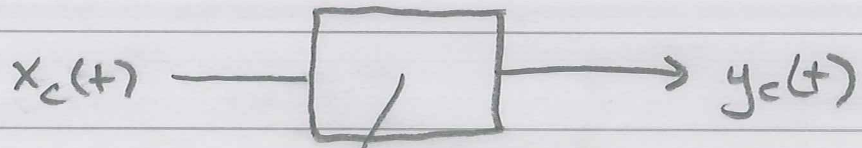
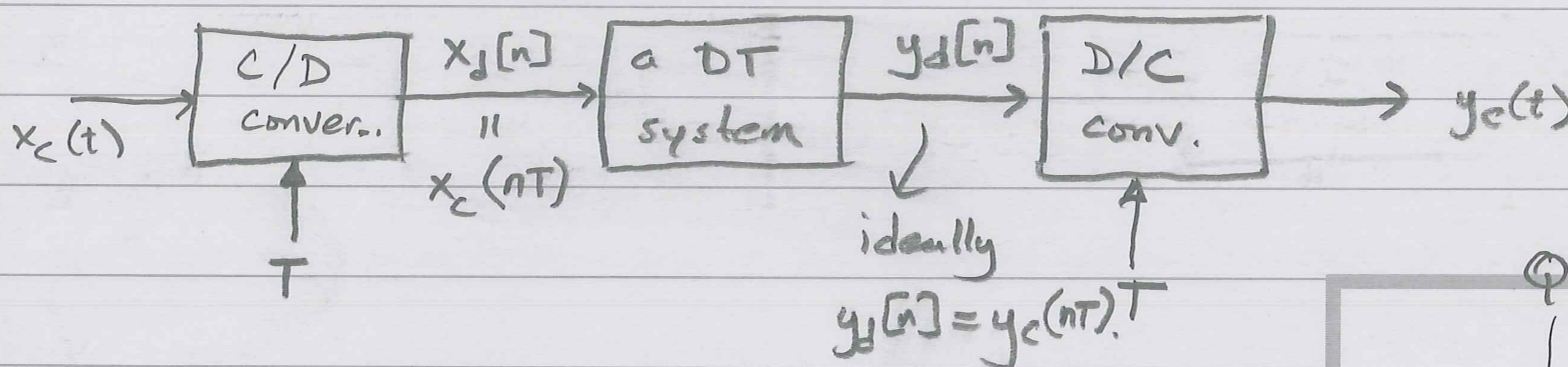
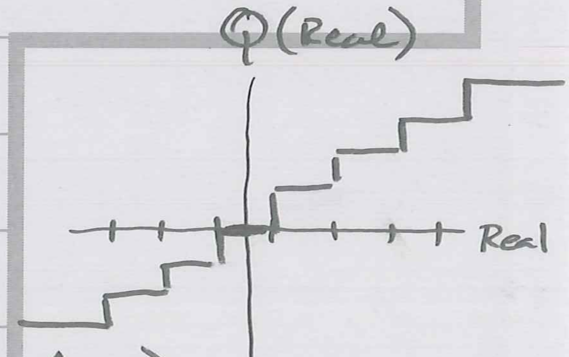
(Week 13 – Friday)

DT Processing of CT Signals

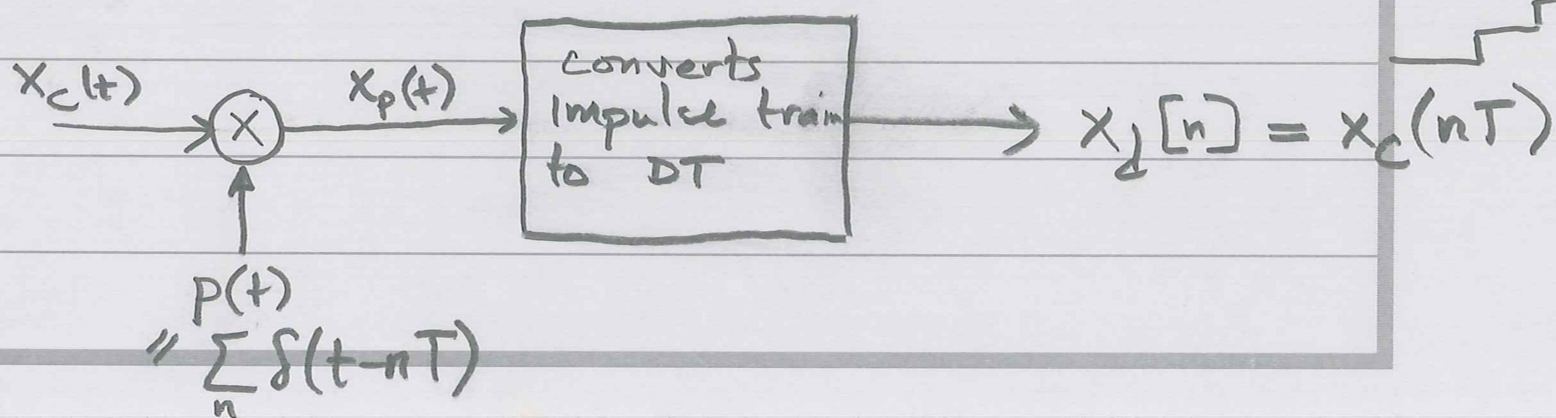


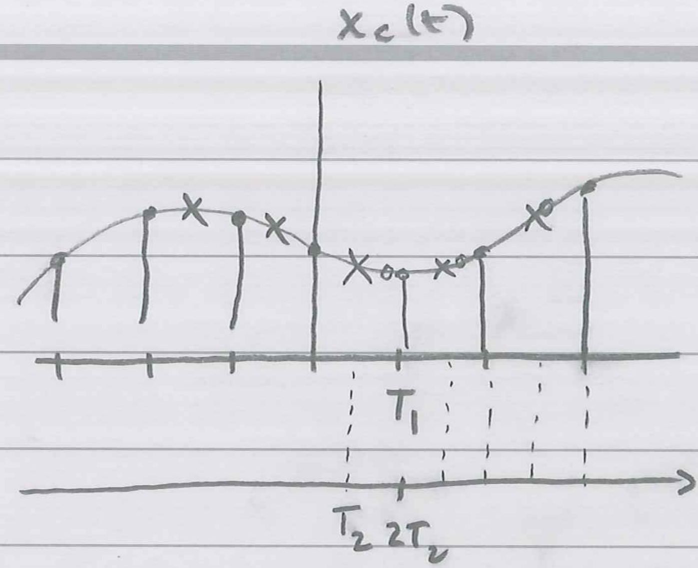
Accomplish some CT processing but using a computer.

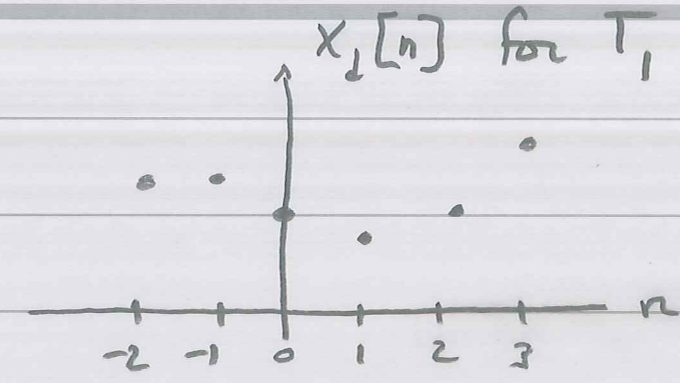


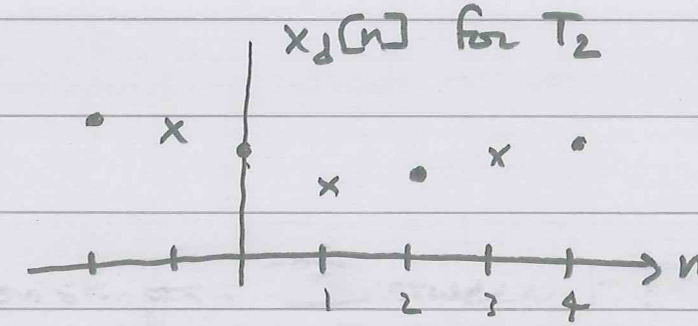


First: A model for C/D









Relationships Between FT & DTFT in this Model

$\omega \longrightarrow$ CT Freq. variable

$\Omega \longrightarrow$ DT Freq variable

$x_c(t) \longleftrightarrow X_c(j\omega)$

$y_c(t) \longleftrightarrow Y_c(j\omega)$

$x_p(t) \longleftrightarrow X_p(j\omega)$

$x_d[n] \longleftrightarrow X_d(e^{j\Omega})$

$y_d[n] \longleftrightarrow Y_d(e^{j\Omega})$.

$$\sum_n \delta(t-nT) \longleftrightarrow \frac{2\pi}{T}\sum_k \delta(\omega - k\omega_s)$$ 3/

$$x_p(t) = \sum_n x_c(nT)\,\delta(t-nT)$$

$$\Updownarrow$$

$$X_p(j\omega) = \frac{1}{T}\sum_k X(j(\omega - k\omega_s))$$

$\omega_s = \frac{2\pi}{T}$ radian sampling freq.

There is another way to express the CTFT here:

$$\delta(t-nT) \longleftrightarrow e^{-j\omega nT}$$

Using linearity

$$X_p(j\omega) = \mathcal{F}\{x_p(t)\} = \mathcal{F}\Big\{\sum_n x_c(nT)\,\delta(t-nT)\Big\}$$

$$= \sum_n x_c(nT)\,\mathcal{F}\{\delta(t-nT)\}$$

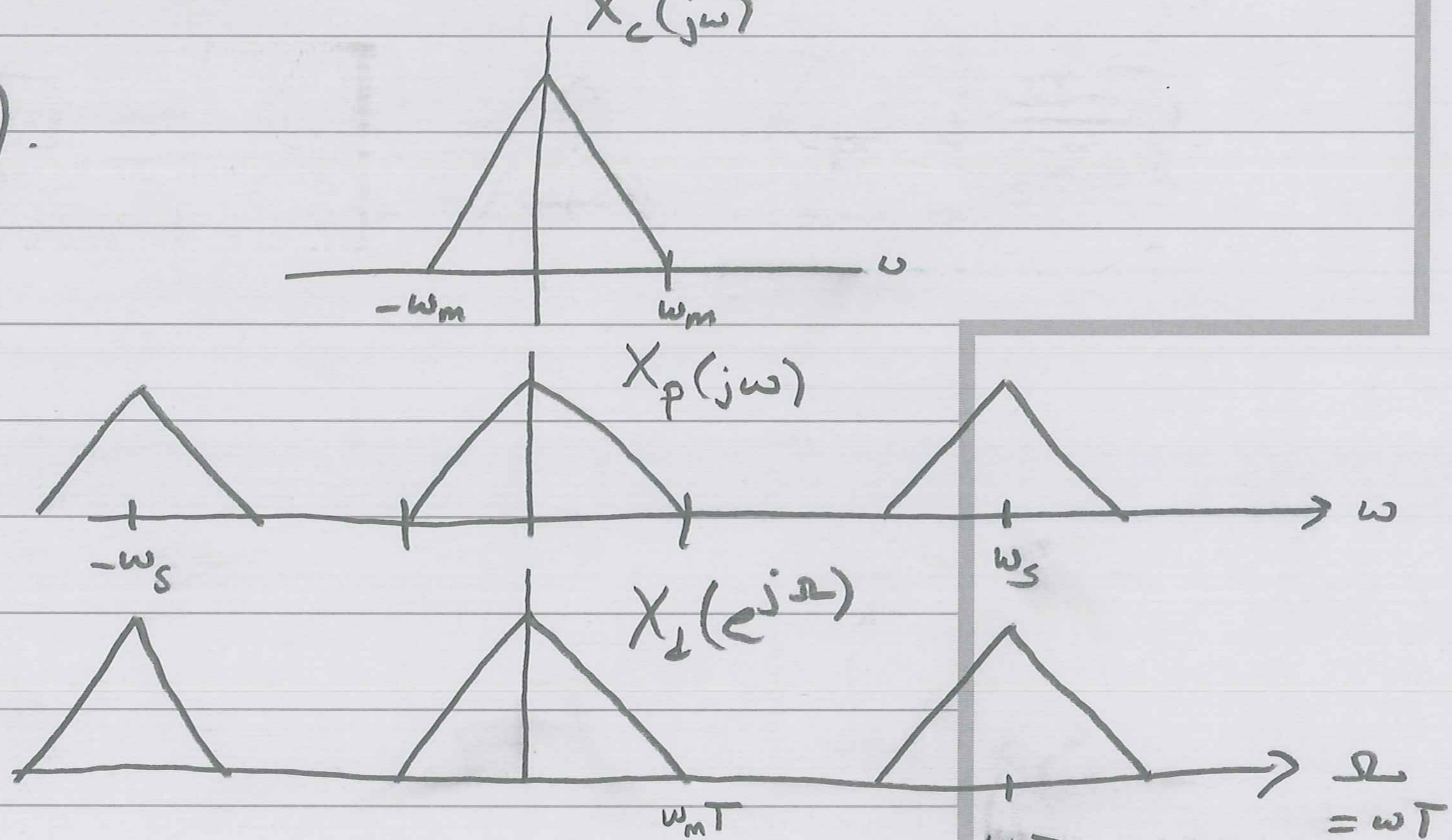
$$= \sum_n x_c(nT)\,e^{-j\omega nT} = \sum_n x_d[n]\,e^{-j\omega nT}$$

$e^{-j\omega Tn}$

$\therefore$ If $\Omega = \omega T$

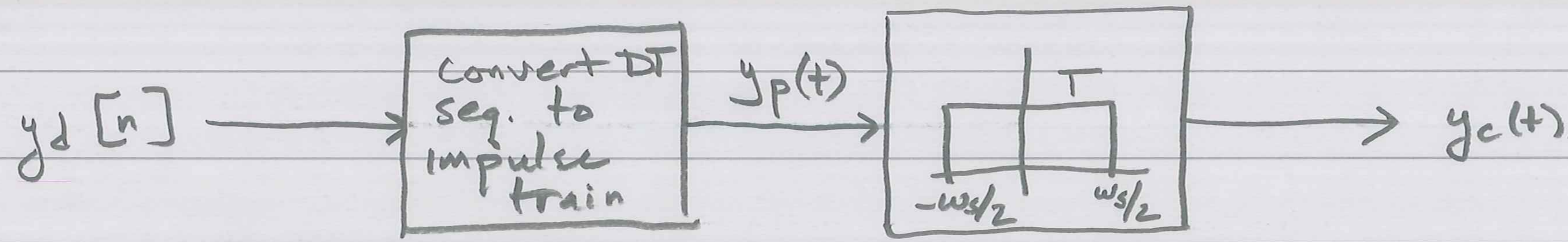
$$X_d(e^{j\Omega}) = \sum_n x_d[n]\, e^{-j\Omega n}$$

$$\therefore \underbrace{X_p(j\omega)}_{X_p\left(j\frac{\Omega}{T}\right)}\Big|_{\omega = \frac{\Omega}{T}} = X_d(e^{j\Omega})$$



$$\omega_s T = \frac{2\pi}{T} \cdot T = 2\pi$$

Now need D/C Conversion



See following notes

Overall System with DT Filtering

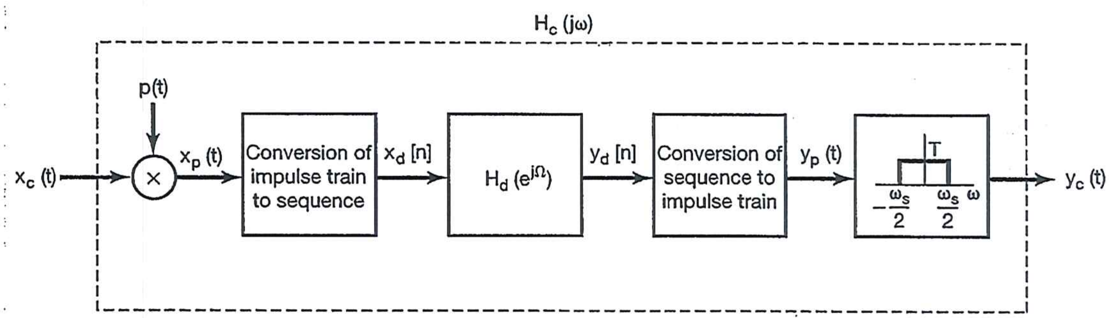


Figure 7.24 Overall system for filtering a continuous-time signal using a discrete-time filter.

Two issues:

- Does above block diagram represent a CT LTI system.
- If so, what is end-to-end transfer function?

$\Rightarrow$ Is LTI only if CT inputs are restricted to be bandlimited to $|\omega| \leq \omega_m < \omega_s/2 = \frac{\pi}{T}$

$\Rightarrow$ For these inputs $H_c(j\omega) = \begin{cases} H_d(e^{j\omega T}) & |\omega| < \omega_s/2 \\ 0 & \text{else.} \end{cases}$

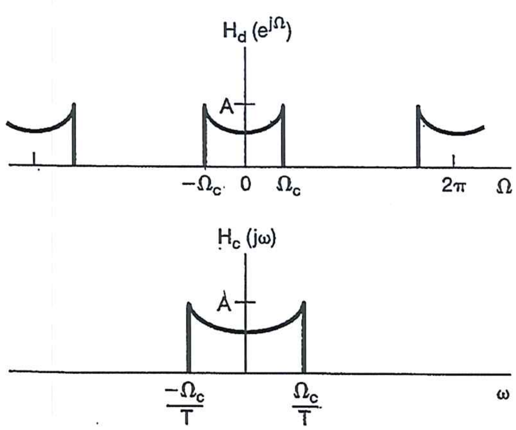


Figure 7.26 Discrete-time frequency response and the equivalent continuous-time frequency response for the system of Figure 7.24.

6

Proof is "By Picture"

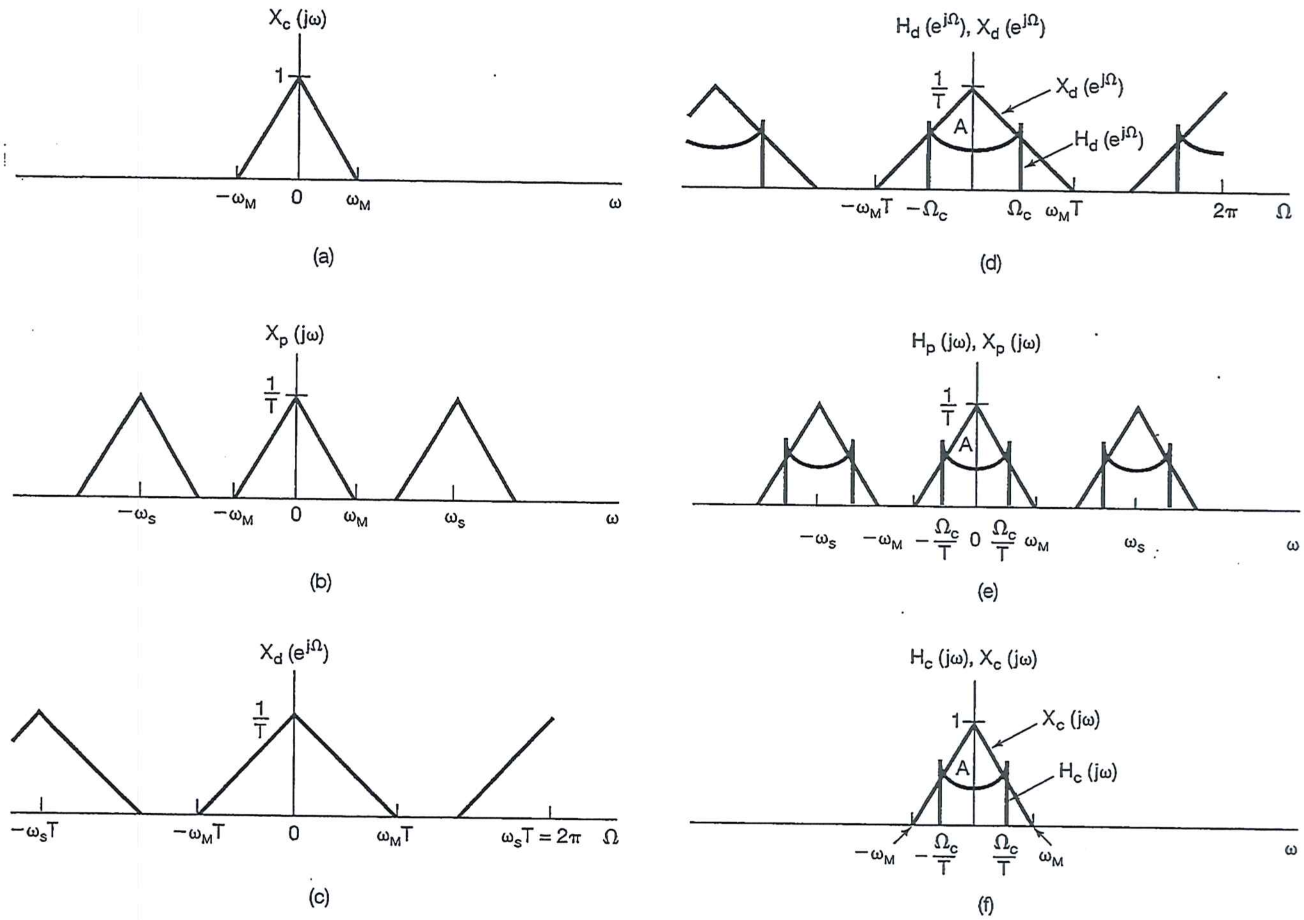


Figure 7.25 Frequency-domain illustration of the system of Figure 7.24: (a) continuous-time spectrum $X_c(j\omega)$; (b) spectrum after impulse-train sampling; (c) spectrum of discrete-time sequence $x_d[n]$; (d) $H_d(e^{j\Omega})$ and $X_d(e^{j\Omega})$ that are multiplied to form $Y_d(e^{j\Omega})$; (e) spectra that are multiplied to form $Y_p(j\omega)$; (f) spectra that are multiplied to form $Y_c(j\omega)$.