DTFT

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{+j\omega n}\, d\omega = x[n]$$

$$\updownarrow$$

$$\sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} = X(e^{j\omega})$$

We had stopped after deriving the DTFT for a signal

$$x[n] = e^{+j\omega_0 n} \longleftrightarrow X(e^{j\omega}) = 2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi l)$$

Note: This is true for any ω_0.

In the special case where $x[n] = x[n+N]$ is periodic can relate DTFT and DTFS.

A periodic DT signal

$$x[n] = \underbrace{\sum_{k=0}^{N-1} X_k e^{+j2\pi kn/N}}$$

sum of N discrete time signals

$$X_k e^{+j2\pi kn/N} \longleftrightarrow 2\pi X_k \sum_{\ell=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N} - 2\pi \ell\right)$$

consider as
a funct of n

Then via linearity

$$x[n] \longleftrightarrow X(e^{j\omega}) = \sum_{k=0}^{N-1} 2\pi X_k \sum_{\ell=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N} - 2\pi \ell\right)$$

$$= 2\pi \sum_{k=0}^{N-1} \sum_{\ell=-\infty}^{\infty} X_k \, \delta\left(\omega - \frac{2\pi k}{N} - 2\pi \ell\right)$$

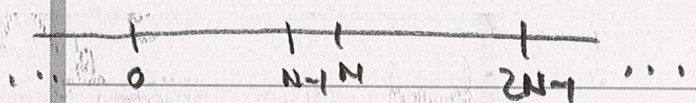
$$\frac{2\pi}{N}(k + \ell N)$$

$$= 2\pi \sum_{k=0}^{N-1} \sum_{\ell=-\infty}^{\infty} X_{k+\ell N}\, \delta\left(\omega - \frac{2\pi}{N}(k+\ell N)\right)$$

$X_k = X_{k+\ell N}$ because a DTFS is per. with N.

$$= 2\pi \sum_{p=-\infty}^{\infty} X_p\, \delta\left(\omega - \frac{2\pi}{N} p\right)$$

OTWEq 5.20



<u>Convolution Prop</u>

$x[n] \longleftrightarrow X(e^{j\omega})$

$h[n] \longleftrightarrow H(e^{j\omega})$

$x[n]$, $h[n]$ are DT signals

$$y[n] = x * h[n] \longleftrightarrow Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$

Example O+W 5.29 (a-ii)

$$h[n] = \left(\tfrac{1}{2}\right)^n u[n]$$

$$x[n] = (n+1)\left(\tfrac{1}{4}\right)^n u[n]$$

Find $y[n] = x * h[n]$ via DTFTs & tables.

$$H(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

$$X(e^{j\omega}) = \frac{1}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2}$$

$$Y(e^{j\omega}) = X(e^{j\omega})\,H(e^{j\omega})$$

$$= \frac{1}{\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)^2}$$

To use PFE I like $v = e^{-j\omega}$

$$\tilde{Y}(v) = Y(v^{-1})$$

$$\tilde{Y}(v) = \frac{1}{\left(1-\frac{1}{2}v\right)\left(1-\frac{1}{4}v\right)^2}$$

$$\tilde{Y}(v) = \frac{B_{11}}{1-\frac{1}{2}v} + \frac{B_{21}}{1-\frac{1}{4}v} + \frac{B_{22}}{\left(1-\frac{1}{4}v\right)^2}$$

Finding the Bs

mult by $1-\frac{1}{2}v$ and eval. @ $v=2$:

$$\tilde{Y}(v)\left(1-\frac{1}{2}v\right)\Big|_{v=2} = \left. B_{11} + \frac{B_{21}}{\left(1-\frac{1}{4}v\right)}\left(1-\frac{1}{2}v\right) + \frac{B_{22}}{\left(1-\frac{1}{4}v\right)^2}\left(1-\frac{1}{2}v\right) \right|_{v=2}$$

$$\left.\frac{1}{\cancel{\left(1-\frac{1}{2}v\right)}\left(1-\frac{1}{4}v\right)^2}\cancel{\left(1-\frac{1}{2}v\right)}\right|_{v=2} = B_{11}$$

$$B_{11} = \frac{1}{\left(1-\frac{1}{2}\right)^2} = 4$$

$$B_{22} = -1$$

In appendix

$$B_{21} = \frac{1}{1!}(-4)^1 \underbrace{\frac{d}{dv}\left\{\left(1-\tfrac{1}{4}v\right)^2 \tilde{Y}(v)\right\}}_{1/2}\Bigg|_{v=4}$$

$$= -2$$

$$\tilde{Y}(v) = \frac{4}{1-\frac{1}{2}v} - \frac{1}{\left(1-\frac{1}{4}v\right)^2} - \frac{2}{1-\frac{1}{4}v}$$

Then let $v = e^{-j\omega}$ again & find in table.

$$\Longrightarrow y[n] = 4\left(\tfrac{1}{2}\right)^n u[n] - 1\cdot(n+1)\left(\tfrac{1}{4}\right)^n u[n] - 2\left(\tfrac{1}{4}\right)^n u[n].$$

Parseval & "Cousin" of Parseval

$x[n] \leftrightarrow X(e^{j\omega})$

$z[n] \leftrightarrow Z(e^{j\omega})$

① $$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 \, d\omega$$

② $$\sum_{n=-\infty}^{\infty} x[n]\, z^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, Z^*(e^{j\omega}) \, d\omega .$$

$$\sum_{n=-\infty}^{\infty} x[n]\, z^*[n+m] = \text{a function of } m.$$

$$z[n] = x[\underbrace{n-N}_{\text{unknown}}] + \text{noise}$$