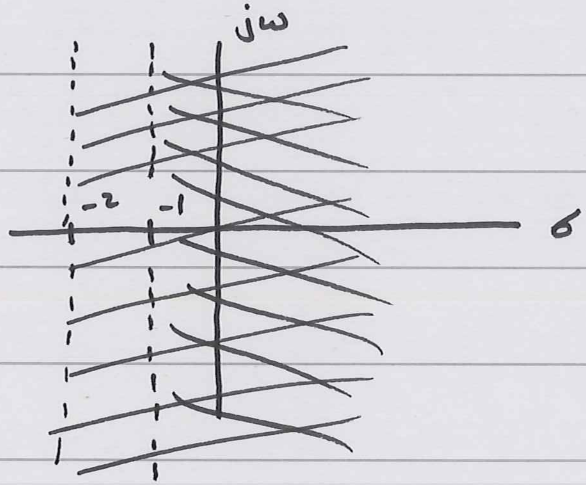
$$X(s) = \frac{1}{s+a} \qquad Re(s) < -a$$



Ex 9.3 $\quad x(t) = 3e^{-2t}u(t) - 2e^{-t}u(t)$

$$3e^{-2t}u(t) \longleftrightarrow \frac{3}{s+2} \qquad Re(s) > -2$$

$$-2e^{-t}u(t) \longleftrightarrow \frac{-2}{s+1} \qquad Re(s) > -1$$



The overall ROC is the intersection of the two:

$$\{s \in \mathbb{C} : Re(s) > -2\} \cap \{s \in \mathbb{C} : Re(s) > -1\}$$

$$= \{s \in \mathbb{C} : Re(s) > -1\}$$

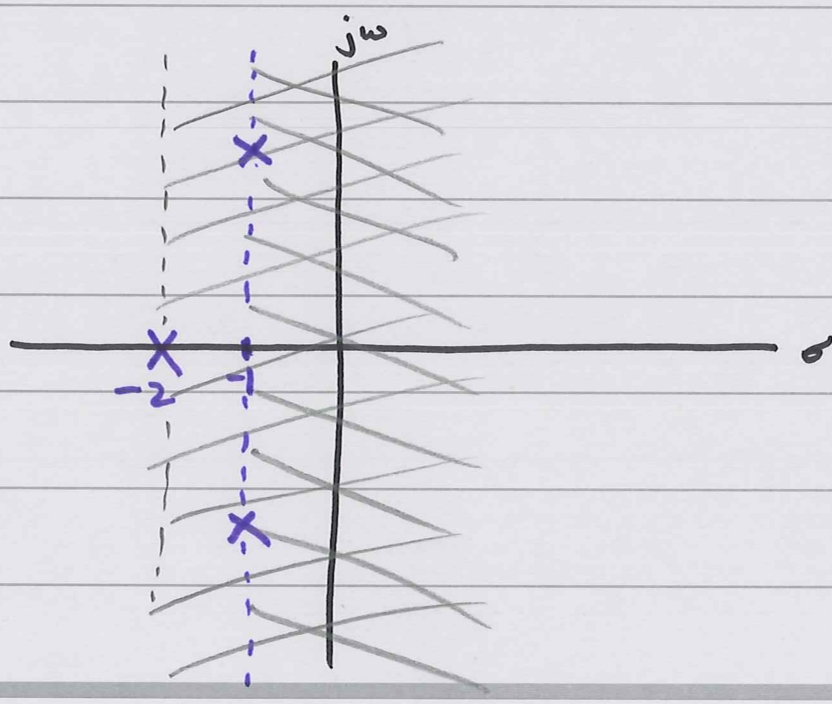
$$\therefore \quad X(s) = \frac{3}{s+2} - \frac{2}{s+1} \qquad Re(s) > -1$$

Ex 9.4 $x(t) = e^{-2t}u(t) + e^{-t}\underbrace{\cos(3t)}_{\frac{e^{j3t}+e^{-j3t}}{2}}u(t) \rightarrow \frac{1}{2}e^{-(1-j3)t} + \frac{1}{2}e^{-(1+j3)t}$

$$X(s) = \frac{1}{s+2} + \frac{1}{2}\frac{1}{s+(1-j3)} + \frac{1}{2}\frac{1}{s+(1+j3)}$$

$Re(s) > -2$ $\quad$ $Re(s) > -1$ $\quad$ $Re(s) > -1.$

$\therefore$ Overall ROC is $Re(s) > -1.$



Note Examples have all had

$$X(s) = \frac{N(s)}{D(s)} \longrightarrow$$ a rational function, a ratio of polynomials.

Suppose all common factors have been cancelled.

$\deg N = n \Longrightarrow N(s)$ has n zeros in the finite $\mathbb{C}$ plane.

$\deg D = d \Longrightarrow D(s)$ has d zeros in " " " "

Zeros of N are the zeros of $X(s)$. Zeros of D are the poles of $X(s)$.

Then

If $n > d \Longrightarrow$ "$X(s) \to \infty$ as $s \to \infty$" $n-d$ poles at infinity

If $d > n \Longrightarrow$ "$X(s) \to 0$ as $s \to \infty$" $d-n$ zeros at infinity

In all of our examples the poles of $X(s)$ lie on the boundaries of the ROC.