(Week-10-Wednesday)

No class Friday → JVK @ funeral

Announcements

Wed Nov 4 is a "reading day" so no official class that day.

However, I will record a lecture on Wed Nov 4 to replace Fri lecture.

$x[n] \rightarrow$ [Syst.] $\rightarrow y[n] = \sum_{k=-|n|}^{2-|n|} x[k+2]\,|k|$

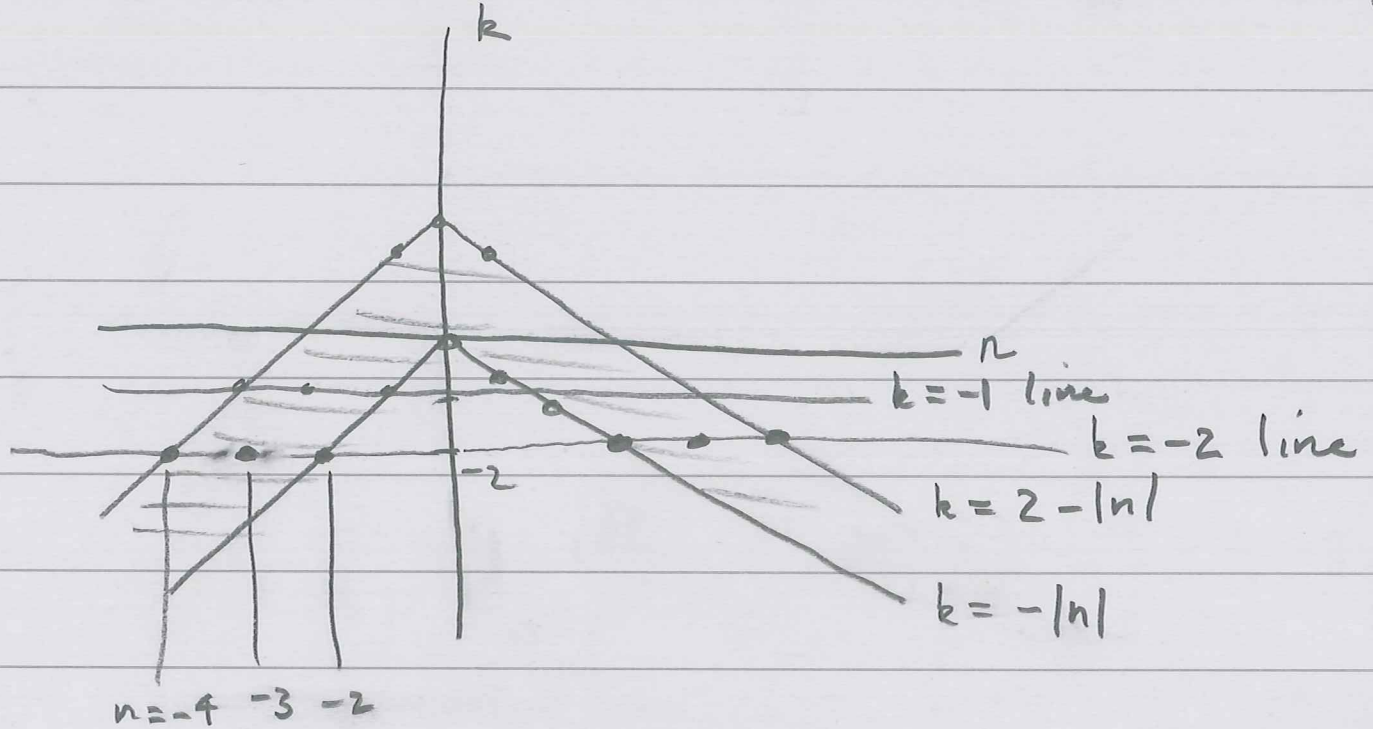
Definitely linear. Looks non-TI, because of the $|k|$ multiplier.

(a) Compute $h[n]$ in resp. to $x[n] = \delta[n]$.

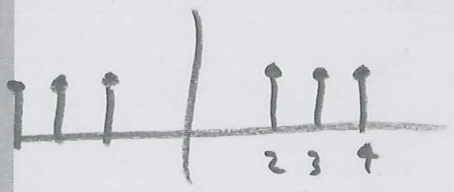
$$h[n] = \sum_{k=-|n|}^{2-|n|} \delta[k+2]\,|k|$$

$$\delta[k+2] = \begin{cases} 1 & \text{if } k = -2 \\ 0 & \text{if } k \neq -2. \end{cases}$$

Look @ the behavior of the summation range:



$$h[n] = \begin{cases} 2 & n = -4, -3, -2 \\ & 2, 3, 4 \\ 0 & \text{else} \end{cases}$$



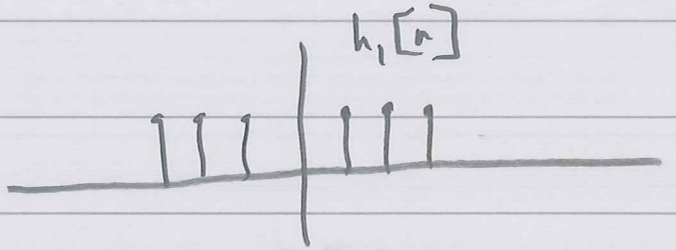
new input

(b) $x[n] = u[n] - u[n-2] = \delta[n] + \delta[n-1]$

$y[n] = h * x[n]$? **No**

Easiest To Do. $x_1[n] = \delta[n-1]$ calculate $h_1[n]$ and show $h_1[n] \neq h[n-1]$.

$$\Rightarrow h_1[n] = \sum_{k=-|n|}^{2-|n|-1} \delta[k+1]\,|k|$$



(c) Suspect this is not invertible ... How to show?

If I can show it is <u>not</u> 1-to-1 then I've done it.

Because syst. is linear $x_0[n] \triangleq 0 \;\forall n \in \mathbb{Z}$ → always maps to $y[n] = 0 \;\forall n \in \mathbb{Z}$

So if I can find an $x_1[n] \neq 0$ on at least one value of n. which maps to all zero output, done.

Try $x[n] = \delta[n-3]$

4/

Question 6: [20%, Multiple Choices, Learning Objective 1]

The following questions are multiple-choice questions and there is no need to justify your answers. Consider the following two systems:

System 1: When the input is $x_1(t)$, the output is

$$y_1(t) = \begin{cases} \int_{-\infty}^{t+1} x(s-1)e^{t-s}ds & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases} \tag{8}$$

System 2: When the input is $x_2[n]$, the output is

$$y_2[n] = y_2[n-1] + \cos(3n) \cdot x_2[n+1] \cdot \mathcal{U}[n-100] \tag{9}$$

and we also known $y_2[-200] = 0$.

Answer the following questions

1. [4%, Learning Objective 1] Is System 1 memoryless? Is System 2 memoryless? No
2. [4%, Learning Objective 1] Is System 1 causal? Is System 2 causal?
3. [4%, Learning Objective 1] Is System 1 stable? Is System 2 stable?
4. [4%, Learning Objective 1] Is System 1 linear? Is System 2 linear? Yes
5. [4%, Learning Objective 1] Is System 1 time-invariant? Is System 2 time-invariant? No

No

Yes

No

Causal?

$$y_1(t) = \underset{t}{\text{funct}}\{x(\lambda) : \lambda \leq t\} \qquad u = s-1 \quad du = ds$$

$$= \int_{-\infty}^{t} x(u)\, e^{t-u-1}\, du \implies \text{Yes causal.}$$

Stable?

$$x(t) = u(t) \implies y_1(t) = \int_0^t 1 \cdot e^{t} e^{-u} e^{-1}\, du \qquad t > 0.$$

$$= e^{-1}e^{t}\left\{-e^{-u}\Big|_0^t\right\} = e^{-1}e^{t}\left\{-e^{-t}+1\right\}$$

$$= e^{-1}\left\{-1+e^{t}\right\} = e^{-1}(e^{t}-1) \quad t>0.$$