(WEEK 10 - Monday) 1/

Last Time

$$X(j\omega) = \cos(4\omega + \pi/3)$$

Re: Student Question

Starting with:

$$\cos t \longleftrightarrow \pi\delta(\omega-1) + \pi\delta(\omega+1)$$

$$y(t) = \cos(4t + \pi/3)$$

$$= \cos(4(t + \pi/12)) \quad ?$$

Two ways to go: ① $\cos(4t + \pi/3) = \dfrac{e^{j(4t+\pi/3)} + e^{-j(4t+\pi/3)}}{2}$

② $\cos 4t \longleftrightarrow \pi\delta(\omega-4) + \pi\delta(\omega+4) = Y(j\omega)$

But $\cos(4t + \pi/3) = \cos(4(t + \pi/12)) \triangleq z(t)$

Time shift prop: $x(t-t_0) \longleftrightarrow e^{-j\omega t_0} X(j\omega)$

$x(t) \longleftrightarrow X(j\omega)$

Applying here: $y(t) = \cos 4t \Rightarrow$ Know $Y(j\omega)$

$\Rightarrow z(t) = y(t + \pi/12) \longleftrightarrow Z(j\omega) = e^{+j\omega\pi/12} Y(j\omega)$

$$Z(j\omega) = e^{+j\omega\pi/12}\, Y(j\omega)$$

$$= e^{+j\omega\pi/12}\left[\pi\,\delta(\omega-4) + \pi\,\delta(\omega+4)\right]$$

$$= \pi\, e^{+j\omega\pi/12}\,\delta(\omega-4) + \pi\, e^{+j\omega\pi/12}\,\delta(\omega+4)$$

Recall: If $f(\omega)$ is a continuous funct. of ω then

$$f(\omega)\,\delta(\omega-\omega_0) = f(\omega_0)\,\delta(\omega-\omega_0)$$

$$= e^{+j4\pi/12}\,\delta(\omega-4) = e^{j\pi/3}\,\delta(\omega-4)$$

$$= e^{j(-4)\pi/12}\,\delta(\omega+4)$$

$$= e^{-j\pi/3}\,\delta(\omega+4).$$

Fourier Transform Properties

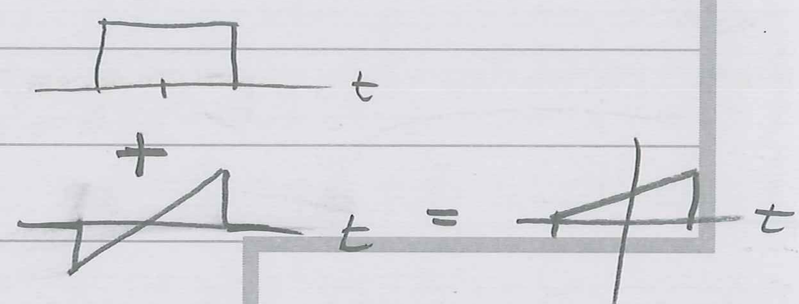
Look @ Tables 4.1, 4.2 in Text.

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega = x(t) \xleftrightarrow{\mathcal{F}} X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

$$\triangleq \mathcal{F}^{-1}\{X(j\omega)\} \qquad\qquad \triangleq \mathcal{F}\{x(t)\}$$

Table 4.1 Props

① Linearity



② Time Shift $x(t) \leftrightarrow X(j\omega)$

$$x(t-t_0) \leftrightarrow e^{-j\omega t_0}X(j\omega)$$

③ Conjugation: $x^*(t) \leftrightarrow X^*(-j\omega) = \mathcal{F}\{x(t)\}^*\Big|_{\omega \mapsto -\omega}$

Symmetry Even/Odd.

Symmetry Even vs. Odd

Assume that $x(t)$ is real-valued.

Because $x(t) = x^*(t)$

$$X(j\omega) = X^*(-j\omega) \Longleftrightarrow X^*(j\omega) = X(-j\omega)$$

ie the F.T. is conjugate symmetric about $\omega = 0$

Thus

$\Longrightarrow$ $X(j\omega) = X_R(j\omega) + j X_I(j\omega)$ → this is the rectangular form of the complex funct.

$$X^*(j\omega) = X_R(j\omega) - j X_I(j\omega)$$

$$\parallel$$

$$X(-j\omega) = X_R(-j\omega) + j X_I(-j\omega)$$

$\Longleftrightarrow$ $X_R(j\omega) = X_R(-j\omega)$ (real part even)

and

$-X_I(j\omega) = X_I(-j\omega)$ (imag part odd).

$\Longleftrightarrow$ (*)

$\Longleftrightarrow$ $|X(j\omega)|$ is even

and

$\angle X(j\omega)$ is odd.

So we just did real $x(t)$

A further result is

$x(t)$ is real and even $\Longleftrightarrow$ $X(j\omega)$ is real and even.

The Glorius Thing to Show: For a general real-valued $x(t)$:

$$\text{even part}\{x(t)\} \longleftrightarrow \text{real part}\{X(j\omega)\}$$

$$\text{odd part}\{x(t)\} \longleftrightarrow j \cdot \text{imag part}\{X(j\omega)\}$$

Differentiation in Time Domain $\quad x(t) \leftrightarrow X(j\omega)$

$$\frac{d}{dt}x(t) = \frac{d}{dt}\left\{\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{+j\omega t}\,d\omega\right\}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\underbrace{\frac{d}{dt}\left\{e^{+j\omega t}\right\}}_{j\omega e^{j\omega t}}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \boxed{j\omega X(j\omega)}\,e^{j\omega t}\,d\omega$$

new function of $j\omega$.

By uniqueness of FTs:

$$\frac{d}{dt}x(t) \longleftrightarrow j\omega X(j\omega)$$

Things that do this in Freq. are called "discriminators"