Don't Blow Your Airlock!
Solution for AAE 334 Extra Credit Project

Jonathan Poggie
School of Aeronautics and Astronautics, Purdue University
jpoggie@purdue.edu

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In the novel "The Martian" by Andy Weir (Crown Publishing, New York, 2014), and in the recent movie (20th Century Fox, 2015), the crew of a Mars-bound spacecraft need to decelerate their ship to rescue their lost crewmate. As a desperate measure to do this, they use a bomb to blow a hole in an airlock, and that produces thrust via a jet of air into space. According to the book, and some statements by the author online, the jet would generate a net change in velocity of 30 m/s and the spacecraft had a mass of 110 Mg. From stills from the movie, the spacecraft looked like a cylinder about 100 m long and 2–3 m in diameter. The airlock looked to be about 2 m in diameter, and the hole would be smaller than that.

To analyze this problem, we'll assume uniform conditions inside the spacecraft, and that the process of emptying the ship is a constant volume, isentropic process. We'll determine the drop in density with time from the continuity equation. Since the flow discharges to vacuum, we'll assume quasi-steady choked flow at the hole in the airlock. These are reasonable assumptions as long as the ship empties on a short time scale compared to the heat transfer rate to space, and over a long time scale relative to the time for an acoustic wave to traverse the inside of the ship. These assumptions permit the assumption of quasi-steady, adiabatic flow, and are consistent with the time scales on the order of minutes portrayed in the book and the movie.

Applying a control volume analysis in the (accelerating) reference frame of the ship, the conser-

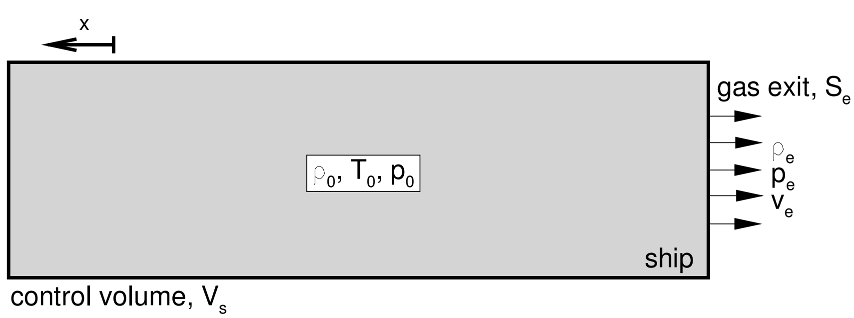

Figure 1: Diagram of the control volume around the ship. Note the the positive x-direction is to the left.

vation of mass gives us:

$$\frac{d}{dt}\int_{\mathcal{V}} \rho \, d\mathcal{V} = -\int_{\mathcal{S}} \rho \, \mathbf{u}_r \cdot \mathbf{n} \, d\mathcal{S} \tag{1}$$

where the relative velocity at the surface $\mathcal{S}$ of the control volume $\mathcal{V}$ is $\mathbf{u}_r = \mathbf{v} - \mathbf{w}$. Here $\mathbf{v}$ is the fluid velocity, $\mathbf{w}$ is the velocity of the control volume's surface, and ρ is the fluid density. We attach our reference frame to the solid shell of the spacecraft, and measure velocities in this reference frame. A schematic diagram of the control volume problem is given in Figure 1. Assuming uniform flow at the exit of the hole, and uniform conditions in the spacecraft, the mass conservation equation (1) becomes:

$$\mathcal{V}_s \frac{d\rho_0}{dt} = -\rho_e v_e S_e \tag{2}$$

The term $\dot{m} = \rho_e v_e S_e$ is the mass flow rate through the hole in the airlock. We'll use a subscript s to indicate quantities associated with the ship, 0 to denote properties of the uniform, still air in the ship, and e to denote properties at the hole where the gas exits.

A corresponding control volume analysis of momentum conservation gives:

$$\frac{d}{dt}\int_{\mathcal{V}} \rho \mathbf{v} \, d\mathcal{V} = -\int_{\mathcal{S}} \rho \mathbf{v} \, \mathbf{u}_r \cdot \mathbf{n} \, d\mathcal{S} + \int_{\mathcal{S}} \mathbf{n} \cdot \mathbb{T} \, d\mathcal{S} + \int_{\mathcal{V}} (\mathbf{f} - \rho \mathbf{a}_r) \, d\mathcal{V} \tag{3}$$

where $\mathbb{T}$ is the stress, $\mathbf{f}$ is the body force, and $\mathbf{a}_r$ is the acceleration of the reference frame (that is, the ship). Under our assumptions, the momentum equation (3) becomes:

$$0 = \rho_e v_e^2 S_e + p_e S_e - m_s \frac{dv_s}{dt} \tag{4}$$

The signs of the terms correspond to a positive x-axis in the direction of the ship's motion (Figure 1). Here we've neglected the motion of the center of mass of the control volume relative to the rocket shell, which would be the main contribution the left hand side of equation (3). Note that the total mass of the ship $m_s = m_{s0} + \rho_0 \mathcal{V}_s$ is allowed to vary with time here, but that effect is very small for our assumptions. The term $F = (\rho_e v_e^2 + p_e) S_e$ is the usual expression for rocket thrust into a vacuum.

We thus have two differential equations for the density and velocity:

$$\frac{d\rho_0}{dt} = -\rho_e v_e S_e / \mathcal{V}_s \tag{5}$$

$$\frac{dv_s}{dt} = \left(\rho_e v_e^2 + p_e\right) S_e / m_s \tag{6}$$

To close the system, we use isentropic relations for the still, uniform gas inside the ship:

$$p_0 = p_{01} \left(\frac{\rho_0}{\rho_{01}}\right)^{\gamma} \tag{7}$$

$$T_0 = T_{01} \left(\frac{\rho_0}{\rho_{01}}\right)^{\gamma - 1} \tag{8}$$

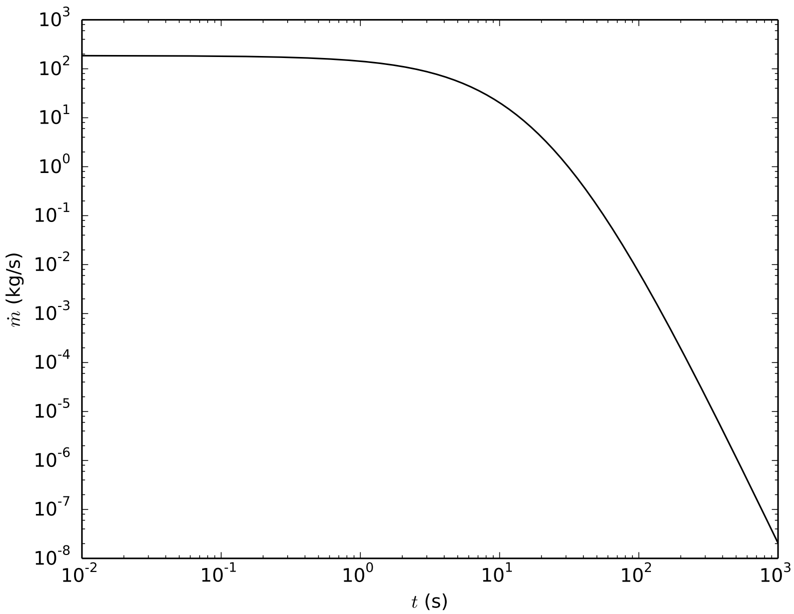

Figure 2: Mass flow rate through the hole in the airlock.

and assume sonic (choked) conditions at the hole in the airlock:

$$\rho_e = \left[\frac{2}{\gamma+1}\right]^{1/(\gamma-1)} \rho_0 \tag{9}$$

$$p_e = \left[\frac{2}{\gamma+1}\right]^{\gamma/(\gamma-1)} p_0 \tag{10}$$

$$v_e = \left[\frac{2}{\gamma+1}\right]^{1/2} \sqrt{\gamma R T_0} \tag{11}$$

Here we'll use $\gamma = 1.4$ and $R = 287$ J/(kg K) for air.

The volume of the ship is $\mathcal{V}_s = \pi D_s^2/4\ L_s$, where $D_s = 3$ m is the diameter and $L_s = 100$ m is the length. We'll assume that the initial pressure and temperature are $p_{01} = 101300$ Pa and $T_{01} = 300$ K. The initial mass of the ship is 1.1×10^5 kg. The area of the hole in the airlock is $S_e = \pi D_e^2/4$, where the diameter of the hole is $D_e = 1$ m.

Integrating the system in time gives the mass flow rate, thrust, and change in velocity of the ship. The mass flow rate through the hole $\dot{m} = \rho_e v_e S_e$ is shown in Figure 2. Because of the rapid change with time, the results are shown on a log-log plot. The mass flow rate is initially quite high, nearly 200 kg/s, but becomes negligible within a couple of minutes. The corresponding thrust $F = (\rho_e v_e^2 + p_e) S_e$ is shown in Figure 3. Again, the initial thrust of around 1×10^5 N drops to essentially zero in a couple of minutes. In the same timeframe, the initially rapid change in velocity (Figure 4) plateaus to $\Delta v_s = 3.4$ m/s.

We can make an order of magnitude estimate of Δv_s from overall conservation of momentum. The initial system consists of the air and the ship together at rest in our frame of reference, and the

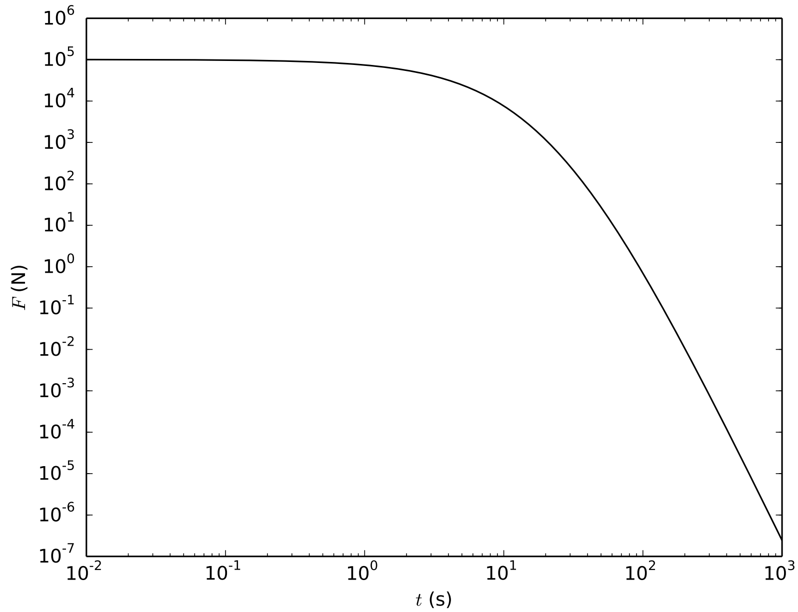

Figure 3: Thrust produced by the jet of air.

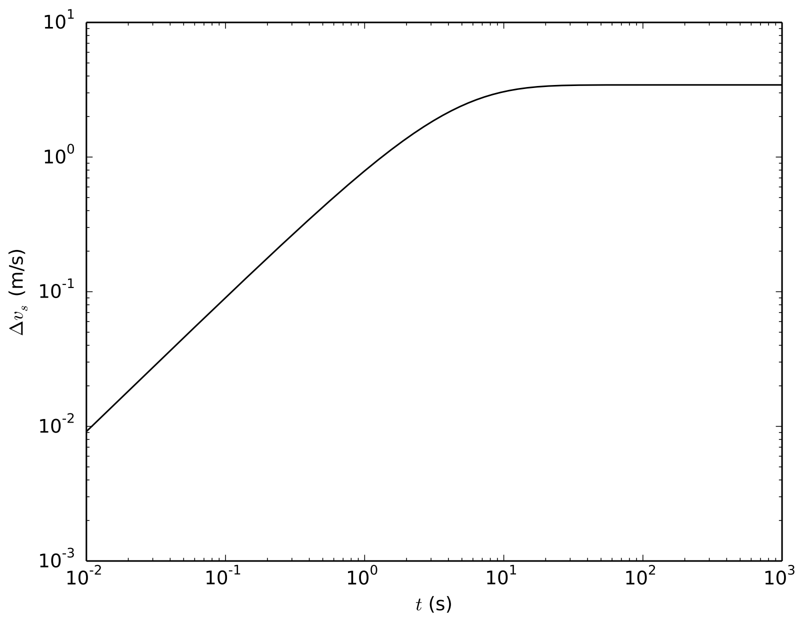

Figure 4: Change in velocity Δv_s for the ship.

final system consists of the two masses traveling apart with equal and opposite momentum. If we assume that the mass of air moves at approximately the initial sound speed, the ship must move in the opposite direction at $v_s = \rho_{01}\mathcal{V}_s a_{e1}/m_{s0} \approx 2$ m/s. So the order of magnitude of Δv_s is about right in our calculation. (The estimate is a bit low because the exit pressure makes a significant contribution to the momentum flux in sonic flow.)

Thus the change in velocity is an order of magnitude lower than that portrayed in the story. Perhaps our initial estimates of the size of the hole or the ship are incorrect. First let's consider the hole. What if the blast destroyed the whole airlock door, so that the diameter of the hole is equal to that of the ship ($D_e = 3$ m)? The result is that the air escapes an order of magnitude faster, but the change in ship's velocity is about the same. If the hole is smaller ($D_e = 0.1$ m), the air release is slower, but Δv_s is again about the same. What if we grossly underestimated the length of the ship? What if it is $L_s = 1000$ m long? In that case, the thrust lasts much longer, and we finally get a value comparable to that portrayed in the story, $\Delta v_s = 35$ m/s.

So the movie misrepresented the ship. It was much larger, and contained more air. Nonetheless, in space travel it is prudent to save some air for the trip home.

Further Reading and Investigation

It would be interesting to do an analysis of the unsteady expansion wave system for the case where the gas left the ship very rapidly, on the order of the time for an acoustic wave to propagate across the ship. As another limiting case, it would be interesting to look at the case where all the initial thermal energy of the gas is converted to kinetic energy.

For further reading on control volumes in arbitrary motion, see Hansen (1965, 1967), Panton (1984, pp. 127–135), Thorpe (1962), Thompson (1972, pp. 39–47), and White (1986, pp. 121–122,142–144). For analysis of the tank emptying problem, see Dutton and Coverdill (1997). Rocket thrust problems are explained nicely in Thompson (1972, pp. 43–47).

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