

# 3-D BUILDING RECONSTRUCTION FROM UNSTRUCTURED DISTINCT POINTS

Chiung-Shiuan Fu

Jie Shan

Geomatics Engineering, School of Civil Engineering, Purdue University  
550 Stadium Mall Drive, West Lafayette, IN 47907-2051, USA

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## ABSTRACT:

This paper presents a model-based approach for 3-D building reconstruction unstructured distinct points. The data structure of a building is an unstructured roof point cloud digitized by operators. Constructive Solid Geometry (CSG) model is applied in our approach for a complex roof which can be decomposed into three types of primitive models. The primitive model is represented as a tree. The leaves contain primitives and the nodes store Boolean set operators to combine primitives to form a whole building. There are three steps in this approach. The key process during the entire reconstructing procedures is rectangulation, which is to form points automatically into horizontal rectangular bases of primitive roof models. In the second step, we determine the primitive roof model by its rectangular bases and the corresponding roof points. A primitive roof or a combination of roof primitives can then be assembled to a polyhedral roof model in the third step. The building can be completely reconstructed by projecting its roof boundary outline to ground. Because of the rectangulation process, our method has limitation in only describing the buildings with right angle corners. Nevertheless, under this assumption, the result shows that our methodology can successfully reconstruct buildings with complex roofs over Purdue University campus. We demonstrate the results with pertinent discussions.

## 1. INTRODUCTION

3-D building reconstruction has been an active research subject in computer vision, computer graphics and photogrammetry since 1980's. An efficient building construction can benefit fast data collection, support effective photorealistic visualization, and facilitate 3-D geospatial query and database creation.

A common strategy for automatic building reconstruction is to utilize building model base. Three most popular model types are parametric models, CAD (Computer Aided Design) models, and generic models. In the parametric model (Lang and Förstner, 1996; Fischer et al, 1998), buildings are described by a number of parameters, whose values are to be determined in the reconstruction process. This model has limitations in describing complex buildings, because the number of parameters is fixed and a complex building may have many levels/floors and complex rooftops with irregular shape. The CAD based modeling approach classifies buildings into different primitive components. Buildings must be described in advance with a fixed geometry and topology. Because of such pre-definition procedure, the exploration of unknown or complex buildings is constrained by the lack of variability in this model. Generic models allow for variation in building structures, which indicates the number of geometric parameters is free. There are three subclasses of generic models, including prismatic models (Wenidner, 1997), polyhedral models (Grün and Wang, 1998) and constructive solid geometry (CSG) models (Gülch et al, 1999; Zlatanova et al, 1998; Norbert and Brenner, 1998). However, the requirement on pre-defined models still limits the types of realistic buildings to be reconstructed.

This paper is focused on using unstructured distinct points for 3-D building reconstruction. Similar studies have been reported by using roof points. Grün and Dan (1997) proposed a topology builder TOBAGO for semi-automatic building reconstruction. The method is a model-based approach that requires the operator measuring all roof points, however, no specific sequence needs to be followed. Each "roof unit" is a complete

point cloud to be proceeded individually. First, a K-Parser classifies the roof into six generic roof models based on the number of ridge points. Then, a G-Parser exploits the geometric criteria within a particular roof class to enclose the 3-D points as a complete CAD building model. However, TOBAGO might fail if no corresponding roof unit is found in the pre-defined model database. Grün and Wang (1998) proposed another generic topology generator CC-Modeler for CyberCity (CC) modeling. The data in CC-Modeler are regarded as 3-D point clouds manually measured by the operator. During data acquisition, 3-D point clouds for each building need to be divided and labeled into two groups: boundary points and interior points. Boundary points for building roofs are digitized by following certain order and interior points can be measured without sequence. For reconstruction, the data is treated by a consistent labeling algorithm based on probabilistic relaxation.

In this paper, our effort is devoted to structuring the unordered distinct points into 3-D buildings. The 3-D point cloud is manually measured from a pair of stereo images. The measurement of 3-D point clouds for all roof corners must be complete, including any hidden ones. For this objective, we focus our attention on reconstructing the buildings only with right angle corners and propose a model-based approach to regularly construct these unstructured points. In this study, CSG modeling rule is applied that a complex building roof can be decomposed into several primitive CAD models. Combinations of primitives are created by Boolean operations using a CSG tree. One primitive or a combination of primitives can be combined to form a polyhedral building. This proposed reconstruction method is formulated as a process of finding 2-D rectangles, forming 3-D polyhedral primitives, and assembling them to a building. Presented in this paper are successfully reconstructed Purdue University campus buildings and their comparison with aerial images. Properties of the proposed approach and its further improvement are also discussed based on our experience.

## 2. RECONSTRUCTION METHOD

In general, buildings show a large diversity in their geometry. Therefore, it is impractical to expect that one strategy can handle all types of complex buildings. In this study, we only focus on building with right angle corners. We first reconstruct roofs and then reconstruct walls of the building, where the former is the focus of our discussion below.

The main idea of roof reconstruction is that most roofs with right angle corners in their horizontal base can be decomposed into an aggregation of simple roof types. Therefore our strategy for 3-D roof reconstruction is to deal with a complex roof as a CSG model. The CSG model can be divided into one or more primitive roof models. Each of the primitive roof models consists of a horizontal rectangular base and the roof can be either one-ridge point, two-ridge points or four-ridge points roof (Figure 1). To reconstruct the primitive model, we need to form a (horizontal) rectangular base and generate polyhedral roof surfaces. The workflow of primitive models reconstruction starts by dividing points in the 3-D space into many 2-D horizontal levels according to the height. Then we connect points in each 2-D level to form rectangle bases. For those points that cannot form rectangles, we classify them as roof ridge points. The next is to reconstruct 3-D primitive models by determining the corresponding rectangle bases for roof points in 3-D space. After reconstructing all possible primitive models, we apply operations such as union and intersection to combine them as a complete building roof.

To reconstruct walls of a building, we need to determine the boundary outline from the roof and project the outline to the ground. These detail steps are described in the following sections.

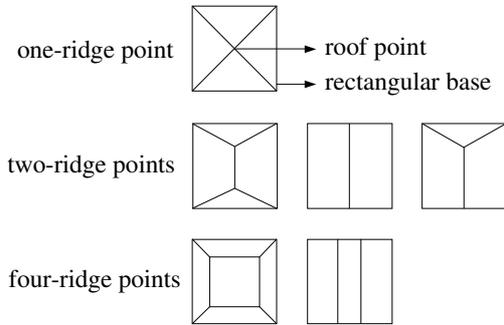


Figure 1. Primitive roof models.

### 2.1 Finding 2-D rectangular bases

To decompose a complex roof into primitive roofs, the first step is to find rectangular bases of primitives. This process is working on each 2-D horizontal level. The roof outlines with perpendicular edges can be treated as 2-D polygons. Essentially, it is a process of rectangling the points as opposed to the well-known triangulation, namely rectangulation. Rectangulation involves rectangles produced from a set of points from a polygon with only perpendicular edges, with the restriction that overlapping rectangles are not allowed. The hypothesis is that such polygon can always be partitioned into a set of rectangles. The purpose of rectangulation here is to find a feasible combination of rectangles for roof outlines. Our approach is described below.

There are two initial steps for rectangulation. The first is that the building data, which is represented by a set of points with 3-D coordinates (Figure 2a), should be separated into different levels based upon height information in their  $z$ -coordinates. The second is to rotate the dominant directions of a building align with  $x$  and  $y$ -axis. Since our building outlines are perpendicular edges, we can find two dominant directions of the building perpendicular to each other. It starts from finding lines among all points by using slope-intercept representation in each level. Loop over the process for all levels. A reasonable assumption is that dominant perpendicular lines are consistent across all the levels and represent dominant directions of the entire building. Once these directions are found, the building can be rotated to align dominant directions with  $x$  and  $y$ -axis, respectively.

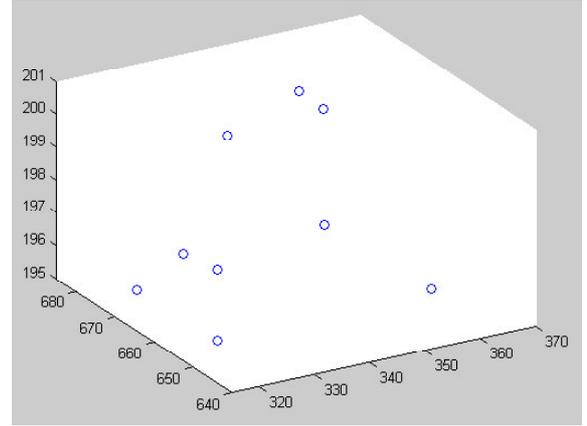


Figure 2a). 3-D points of a building.

We now can proceed to rectangulation in each horizontal level separately. Processing starts by examining the set of points in the  $i$ -th level, which is denoted by  $L_i$ . If  $|L_i| < 4$ , this level is categorized as a set of roof points  $L_r$  (Figure 2b). If  $|L_i| \geq 4$ , the level is classified as an intermediate level (Figure 2c). Points in intermediate level could be either roof points or base points, therefore, further processing is needed. We decompose points in intermediate level into rectangular bases automatically as below.

1. We start from the lower left point  $p_s(x_s, y_s)$  specified by Eq (1), where  $P_{ij} \in L_i$  represents the point set during the  $j$ -th round of rectangulation process on the  $i$ -th level.

$$p_s(x_s, y_s) = \min_y \{ \min_x \{ P_{ij}(x, y) \} \} \quad (1)$$

2. After  $p_s(x_s, y_s)$  is determined, we start looking for the closest points located along the east and north direction, which are denoted as  $p_e(x_e, y_e)$  and  $p_n(x_n, y_n)$ , respectively. Their coordinates are determined via

$$x_e = \min_x \{ P_{ij}(x, y) - p_s \}, y_e = y_s \quad (2a)$$

$$x_n = x_s, y_n = \min_y \{ P_{ij}(x, y) - p_s \} \quad (2b)$$

3. After  $p_e$  and  $p_n$  are identified, move toward the corresponding perpendicular direction to find  $p_j(x_j, y_j)$  which is diagonal to  $p_s$  and determined by

$$x_j = x_e, \quad y_j = y_n \quad (2c)$$

4. Once  $p_s, p_e, p_n$  and  $p_j$  are determined, the  $j$ -th rectangle is formed as  $B_{ij} = \{p_s, p_e, p_j, p_n\}$ .
5. If the two lines starting from  $p_e$  and  $p_n$  cannot join together at the same point  $p_j$ , we temporarily create an auxiliary point  $p_a(x_a, y_a)$  for rectangulation (Figure 2d).

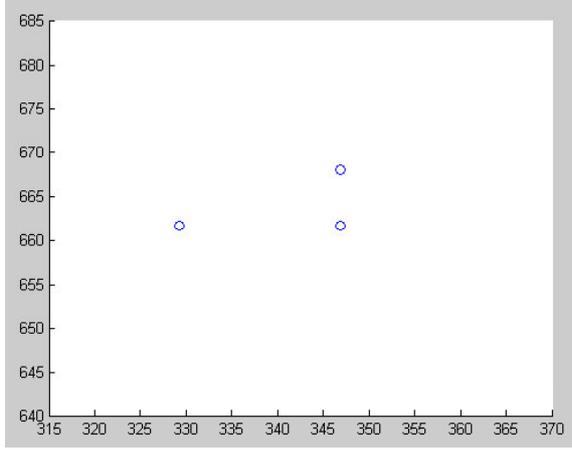


Figure 2b). Roof points.

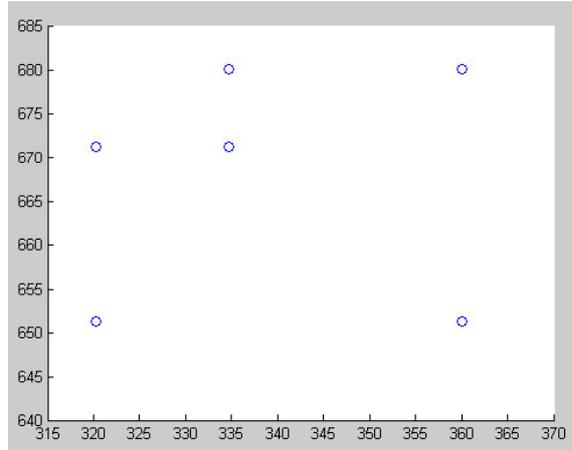


Figure 2c). Intermediate points.

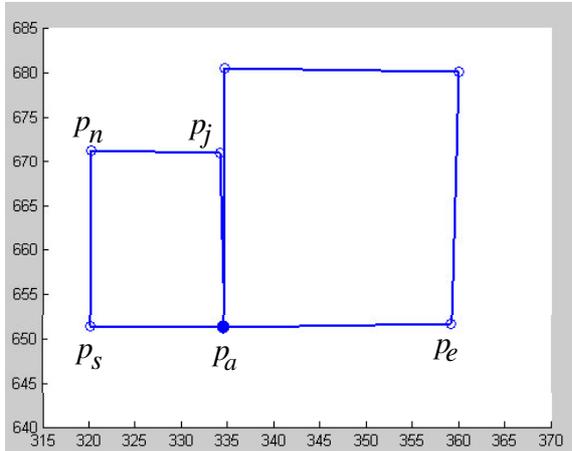


Figure 2d). Rectangulation.

6. If there is no auxiliary point created, repeat step 1 to step 5 to form the  $(j+1)$ -th rectangle on this level by  $P_{i(j+1)} = P_{ij} - B_{ij}$ .
7. Once auxiliary points are created, repeat step 1 to step 5 by  $P_{i(j+1)} = P_{ij} - B_{ij} + P_{aj}$  where  $P_{aj}$  denotes auxiliary points in the  $j$ -th round.
8. After the rectangulation process continually repeats, if the number of the rest points is less than four, or if there are points that cannot form a rectangle, what remains can be classified as roof points.

## 2.2 Forming 3-D primitives

After finding rectangular bases of primitive roofs, the next step is to reconstruct 3-D polyhedral roof primitives. This process is working on the 3-D space. However, to reconstruct these surfaces, we first apply a 2-D range query to find roof points for each rectangular base. This is a 2-D check across different levels. The range query is to check the  $x, y$  coordinates of roof points  $P_r$  and base points. If the roof points lie inside the base in the nearest level above the base, they can form a roof primitive. The rectangular base can be represented by  $B_{ij} = [x: x'] \times [y: y']$  and  $P_r$  lying inside  $B_{ij}$  if

$$x_r \in [x: x'] \text{ and } y_r \in [y: y'] \quad (3)$$

The primitive is determined according to  $|P_r|$ . If  $|P_r| = 1$  and  $2$ , it is a one-ridge point roof and two-ridge points roof, respectively (Figure 2e). If an upper rectangular base  $B_{hx, h>i}$  in the nearest level exists, and satisfies

$$B_{hx} \subseteq B_{ij} \quad (4)$$

a four-ridge points roof can be formed. Then the 3-D polyhedral surfaces of a roof primitive can be formed by connecting roof points and base points (Figure 2f).

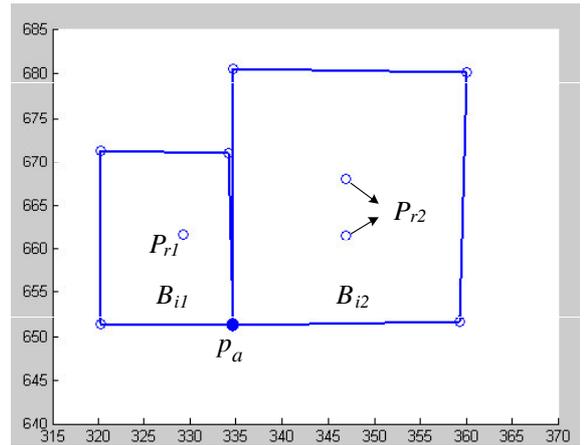


Figure 2e). Roof points inside bases.

## 2.3 Combining 3-D primitives

Since the primitive roof models have been reconstructed in the previous step, the following steps are combining primitive models by Boolean operators in order to obtain a complete building. This process is first working on 2-D horizontal level for rectangular bases and roof points separately.

1. To merge rectangular bases, if two rectangular bases  $B_{i1}$  and  $B_{i2}$  share the same auxiliary point, they can union as a new base represented as  $B_{i1} \leftarrow B_{i1} \cup B_{i2}$ . Because the auxiliary point set is created during rectangulation, these auxiliary points have to be removed.
2. Remove roof surfaces that contain auxiliary points.
3. In addition to merge roof points, if bases  $B_{i1}$  and  $B_{i2}$  belong to primitive roofs, consider linking the roof points in  $B_{i1}$  and  $B_{i2}$  if they have the same height and identical  $x$  value or  $y$  value. For example, Fig 2g shows  $y_{r1} = y_{r2}$ , where  $P_{r1} \in B_{i1}$  and  $P_{r2} \in B_{i2}$ ; hence line  $\overline{P_{r1}P_{r2}}$  indicates a roof outline.

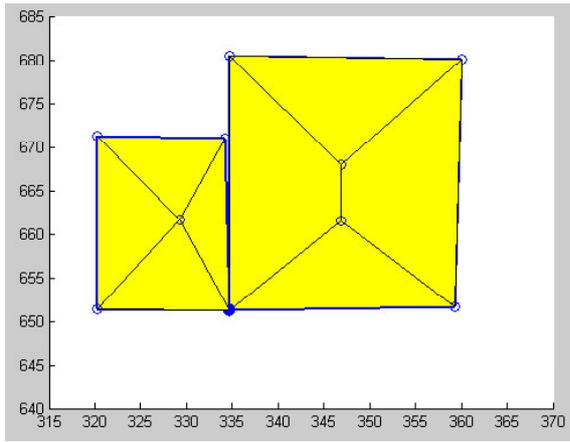


Figure 2f). Primitive roofs reconstruction.

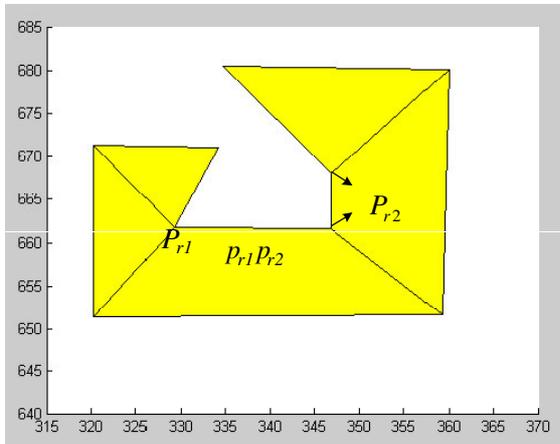


Figure 2g). Roof merging.

After merging rectangular bases and roof points, we have to combining polyhedral roof surfaces in 3-D space. Reconstruct roof surfaces to the base that merge in step 1. The assumption here is the number of lines connected from each roof ridge point to the base points is determined by  $|B_i|/|P_r|$  (Figure 2h). In this example, the number of base points is six and the number of roof ridge points is three. Therefore each roof point connects to two base points. According to this assumption, we rearrange the polyhedral surfaces as Figure 2i.

After combining primitive models together, the building can be completed by projecting the boundary outlines of roof to the

ground. Here, operators measure one footprint point of the building on the ground to indicate the height (Figure 2j).

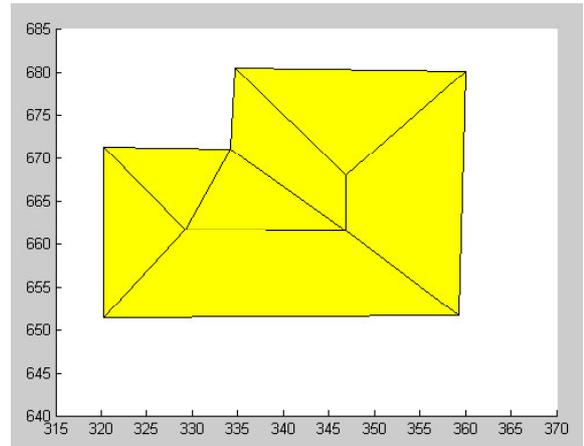


Figure 2h). Rearrange polyhedral surfaces

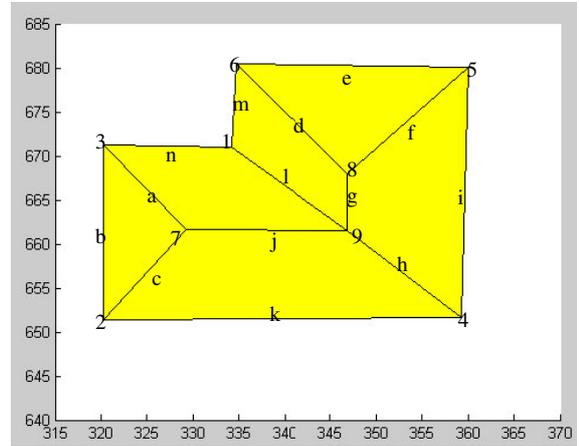


Figure 2i). Roof with labeling.

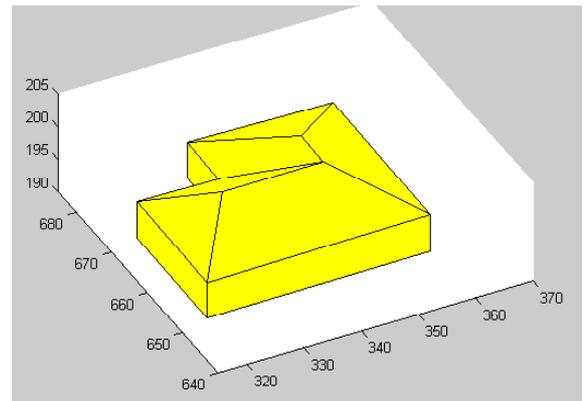


Figure 2j). Project roof to the ground.

After reconstructing the roof completely, the topologic data structure of this roof is described in Table 1. The data structure gives information about surface shapes, positions and how they are joined together. This table stores the 3-D topologic relationships among the points, edges and the polygons. The 3-D object is composed of labeling basic units including points and edges (Figure 2i). In this table, polygons are described by a sequence of edges, and edges are represented by two points without sequence. Based on this table, the roof can be

reconstructed with topological integrity. This table is a 3-D surface generalization of the well-known dual independent map encoding (DIME) structure in 2-D topology expression.

Table 1. Topological relationship of roof Figure 2i.

Polygon	Edges	Point - Point
1	a	7 - 3
	b	3 - 2
	c	2 - 7
2	d	8 - 6
	e	6 - 5
	f	5 - 8
3	g	8 - 9
	h	9 - 4
	i	4 - 5
4	f	5 - 8
	j	7 - 9
	k	9 - 4
5	c	2 - 7
	g	8 - 9
	l	9 - 1
6	m	1 - 6
	d	6 - 8
	j	7 - 9
7	l	9 - 1
	n	1 - 3
	a	3 - 7

### 3. TEST AND RESULTS

In this study, the test data results from a pair of stereo images at the scale of 1:4000. The location is Purdue University campus. These distinct building points were measured manually under stereo mode. During the measurement, operators need to estimate the location of points in hidden areas, and all roof corners must be completely denoted. In addition to these roof points, operators also need to obtain one footprint of the building on the ground to define the building base height. The height information is a key issue before the 3-D reconstruction process, because our approach needs to initially level the point clouds and finally project the building outlines to the ground. After the data collection, each building is one unit and is reconstructed independently according to the procedures described in last section. During the reconstructing process, selecting tolerance parameters is necessary because the digitized data may not be perfectly accurate. For the data measurement error in most buildings, we apply a height tolerance 0.7 m, beyond which data will be separated to different levels. Moreover, in 2-D XY-plane, we apply a value of  $\pm 5$  degree for deviation from perpendicularity, respectively in x and y directions. Figure 3 shows four building images and their building structures reconstructed by using our approach.

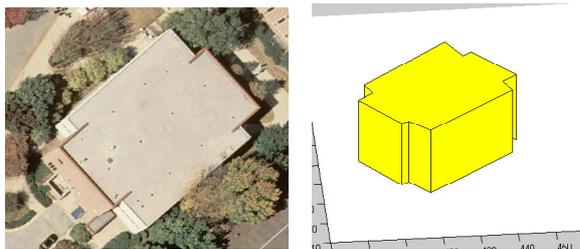


Figure 3a)

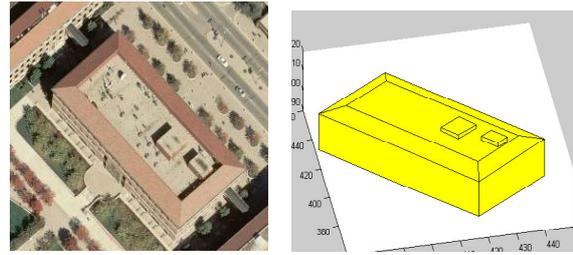


Figure 3b)

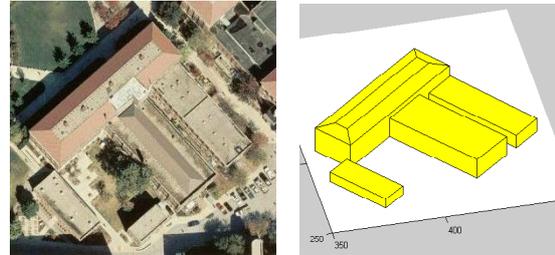


Figure 3c)

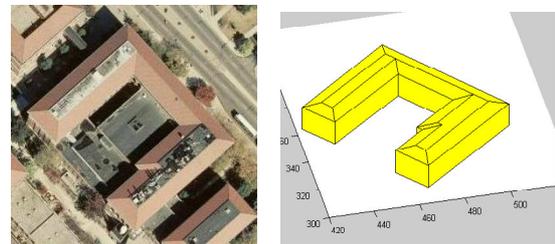


Figure 3d)

Figure 3. Examples of reconstructed buildings.

In Figure 3a, the building is a simple polygon with one level. Therefore, this level is partitioned into three rectangular bases, which are then merged together to form the outline. Its outline is directly projected into the ground to form the vertical walls. Figure 3b shows a standard four-ridge points roof contains two small rectangular rooftops. After rectangulation process, the highest level has two rectangular bases contained by the rectangular base just below it. Since our primitive models did not include this situation, we make an assumption that these belong to rooftops. In this case, we project these two polygons directly to the level below them to form two small structures. A more complicated case is shown in Figure 3c. Notice that the building union in Figure 3c includes four structures. They are one four-ridge roof model and three simple flat polygons. These rectangular bases in this union can be distinguished and reconstructed correctly and simultaneously by our approach. Another complex building is shown in Figure 3d. In the building image, this is a combination of two-ridge points roofs and four-ridge points roof structure. Nevertheless, because we suitably adopt auxiliary points during rectangulation and remove the corresponding auxiliary lines after merging, all the roof outlines are still illustrated well by merging different roof models. Once the roof is reconstructed, the connectivity between roofs and boundary outlines are correctly performed. These examples indicate that our method for building outlines with right angle corners presents quite satisfactory results.

Figure 4 shows the results obtained by applying our methodology to the Purdue campus. Most campus buildings in Figure 4a can be decomposed into several parts, and each part is well reconstructed by our method. Notice that detail structures, such as small rooftop structures, have also been reconstructed.

The topological structure of boundary part is unique when there is exactly one given boundary model if coplanar bases are merged into one so that the base is forced to have maximum extent. Since some campus buildings include combinations of boundaries with different heights, boundary models are not closed under set operations. The union of two different height level boundary models does not result in a new valid boundary model. The edges or the polygon of one solid boundary touches any element of the other. Under this circumstance, these Boolean set operations become very difficult dealing with boundary walls. However, the data structure is still useful for modeling a man-made object including visualization tasks, architectural reconstruction and geospatial query. Figure 4b shows the residential buildings around the campus. Most buildings are well reconstructed. However, roofs of residential buildings show much more diversity than campus building. Some complicated roofs have one rectangular base, but roof points are not included in our primitive models. The approach fails to reconstruct buildings in those cases. This can be improved by applying more complex roof models in the future. The other reason for those features that cannot be correctly reconstructed is the erroneous data or missing points occur during measurement. Such faultiness certainly can be minimized through more careful data acquisition.

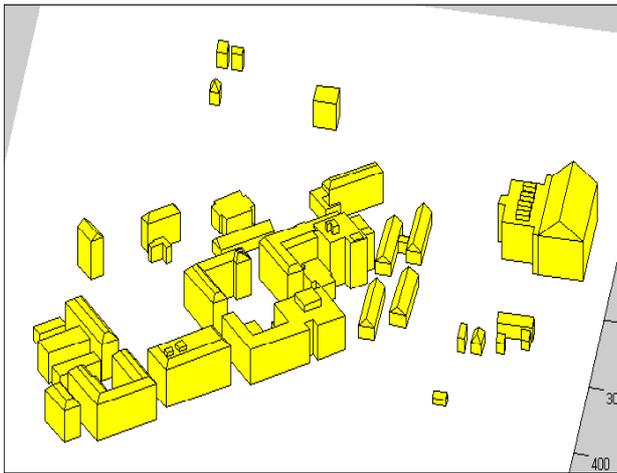


Figure 4a). School buildings

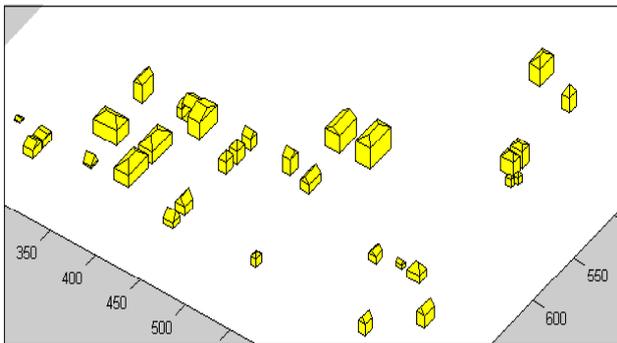


Figure 4b). Residence houses

Figure 4. Reconstructed Purdue University campus buildings

#### 4. CONCLUSION

In this study, we have developed a methodology for 3-D building reconstruction from unstructured distinct points. The

underlying mechanism is that the buildings with right angle corners can be rectangulated on each height level. Based on this, we propose a novel rectangulation approach to regularly construct these unstructured points.

The rectangulation process facilitates the model identification of roof structures according to the construction between the roof points and corresponding closest rectangular bases. Moreover, once the roof models are determined, merging the rectangular bases also can easily outline the building boundary and finally reconstruct the building by including the vertical walls. In this paper, such a rectangulation approach for 3-D building reconstruction is introduced through regularization procedures. Through this key issue, the unstructured data points initially without any sequence can be constructed step by step from model identification to building-boundary projection.

To demonstrate our methodology, we present successfully reconstructed Purdue University campus building and their comparison with aerial images. Because we consider three type CAD roof models only, some complicated roof types require more primitive models included. Our experience also shows that a reliable rectangulation approach is necessary for model classification. Buildings with non-right angle corners need certain modification and adaptation of the reported hierarchical methodology.

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