

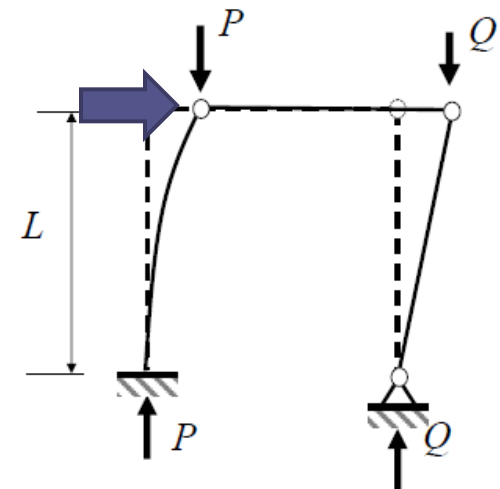
# Design for Stability

CE470 Spring 2014

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# What is Stability?

- “the capacity of a compression member, element, or frame to remain in position and support load, even if forced slightly out of line or position by an added lateral force” (Galambos, 1998)



# 5 factors that influence stability

- Member, component, and connection deformations
- Second-order effects ( $P-\Delta$  and  $P-\delta$ )
- Geometric imperfections (out-of-plumbness or out-of-straightness)
- Stiffness reductions due to inelasticity (residual stresses)
- Variability in component and system stiffness

# AISC Chapter C: Design for Stability

- Direct Analysis Method of Design
  - Calculation of required strengths – Section C2
  - Calculation of available strengths – Section C3
- Alternative Methods of Design
  - “Effective Length” method and first-order analysis method permitted for structures that satisfy the constraints in Appendix 7
- Use of approximate method of second-order analysis (B1, B2) permitted as an alternative (Appendix 8) to “rigorous” second-order analysis

# Appendix 7 - Constraints

## 7.2. EFFECTIVE LENGTH METHOD

### 1. Limitations

$$B_2 = \frac{\Delta_{2nd-order}}{\Delta_{1st-order}} \leq 1.5$$

The use of the *effective length* method shall be limited to the following conditions:

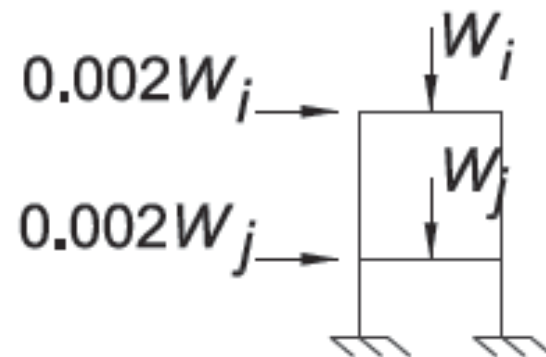
- (1) The structure supports *gravity loads* primarily through nominally vertical *columns*, walls or frames.
- (2) The ratio of maximum second-order *drift* to maximum first-order drift (both determined for *LRFD load combinations* or 1.6 times *ASD load combinations*) in all stories is equal to or less than 1.5.

**User Note:** The ratio of second-order drift to first-order drift in a story may be taken as the  $B_2$  multiplier, calculated as specified in Appendix 8.

# Appendix 7 - Required Strengths

## 2. Required Strengths

The *required strengths* of components shall be determined from analysis conforming to the requirements of Section C2.1, except that the *stiffness* reduction indicated in Section C2.3 shall not be applied; the nominal stiffnesses of all *structural steel* components shall be used. *Notional loads* shall be applied in the analysis in accordance with Section C2.2b.



# Appendix 7 - Available Strengths

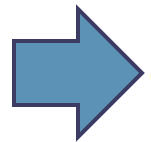
- (b) In *moment frame* systems and other structural systems in which the flexural stiffnesses of columns are considered to contribute to lateral stability and resistance to lateral loads, the effective length factor,  $K$ , or elastic critical *buckling* stress,  $F_e$ , of those columns whose flexural stiffnesses are considered to contribute to lateral stability and resistance to lateral loads shall be determined from a *side-sway buckling* analysis of the structure;  $K$  shall be taken as 1.0 for columns whose flexural stiffnesses are not considered to contribute to lateral stability and resistance to lateral loads.

Exception: It is permitted to use  $K = 1.0$  in the design of all columns if the ratio of maximum second-order *drift* to maximum first-order drift (both determined for *LRFD load combinations* or 1.6 times *ASD load combinations*) in all stories is equal to or less than 1.1.

**User Note:** Methods of calculating the effective length factor,  $K$ , are discussed in the *Commentary*.

# Effective Length Method

- Design process



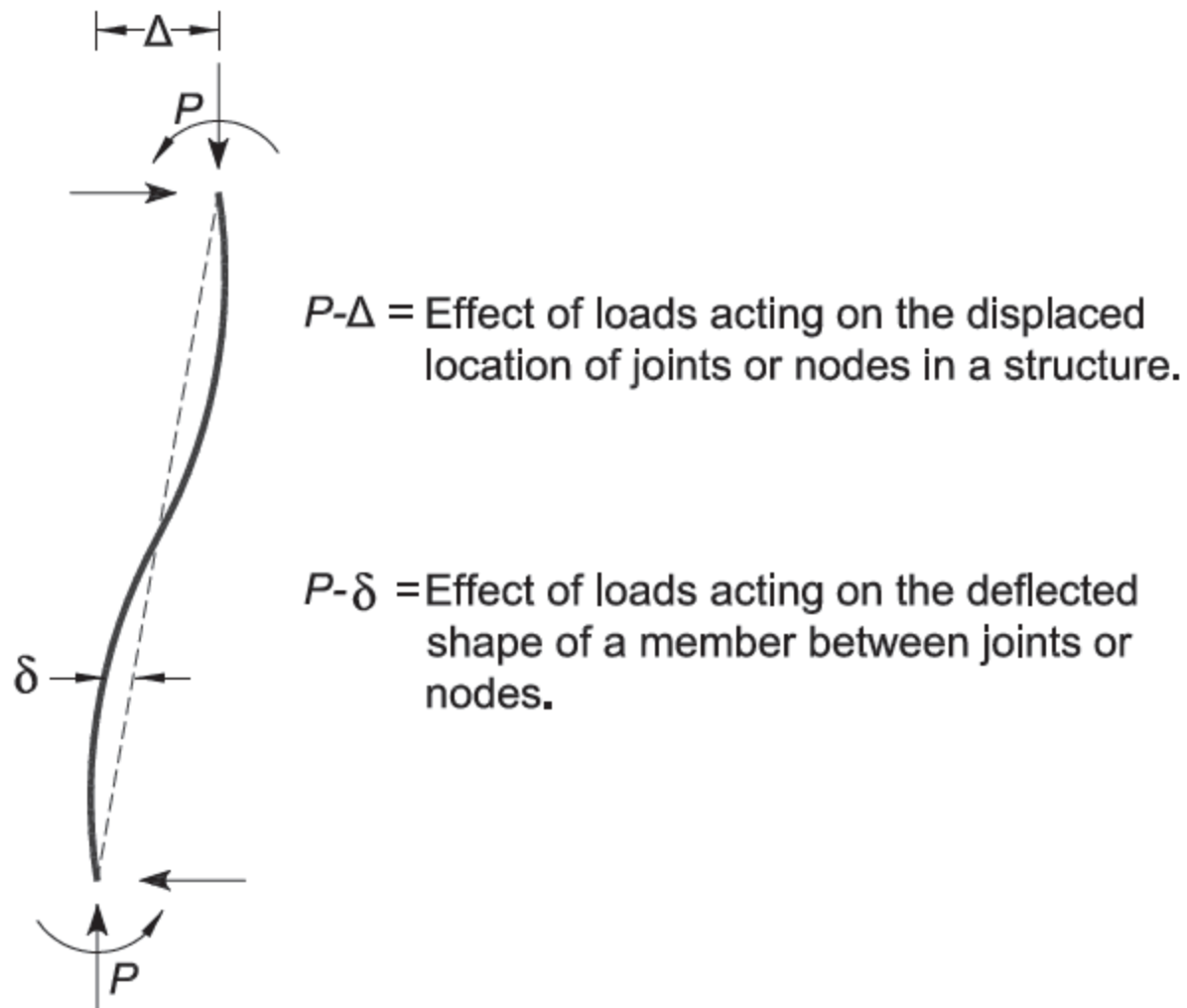
- Apply notional loads,  $N_i = 0.002\alpha Y_i$  in the gravity only load cases

*“B1, B2” OK*

- Perform a second-order elastic analysis
- Determine  $K$ -factor from a sidesway buckling analysis
- Check strength using interaction equations

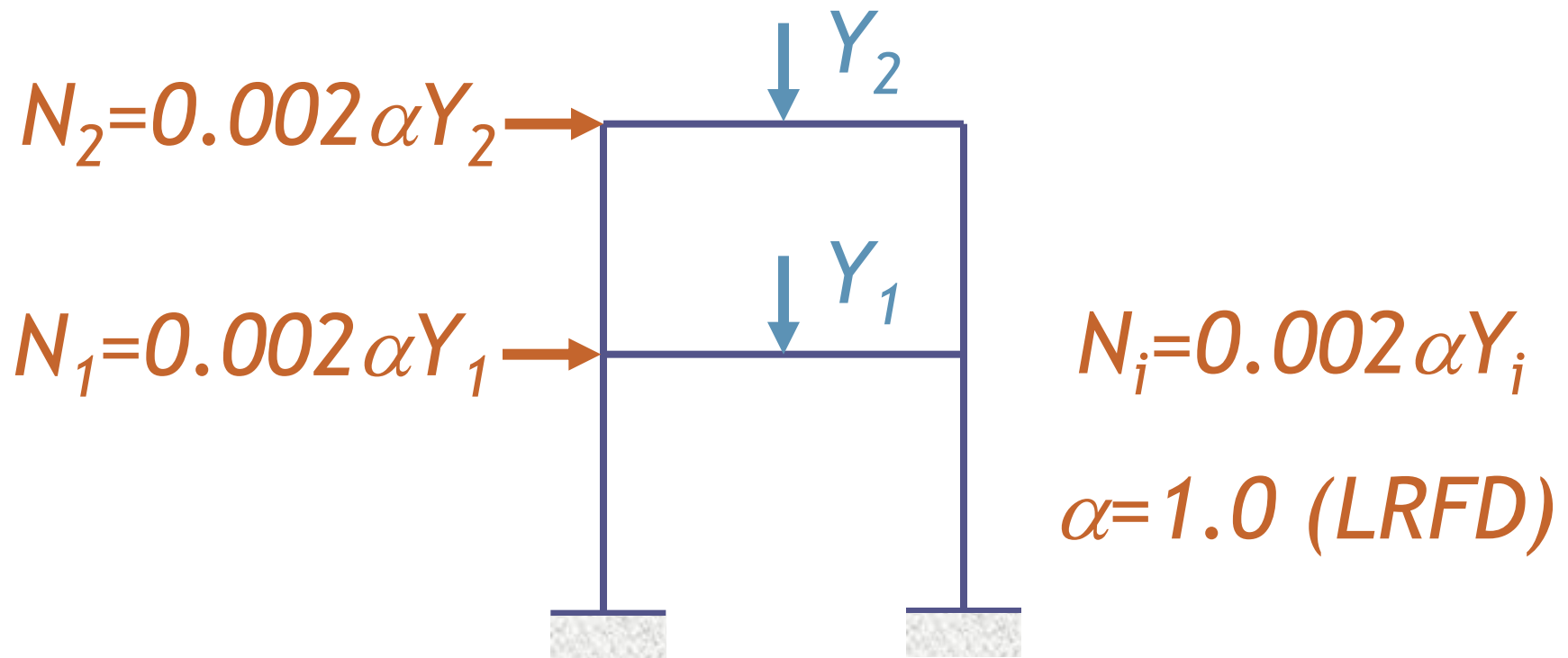




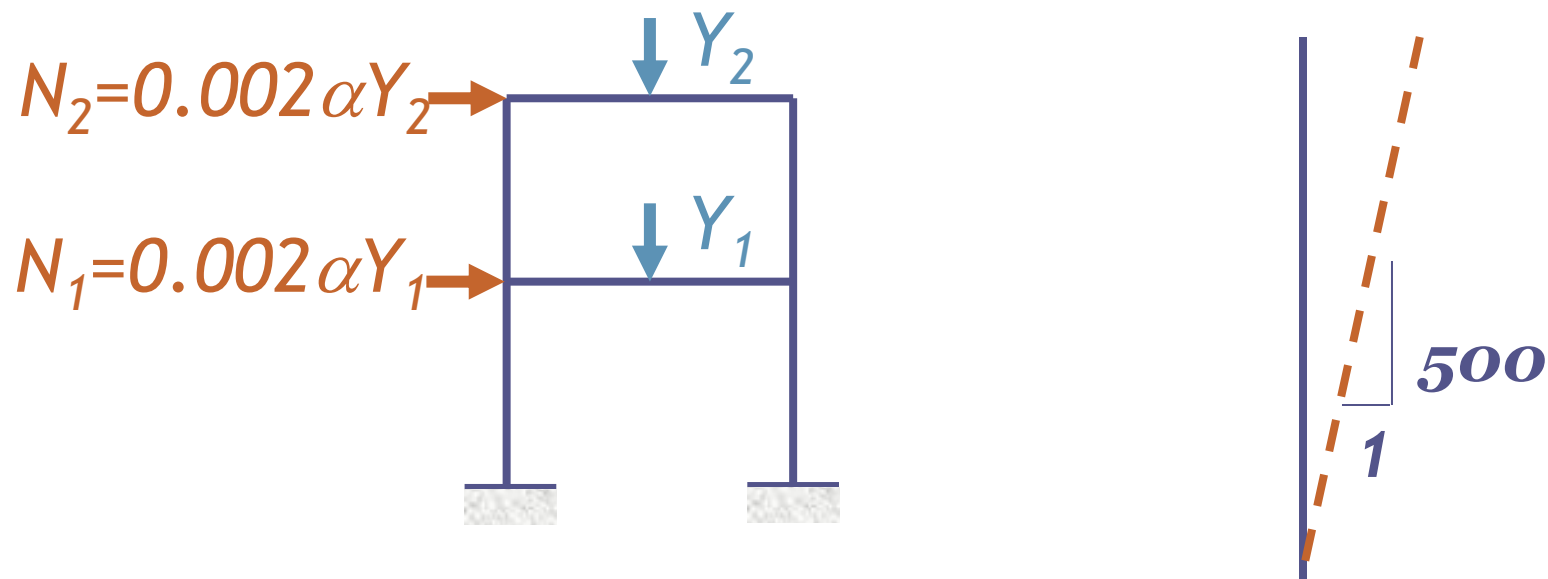


*Fig. C-C2.1. P- $\Delta$  and P- $\delta$  effects in beam-columns.*

# Notional Loads

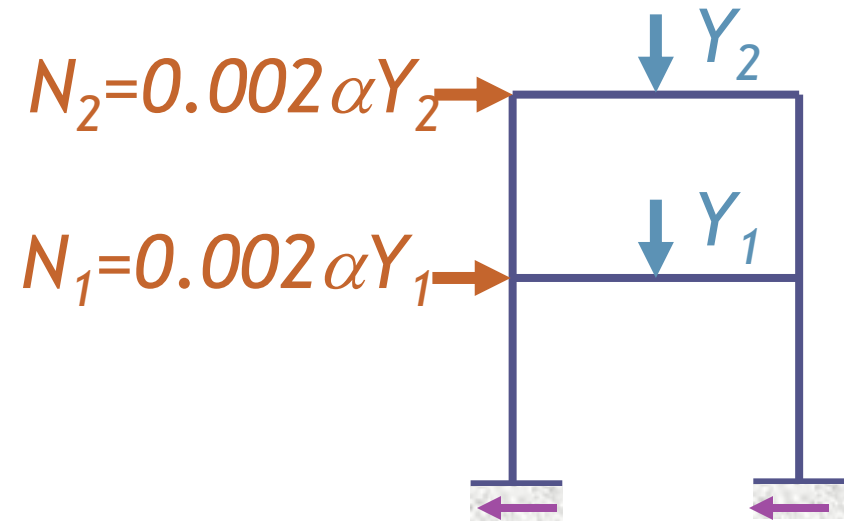


# Notional Loads



Member “out-of-plumbness”

# Notional Loads



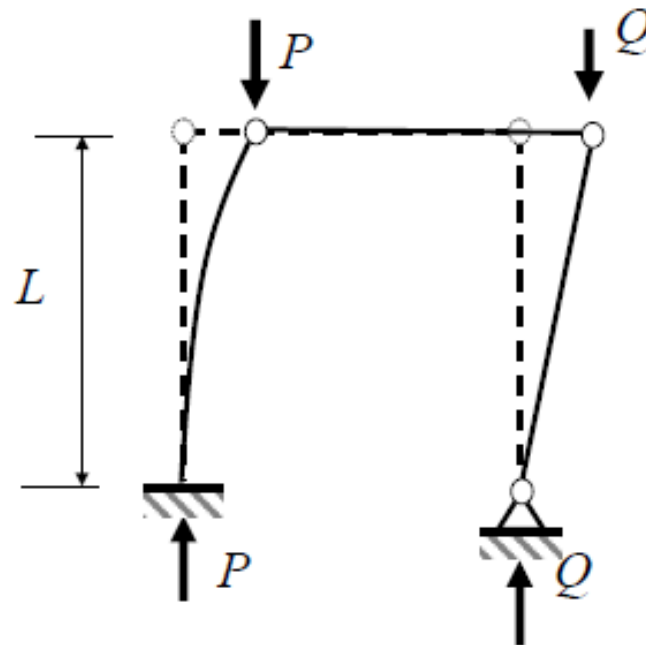
**User Note:** The notional loads can lead to additional (generally small) fictitious base shears in the structure. The correct horizontal reactions at the foundation may be obtained by applying an additional horizontal force at the base of the structure, equal and opposite in direction to the sum of all notional loads, distributed among vertical load-carrying elements in the same proportion as the gravity load supported by those elements. The notional loads can also lead to additional overturning effects, which are not fictitious.

# Direct Analysis Method - Advantages

- No K-factor calculations ( $K = 1.0$ )
  - Good! K often difficult to predict; alignment charts utilize many assumptions
- Internal forces more accurately represented
  - At ultimate capacity (e.g., inelasticity)
- Method applies logically & consistently to all framing systems
  - Braced frames, moment frames, trusses, etc.
- More economical in some cases

Just one example for K ...

## Buckling with Gravity Only Columns



$$K_n = K_o \sqrt{1 + \frac{Q}{P}}$$

No load on gravity only column  
 $Q = 0, K = 2.0$

Equal loads on restraining and  
gravity only column  
 $Q/P = 1, K = 2.8$

Other combinations  
 $Q/P = 2, K = 3.46$

$Q/P = 10, K = 6.63$

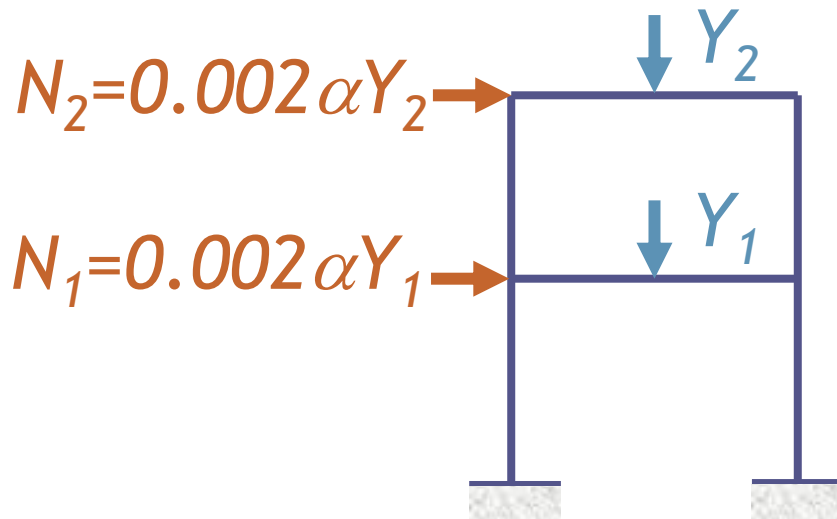


NASCC: The Steel Conference, Toronto, March, 2014  
There's always a solution in steel!

What would you do if there  
were no load  $P$ ?

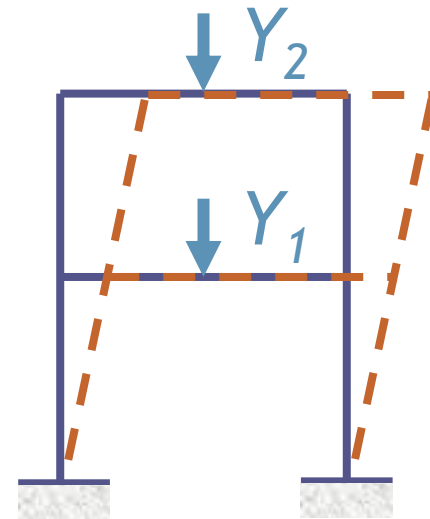
N40.73

# Direct Analysis Method



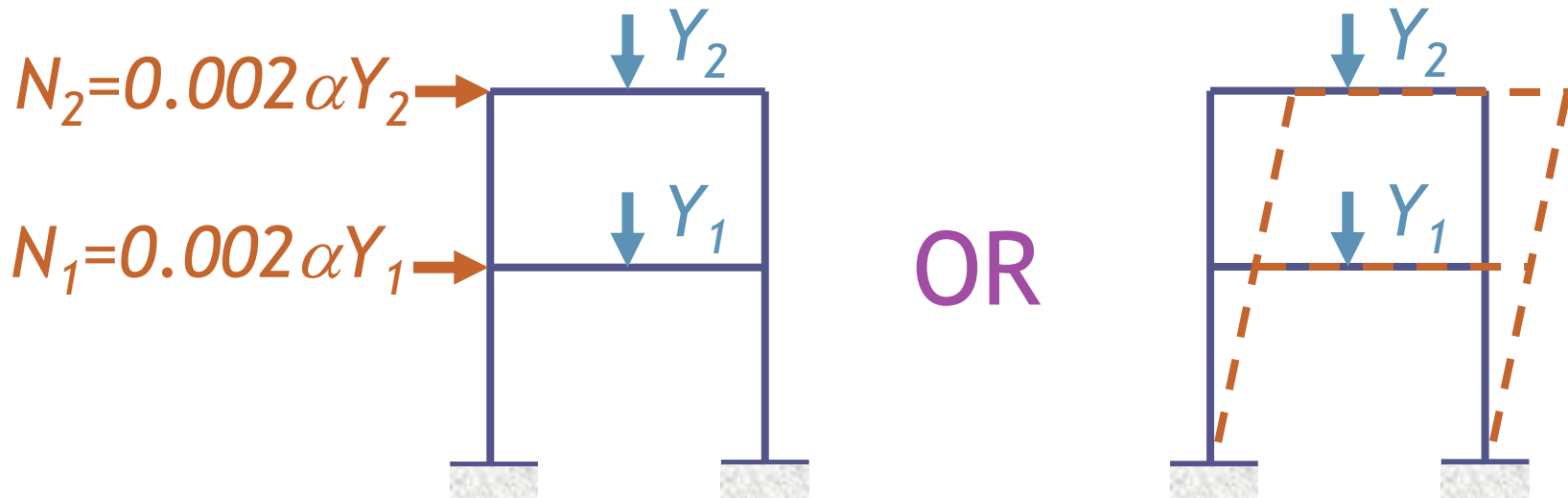
Notional Loads

OR



Direct Modeling of Imperfections

# Adjustments to Stiffness



Reduced stiffnesses  $0.8EA$  and  $0.8\tau_b EI$   
(including in  $B_1$ - $B_2$  amplification, if used)



# Adjustments to Stiffness

(a) When  $\alpha P_r/P_y \leq 0.5$

$$\tau_b = 1.0 \quad (\text{C2-2a})$$

(b) When  $\alpha P_r/P_y > 0.5$

$$\tau_b = 4(\alpha P_r/P_y)[1 - (\alpha P_r/P_y)] \quad (\text{C2-2b})$$

where

$\alpha = 1.0$  (LRFD);  $\alpha = 1.6$  (ASD)

$P_r$  = required axial compressive strength using *LRFD* or *ASD load combinations*, kips (N)

$P_y$  = axial *yield strength* ( $= F_y A_g$ ), kips (N)

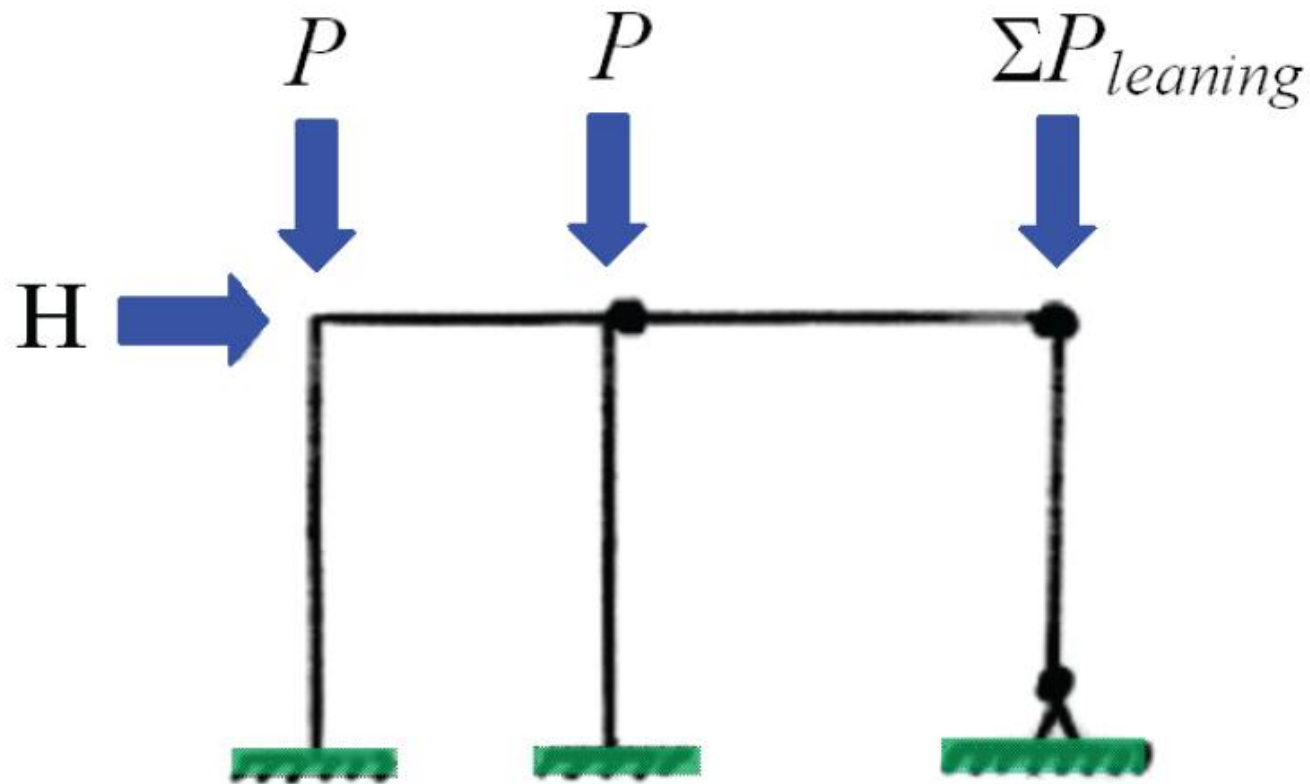


Figure 3. 2D frame model that captures leaning column effects.

“Stability Analysis: It’s not as Hard as you Think,” C.M. Hewitt, 2008.

# 5 factors that influence stability

## Effective Length Method

- Member, component, and connection deformations – addressed directly in analysis
- Second-order effects ( $P-\Delta$  and  $P-\delta$ ) – rigorous second-order analysis or first-order with B1-B2
- Geometric imperfections (out-of-plumbness [notional loads] or out-of-straightness [column design equations for effect on member strength])

## 5 factors that influence stability

### Effective Length Method, cont'd.

- Stiffness reductions due to inelasticity (residual stresses) -- column design equations for effect on member strength
- Variability in component and system stiffness — in resistance and safety factors!

But  $K$  must be calculated and used to account for effects on structure stiffness of member out-of-straightness, residual stresses, and variability in component and system stiffness.

# 5 factors that influence stability

## Direct Analysis Method

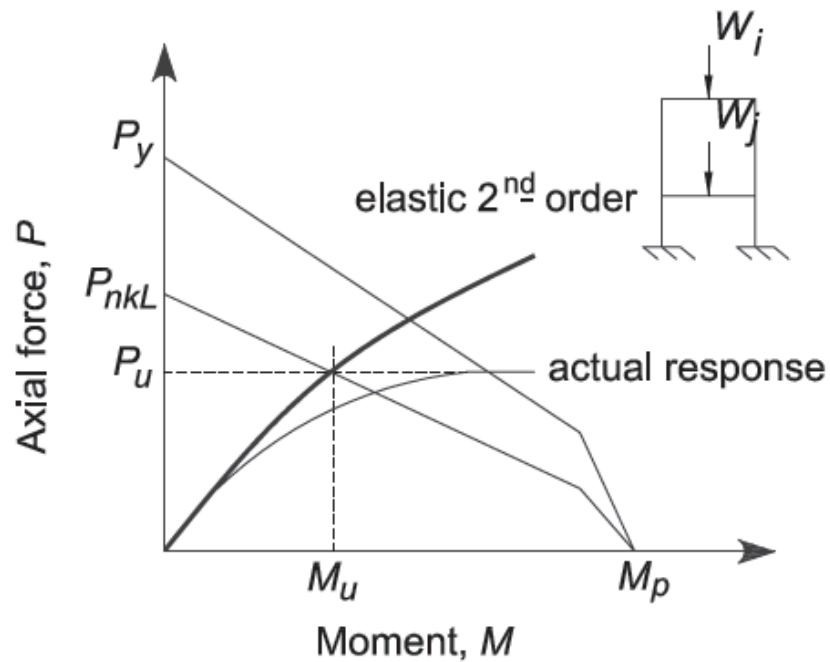
- Member, component, and connection deformations – **addressed directly in analysis**
- Second-order effects ( $P-\Delta$  and  $P-\delta$ ) – **rigorous second-order analysis or first-order with B1-B2**
- Geometric imperfections (out-of-plumbness [**notional loads** or direct modeling] or out-of-straightness [**column design equations; and reduced stiffness for effect on structure stiffness**])

## 5 factors that influence stability

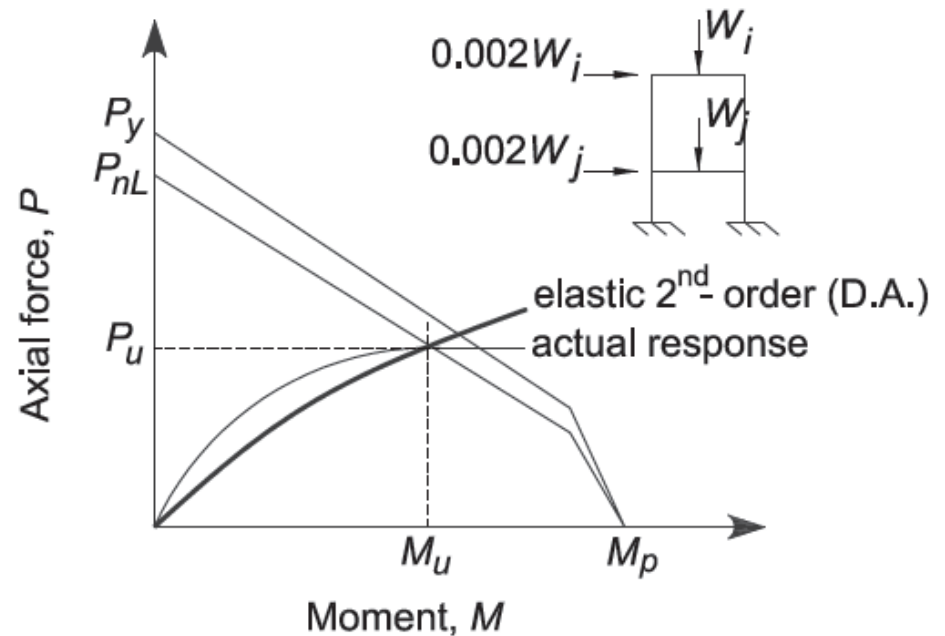
### Direct Analysis Method, cont'd.

- Stiffness reductions due to inelasticity (residual stresses) -- **column design equations**; and reduced stiffness for effect on structure stiffness
- Variability in component and system stiffness — **in resistance and safety factors!**

## AISC C-C2.3

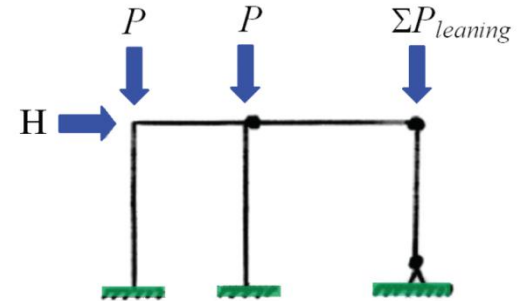


(a) Effective length method



(b) Direct analysis method

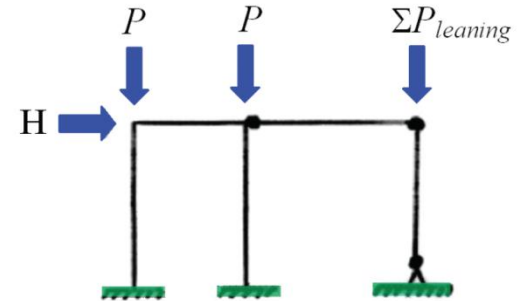
# Direct Analysis Method “Step-by-Step”



- Create a model of the lateral force resisting frame, including the leaning columns.
- Reduce the stiffnesses (modify  $E$ ) of the lateral framing members in your model.
- Apply notional loads or directly model the imperfections
- Conduct a second-order analysis (“rigorous” or B1-B2 amplification on first-order)



## “Step-by-Step” cont’d.



- Design members using the AISC Specification and  $K=1.0$ .
  - If using structural analysis program to do this, reset  $E$  to 29,000 ksi.
- Check lateral drift limits for wind and seismic.