Important Example (O+W Prob. 4.34)

Given a causal + stable LTI freq. response ...

$$H(j\omega) = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega}$$

Find : ⓐ differential eqn model of system

ⓑ impulse response of system

ⓒ output of system when input is

$$x(t) = e^{-4t}u(t) - te^{-4t}u(t)$$

Re a Find the relationship between a diff eqn. model & a transf. funct. model ...

$x(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow y(t) \iff$ The input and output are governed by

$$\sum_{k=0}^{N} a_k y^{(k)}(t) = \sum_{k=0}^{M} b_k x^{(k)}(t)$$

Taking the F.T. of both sides ...

Then $$\sum_{k=0}^{N} a_k (j\omega)^k Y(j\omega) = \sum_{k=0}^{M} b_k (j\omega)^k X(j\omega)$$

$$\Longrightarrow \quad H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{\sum_{k=0}^{M} b_k (j\omega)^k}{\sum_{k=0}^{N} a_k (j\omega)^k}$$

Now lets specialize for the problem at hand ...

$$H(j\omega) = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega}$$

note : $(j\omega)^2 = -\omega^2$

$$= \frac{j\omega + 4}{(j\omega)^2 + 5(j\omega) + 6} \Longrightarrow \begin{array}{l} M = 1, \; b_1 = 1, \; b_0 = 4 \\ N = 2, \; a_2 = 1, \; a_1 = 5, \; a_0 = 6 \end{array}$$

$$\therefore \quad y^{(2)}(t) + 5y^{(1)}(t) + 6y(t) = x^{(1)}(t) + 4x(t) \qquad a$$

(b) Find $h(t) \longleftrightarrow H(j\omega)$ Method: Use Tables of Transforms and Partial Fract. Expansion

$$H(j\omega) = \frac{j\omega + 4}{(j\omega)^2 + 5(j\omega) + 6}$$

1st step by writing $v = j\omega$ $\Longrightarrow H(v) = \dfrac{v+4}{v^2+5v+6}$

2nd step: factor the denom. poly: $v^2 + 5v + 6 = (v+3)(v+2)$

PFE form: $$H(v) = \frac{v+4}{(v+3)(v+2)} = \frac{A}{v+3} + \frac{B}{v+2}$$

Find A, B: For A $$\frac{v+4}{\cancel{(v+3)}(v+2)}\cancel{(v+3)} = A + \frac{B}{v+2}(v+3)$$

$$A = \left.\frac{v+4}{v+2}\right|_{v=-3} = -1$$

For B $$\left.\frac{v+4}{v+3}\right|_{v=-2} = B = 2$$

$$\therefore H(v) = \frac{v+4}{(v+3)(v+2)} = \frac{-1}{v+3} + \frac{2}{v+2}$$

Now replace ν by $j\omega$:

$$H(j\omega) = \frac{-1}{j\omega+3} + \frac{2}{j\omega+2}$$

$$\Updownarrow$$

$$h(t) = -e^{-3t}u(t) + 2e^{-2t}u(t)$$

(c) Do this via transforms: $Y(j\omega) = H(j\omega)X(j\omega)$

here $x(t) = e^{-4t}u(t) - te^{-4t}u(t)$

$$\Updownarrow$$

$$X(j\omega) = \frac{1}{j\omega+4} - \left(\frac{1}{j\omega+4}\right)^2$$

$$= \frac{j\omega+3}{(j\omega+4)^2}$$

$$Y(j\omega) = \frac{\cancel{j\omega+4}}{\cancel{(j\omega+3)}(j\omega+2)} \cdot \frac{\cancel{j\omega+3}}{(j\omega+4)^{\cancel{2}}} = \frac{1}{(j\omega+2)(j\omega+4)}$$

Use $v = j\omega$

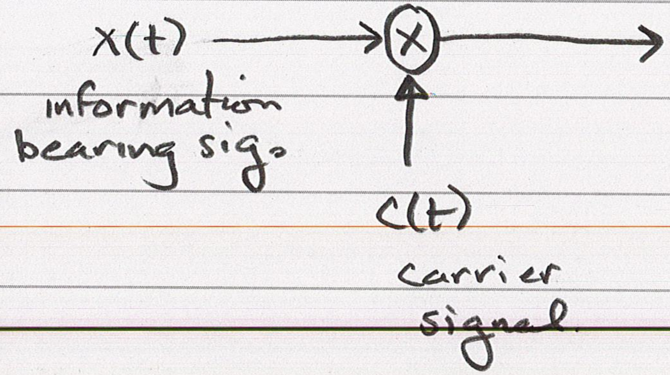
$$Y(v) = \frac{1}{(v+2)(v+4)} = \frac{A}{v+2} + \frac{B}{v+4}$$

$$A = \tfrac{1}{2} \qquad B = -\tfrac{1}{2}$$

$$\therefore \quad Y(j\omega) = \frac{1/2}{j\omega + 2} - \frac{1/2}{j\omega + 4}$$

$$y(t) = \frac{1}{2} e^{-2t} u(t) - \frac{1}{2} e^{-4t} u(t).$$

General Amplitude Modulation Property



$y(t) = x(t)\,c(t) \triangleq$ modulated signal.

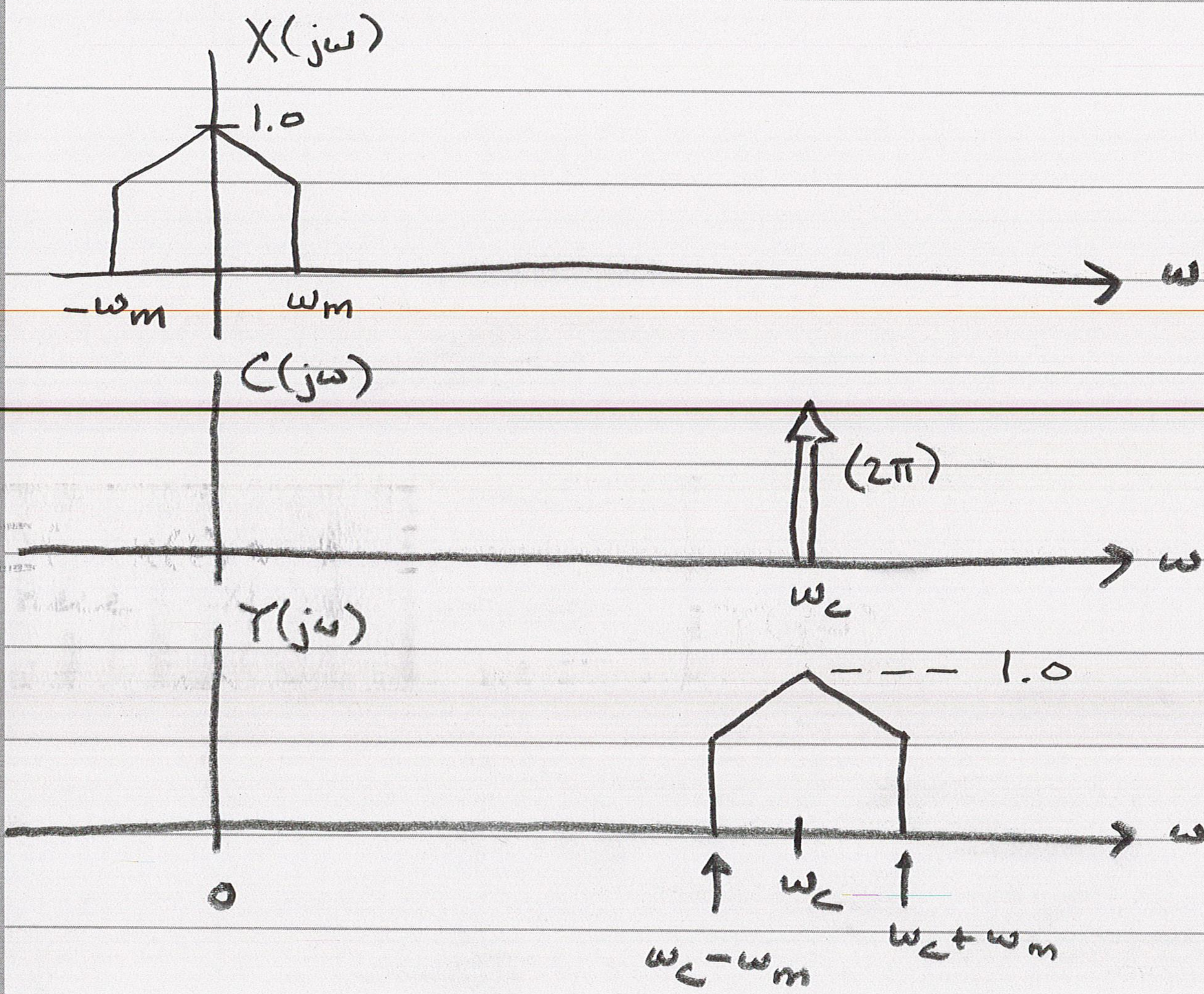
If $x(t) \longleftrightarrow X(j\omega)$ and $c(t) \longleftrightarrow C(j\omega)$ then

$$y(t) \longleftrightarrow Y(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)\,C(j(\omega-\theta))\,d\theta$$

<u>Case</u> $c(t) = e^{j\omega_c t}$ Have $C(j\omega) = 2\pi\,\delta(\omega-\omega_c)$

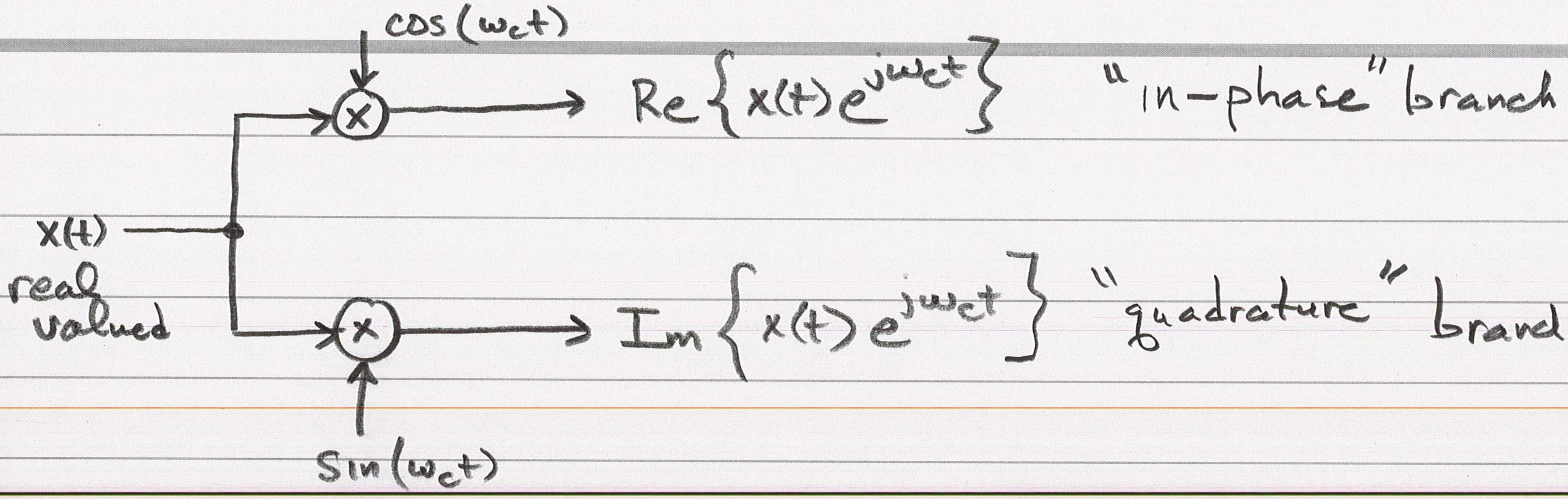
$$\Rightarrow Y(j\omega) = X(j(\omega-\omega_c))$$

Typical Pictures



When we implement the above with real arithmetic we are using

$$e^{j\omega_c t} = \cos\omega_c t + j\sin\omega_c t$$



Quadrature Modulator.

Demodulation: Process of recovering the information bearing signal from the modulated signal.

$x(t)$

$y(t) = x(t)\,e^{j\omega_c t} \implies$ Can demod by

$x(t) = y(t)\,e^{-j\omega_c t}$.