Local State-Feedback Stabilization of Nonlinear Fuzzy Systems with Measurement Errors in the State

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***Abstract*— This paper deals with the local state-feedback control design problems for continuous-time Takagi–Sugeno (T–S) fuzzy systems with the measurement errors in the state. The state-feedback controller is designed in such a way that the closed-loop system is locally asymptotically stable. The stability is guaranteed to be robust against the measurement errors in the state. In addition, an inner estimation of the domain of attraction (DA) is obtained as a sublevel set of the obtained quadratic Lyapunov function. The design procedure is formulated as optimizations subject to linear matrix inequalities (LMIs).**

I. Introduction

The Takagi–Sugeno (T–S) fuzzy system is a nonlinear system description, which can express a general nonlinear system locally as a convex combination of linear subsystems. One of the benefits of using the T–S fuzzy system is that in conjunction with the Lyapunov stability theory, many nonlinear system analysis and control synthesis problems can be formulated as convex linear matrix inequalities (LMIs), for which several optimization tools are available [1], [2]. For this reason, the nonlinear control system design and analysis based on the T–S fuzzy system have attracted attention in the last decades.

The simplest Lyapunov approach is to use the quadratic Lyapunov function [3]–[5], which is in general overly conservative, because a common Lyapunov matrix should be found for all subsystems of the fuzzy systems. To reduce the conservatism, many efforts have been made to date. Nowadays, there is immense literature addressing the relaxation problem through various approaches, for instance, the slack variable approaches [6], [7]; LMI conditions using information on the membership functions' shape [8], [9]; convergent LMI relaxation techniques that exploit Pólya's theorem [10], approaches based on piecewise Lyapunov functions [11]–[13], the fuzzy Lyapunov functions [14]–[18], a class of Lyapunov functions using line integral [19], the polynomial Lyapunov functions [20]–[23], the switching polynomial Lyapunov function [24], the polynomial fuzzy Lyapunov functions [25]–[29], the multiple samples variation approaches [30], [31]; approaches using new bounding techniques on the time derivative of the membership functions [32]–[34]; the local stability and stabilization approaches [35]–[38] that guarantee the asymptotic stability only in some local region of the state space.

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The goal of this paper is to develop a locally stabilizing state-feedback control design method for the continuous-time nonlinear fuzzy systems with measurement errors in the state. Motivated by the recent development on the local stability of the discrete-time nonlinear fuzzy systems in [38], an LMI-based optimization procedure is developed to design a locally stabilizing state-feedback controller. The state-feedback controller is robust against the errors in the state. Moreover, the design procedure provides a way to compute an inner estimate of the domain of attraction (DA) as a sublevel set of the Lyapunov function. Examples will be given to illustrate the proposed method.

II. Preliminaries

A. Notation

The adopted notation is as follows: A^T: transpose of matrix A; ($A \prec 0$, $A \succeq 0$, and $A \preceq 0$, respectively): symmetric positive definite (negative definite, positive semi-definite, and negative semi-definite, respectively) matrix A; I_n and $0_{n\times m}$: identity matrix and $n \times m$ zero matrix, respectively; 0_n: origin of $\mathbb{R}^n$; $\mathrm{He}\{A\}$: short hand notion for $A^T + A$; $\mathrm{co}\{\cdot\}$: convex hull [41]; * inside a matrix: transpose of its symmetric term; $\mathcal{I}_N := \{1, 2, \ldots, N\}$; $\mathrm{diag}\{A, B\}$: block diagonal matrix with blocks A and B. Throughout the paper, the following shorthand is used for ease of notation.

$$\alpha(z(t)) := \begin{bmatrix} \alpha_1(z(t)) & \alpha_2(z(t)) & \cdots & \alpha_N(z(t)) \end{bmatrix}^T \in \mathbb{R}^N$$

$$\Upsilon(\alpha) := \sum\nolimits_{i=1}^{N} \alpha_i(z(t))\Upsilon_i$$

$$\Upsilon(\alpha^2) := \sum\nolimits_{i=1}^{N}\sum\nolimits_{j=1}^{N} \alpha_i(z(t))\alpha_j(z(t))\Upsilon_{ij}$$

For any function $f : \mathbb{R}^p \to \mathbb{R}$, the gradient is denoted by

$$\nabla f(z) = \frac{\partial f(z)}{\partial z} := \begin{bmatrix} \partial f(z)/\partial z_1 & \partial f(z)/\partial z_2 & \cdots & \partial f(z)/\partial z_p \end{bmatrix}.$$

B. Problem Formulation

Consider the continuous-time nonlinear system

$$\dot{x}(t) = f(x(t), u(t)), \tag{1}$$

where $x(t) := \begin{bmatrix} x_1(t) & \cdots & x_n(t) \end{bmatrix}^T \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is a nonlinear function such that $f(0_n, 0_m) = 0_n$, i.e., the origin is an equilibrium point of (1). By the local sector nonlinearity approach [4], some classes of nonlinear systems can be represented by the T–S fuzzy system

$$\dot{x}(t) = \sum_{i=1}^{N} \alpha_i(z(t))(A_i x(t) + B_i u(t)), \quad \forall x(t) \in \mathcal{L}, \tag{2}$$

where $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$ are constant matrices, $i \in \mathcal{I}_N := \{1, 2, \ldots, N\}$ is the rule number, $z(t) := \begin{bmatrix} z_1(t) & \cdots & z_p(t)\end{bmatrix}^T \in \mathbb{R}^p$ is the vector containing premise variables in the fuzzy inference rule, $\alpha_i(z(t))$ is the MF (membership function) for each rule, and the vector of the MFs $\alpha(z(t)) := \begin{bmatrix}\alpha_1(z(t)) & \cdots & \alpha_N(z(t))\end{bmatrix}^T \in \mathbb{R}^N$ lies in the unit simplex Λ_N for all $(t, x(t)) \in \mathbb{R}_{\geq 0} \times \mathcal{L}$, where

$$\Lambda_N := \left\{\delta \in \mathbb{R}^n : \sum_{i=1}^N \delta_i = 1, \quad 0 \leq \delta_i \leq 1, \quad i \in \mathcal{I}_N\right\}.$$

In this paper, we assume that

$$z(t) = \mathcal{T}x(t) = \begin{bmatrix}\mathcal{T}_1\\ \vdots \\ \mathcal{T}_p\end{bmatrix} x(t) \in \mathbb{R}^p, \quad \mathcal{T} \in \mathbb{R}^{p\times n},$$

i.e., the premise variables are linear combinations of the state variables. In addition, $\mathcal{L} \subseteq \mathbb{R}^n$ is a set of state variables satisfying

$$\mathcal{L} \subseteq \{x \in \mathbb{R}^n : f(x, u) = A(\alpha(z))x + B(\alpha(z))u,$$
$$\alpha(z) \in \Lambda_N, z = \mathcal{T}x, u \in \mathbb{R}^m\}.$$

In other words, $\mathcal{L}$ is a modeling region where the T–S fuzzy model is defined. In this paper, it is assumed that $\mathcal{L}$ is described as

$$\mathcal{L} := \{x \in \mathbb{R}^n : \mathcal{T}_l x \in [-z_{l,\max}, z_{l,\max}], \quad l \in \mathcal{I}_p\},$$

where $z_{l,\max} > 0$, $l \in \mathcal{I}_p$ are a priori given real numbers. For $Q \succ 0$, let $V(x) := x^T Q x$, $x \in \mathbb{R}^n$ be a candidate of quadratic Lyapunov functions. In addition, consider the parallel distributed compensation state-feedback control law developed in [3]:

$$u(t) = \left(\sum_{i=1}^N \alpha_i(z(t))F_i\right) x(t) = F(\alpha)x(t).$$

When considering the state-feedback control, one typically assumes that the state vector for the state-feedback control law is identical to the real state vector $x(t)$ without errors. Generally, the state can include the measurement or estimation errors, i.e., $\tilde{x}(t) = x(t) + e(t)$, where $e(t)$ is the error vector. In this case, the closed-loop system can be expressed as

$$\dot{x}(t) = (A(\alpha) + B(\alpha)F(\tilde{\alpha}))\tilde{x}(t), \quad \forall (x(t), \tilde{x}(t)) \in \mathcal{L}\times\mathcal{L}, \tag{3}$$

where

$$F(\tilde{\alpha}) := \sum_{i=1}^N \alpha_i(\tilde{z}(t))F_i = \sum_{i=1}^N \alpha_i(\tilde{z}(t))F_i = \sum_{i=1}^N \alpha_i(\mathcal{T}\tilde{x}(t))F_i$$

is the state-dependent feedback gain. Assume that the error is of the following form:

$$e(t) = \lambda^{-1}\Delta x(t), \quad \forall \Delta \in \mathcal{D} := \{D \in \mathbb{R}^{n\times n} : D^T D \preceq I_n\}.$$

The problem addressed in this paper can be stated as follows.

Problem 1 (Local stability):

1) Determine the state-feedback gain matrices $F_i \in \mathbb{R}^{m\times n}$, $i \in \mathcal{I}_N$ such that the zero equilibrium point of (2) is locally asymptotically stable;
2) Estimate an invariant subset of the domain of attraction (DA).

III. Main Result

We present LMI-based optimization procedures to solve Problem 1. For the development, define the following sets:

- $\mathcal{H}(b) := \{x \in \mathcal{L} : |\alpha_i(\mathcal{T}x) - \alpha_i(\mathcal{T}\tilde{x})| \leq b, \forall i \in \mathcal{I}_N$ $\tilde{x} = x + \lambda^{-1}\Delta x, \forall \Delta \in \mathcal{D}\}$;
- $\Omega(\gamma) := \{x \in \mathcal{L} : V(x) \leq \gamma\}$;
- $\mathcal{D} := \{D \in \mathbb{R}^{n\times n} : D^T D \leq I_n\}$;
- $\mathcal{R} := \{x \in \mathcal{L} : \tilde{x} = (I_n + \lambda^{-1}\Delta)x \in \mathcal{L}, \forall \Delta \in \mathcal{D}\}$;
- $\mathcal{V}(b)$: set of vertices of the cubic $\{v \in \mathbb{R}^N : -b \leq v_i \leq b\}$;
- $\mathcal{G}_i$: set of vertices of a polytope that includes $\partial\alpha_i(\xi)/\partial\xi$ for all $\xi = \mathcal{T}x$, $x \in \mathcal{L}$, where $b \in \mathbb{R}_{>0}$ is the upper bound on the variations of the membership functions.

To proceed further, let us recall the following lemmas.

Lemma 1: ([7, Theorem 2.2]) Given symmetric matrices Υ_{ij}, $(i, j) \in \mathcal{I}_N^2$, $\sum_{i=1}^r\sum_{j=1}^r a_i(z(t))a_j(z(t))\Upsilon_{ij} \prec 0$ holds for all $x(t) \in \mathcal{L}$ if LMIs $\Upsilon_{ii} \prec 0, \forall i \in \mathcal{I}_N$ and $(2/(r-1))\Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} \prec 0$, $i \neq j$, $\forall (i, j) \in \mathcal{I}_N^2$ are fulfilled.

Lemma 2: ([42, Lemma 3.1]) Given matrices U, V, $\Gamma = \Gamma^T$ of appropriate dimensions, the following statements are equivalent:

1) $\Gamma + \mathrm{He}\{U\Delta V\} < 0, \quad \forall \Delta \in \mathcal{D}$;
2) There exists a real number $\kappa > 0$ such that $\Gamma + \kappa^{-1}V^T V + \kappa U U^T \prec 0$ holds;
3) There exists $\kappa \in \mathbb{R}$ such that $\begin{bmatrix}\Gamma + \kappa U U^T & * \\ V & -\kappa I\end{bmatrix} \prec 0$.

Proof: The proof for $a) \Leftrightarrow b)$ is given in [42, Lemma 3.1], and $b) \Leftrightarrow c)$ can be proven by the standard Schur complement argument [41]. ■

Lemma 3: ([43, mean value theorem in several variables]) Let $U \in \mathbb{R}^p$ be a convex set, and suppose $f : U \to \mathbb{R}$ is continuously differentiable. Then, for any $x, y \in U$, there exists a real number $c \in [0, 1]$ such that

$$f(y) - f(x) = \frac{\partial f((1-c)y + cx)}{\partial x}(y - x).$$

We now introduce an LMI-based optimization for solving Problem 1.

Problem 2: Let scalars $b > 0$, and $\lambda > 0$ be given. Solve for $P = P^T \in \mathbb{R}^{n\times n}$, $K_i \in \mathbb{R}^{m\times n}$, $M_{1j} \in \mathbb{R}^{m\times n}$, $M_{2j} \in \mathbb{R}^{m\times n}$, and $\kappa_1, \kappa_{2ij}, \eta, \beta \in \mathbb{R}$, $(i, j) \in \mathcal{I}_N \times \mathcal{I}_N$ the following optimization problem:

$$\min_{P_i, K_i, M_{1j}, M_{2j}, \kappa_1, \kappa_{2ij}, \eta, \beta} \quad \beta \quad \text{s.t.}$$

$$\begin{bmatrix} -P & * \\ z_{l,\max}^{-1}\mathcal{T}_l P & -1\end{bmatrix} \prec 0, \quad \forall l \in \mathcal{I}_p, \tag{4}$$

$$\begin{bmatrix} \left(\begin{bmatrix} -P & * \\ z_{l,\max}^{-1}\mathcal{T}_l P & -1 \end{bmatrix} + \kappa_1 \begin{bmatrix} 0 & 0 \\ 0 & \lambda^{-2} z_{l,\max}^{-2}\mathcal{T}_l^T\mathcal{T}_l \end{bmatrix} \right) & * \\ \begin{bmatrix} P & 0 \end{bmatrix} & -\kappa I_n \end{bmatrix} \prec 0,$$

$$\forall l \in \mathcal{I}_p, \tag{5}$$

$$\begin{bmatrix} -P & P \\ P & -\eta I_n \end{bmatrix} \prec 0, \tag{6}$$

$$\begin{bmatrix} -\beta I_n & I_n \\ I_n & -P \end{bmatrix} \prec 0, \tag{7}$$

$$\begin{bmatrix} -\lambda^2 I_n & * \\ \eta g_i \mathcal{T} & -\eta b^2 \end{bmatrix} \prec 0, \quad \forall g_i \in \mathcal{G}_i, \forall i \in \mathcal{I}_N, \tag{8}$$

$$\begin{cases} \Upsilon_{ii}(v) \prec 0, \quad \forall i \in \mathcal{I}_N, \quad \forall v \in \mathcal{V}(b), \\ \frac{2}{r-1}\Upsilon_{ii}(v) + \Upsilon_{ij}(v) + \Upsilon_{ji}(v) \prec 0, \\ \forall (i,j) \in \{(i,j) \in \mathcal{I}_N \times \mathcal{I}_N : i \neq j\}, \quad \forall v \in \mathcal{V}(b), \end{cases} \tag{9}$$

where $\Upsilon_{ij}(v)$ is defined in (10) at the top of the next page.

It can be proved that the LMI optimization in Problem 2 can be used to solve Problem 1.

Theorem 1: Let scalars $b > 0$ and $\lambda > 0$ be given. Suppose that there exist matrices $P = P^T \in \mathbb{R}^{n\times n}$, $K_i \in \mathbb{R}^{m\times n}$, $M_{1j} \in \mathbb{R}^{m\times n}$, $M_{2j} \in \mathbb{R}^{m\times n}$, and scalars $\kappa_1, \kappa_{2ij}, \eta, \beta \in \mathbb{R}$, $(i,j) \in \mathcal{I}_N \times \mathcal{I}_N$ which are feasible solutions to the optimization problem in Problem 2. Consider the quadratic Lyapunov function candidate $V(x) = x^T P^{-1} x$. Then, an invariant subset of the DA for the closed-loop system (3) is given by $\Omega(1)$.

In order to prove Theorem 1, some intermediate results should be established first.

a) $\Omega(1) \subset \mathcal{L}$;
b) $\Omega(1) \subseteq \mathcal{R}$;
c) $\Omega(1) \subset \{x \in \mathbb{R}^n : x^T x \le \eta\}$;
d) $\Omega(1) \subseteq \{x \in \mathbb{R}^n : e^T e \le \eta\lambda^{-2}, e = \lambda^{-1}\Delta x, \Delta \in \mathcal{D}\}$;
e) $\{x \in \mathbb{R}^n : x^T x \le 1/\beta\} \subset \Omega(1)$;
f) $\Omega(1) \subseteq \mathcal{H}(b)$;
g) $\Omega(1) \subseteq \{x \in \mathcal{L} : \dot{V}(x) < 0\}$.

Proof for statement a): Applying the congruence transformation to (4) with $\mathrm{diag}\{P^{-1}, 1\}$ and using the Schur complement, we have

$$\begin{aligned} & z_{l,\max}^{-2}\mathcal{T}_l^T\mathcal{T}_l \prec P^{-1}, \quad \forall l \in \mathcal{I}_p \\ & \Leftrightarrow z_{l,\max}^{-2} x^T \mathcal{T}_l^T\mathcal{T}_l x < x^T P^{-1} x, \quad \forall l \in \mathcal{I}_p \\ & \Leftrightarrow z_{l,\max}^{-2} z_l^2 < x^T P^{-1} x, \quad \forall l \in \mathcal{I}_p \\ & \Rightarrow \{x \in \mathbb{R}^n : x^T P^{-1} x \le 1\} \\ & \subset \{x \in \mathbb{R}^n : z_l \in [-z_{l,\max}, z_{l,\max}], \forall l \in \mathcal{I}_p\} \\ & \Rightarrow \Omega(1) \subset \mathcal{L}. \end{aligned}$$

□

Proof for statement b): Using Lemma 2, it follows form (5) that

$$\begin{bmatrix} -P & * \\ z_{l,\max}^{-1}\mathcal{T}_l P & -1 \end{bmatrix} + \mathrm{He}\left\{ \begin{bmatrix} 0 \\ \lambda^{-1} z_{l,\max}^{-1}\mathcal{T}_l \end{bmatrix} \Delta \begin{bmatrix} P & 0 \end{bmatrix} \right\}$$

$$\begin{aligned} &= \begin{bmatrix} -P & * \\ (z_{l,\max}^{-1}\mathcal{T}_l P + \lambda^{-1} z_{l,\max}^{-1}\mathcal{T}_l \Delta P) & -1 \end{bmatrix} \\ &= \begin{bmatrix} -P & * \\ z_{l,\max}^{-1}\mathcal{T}_l (I + \lambda^{-1}\Delta) P & -1 \end{bmatrix} \prec 0, \\ & \forall l \in \mathcal{I}_p, \ \Delta \in \mathcal{D} \end{aligned}$$

Applying the congruence transformation to the last inequality with $\mathrm{diag}\{P^{-1}, 1\}$ and using the Schur complement yield

$$\begin{aligned} & z_{l,\max}^{-2}(I_n + \lambda^{-1}\Delta)^T \mathcal{T}_l^T \mathcal{T}_l (I_n + \lambda^{-1}\Delta) \prec P^{-1}, \\ & \forall l \in \mathcal{I}_p, \quad \forall \Delta \in \mathcal{D} \\ & \Leftrightarrow z_{l,\max}^{-2} \tilde{x}^T \mathcal{T}_l^T \mathcal{T}_l \tilde{x} \le x^T P^{-1} x \le 1, \quad \forall x \in \Omega(1), \forall l \in \mathcal{I}_p \\ & \Leftrightarrow z_{l,\max}^{-2} \tilde{z}_l^2 \le 1, \quad \forall x \in \Omega(1), \forall l \in \mathcal{I}_p \\ & \Leftrightarrow |\tilde{z}_l| \le z_{l,\max}, \quad \forall x \in \Omega(1), \forall l \in \mathcal{I}_p \\ & \Leftrightarrow \tilde{x} \in \mathcal{L}, \quad \forall x \in \Omega(1) \\ & \Rightarrow \Omega(1) \subseteq \mathcal{R} \end{aligned}$$

□

Proof for statement c): Apply the Schur complement to (6) to obtain

$$\begin{aligned} & \eta^{-1} PP - P \prec 0 \\ & \Leftrightarrow P^{-1}(\eta^{-1}PP - P)P^{-1} = \eta^{-1} I_n - P^{-1} \prec 0 \\ & \Leftrightarrow \eta^{-1} x^T x < x^T P^{-1} x \\ & \Leftrightarrow \eta^{-1} x^T x - 1 < x^T P^{-1} x - 1 \\ & \Leftrightarrow \eta^{-1}(x^T x - \eta) < x^T P^{-1} x - 1 \\ & \Leftrightarrow (x^T x - \eta) < \eta(x^T P^{-1} x - 1) \\ & \Rightarrow \Omega(1) \subset \{x \in \mathbb{R}^n : x^T x \le \eta\} \end{aligned}$$

□

Proof for statement d): Since $\Omega(1) \subset \{x \in \mathbb{R}^n : x^T x \le \eta\}$ from statement c), one concludes $x^T x \le \eta$ for all $x \in \Omega(1)$. Therefore, we have

$$\begin{aligned} & e^T e = \lambda^{-2} x^T \Delta^T \Delta x \le \lambda^{-2} x^T x \le \eta\lambda^{-2}, \\ & \forall x \in \Omega(1), \forall \Delta \in \mathcal{D} \end{aligned}$$

which implies $\Omega(1) \subseteq \{x \in \mathbb{R}^n : e^T e \le \eta\lambda^{-2}, e = \lambda^{-1}\Delta x, \Delta \in \mathcal{D}\}$. □

Proof for statement e): Applying the congruence transformation to (7) with $\mathrm{diag}\{I_n, P^{-1}\}$ and using the Schur complement, one gets

$$\begin{aligned} & -\beta I + P^{-1} \prec 0 \\ & \Leftrightarrow x^T P^{-1} x < \beta x^T x \\ & \Leftrightarrow x^T P^{-1} x - 1 < \beta(x^T x - 1/\beta) \\ & \Rightarrow \{x \in \mathbb{R}^n : x^T x \le 1/\beta\} \subset \Omega(1) \end{aligned}$$

□

Proof for statement f): First of all, it can be seen that () ensures

$$\begin{bmatrix} -\eta^{-1}\lambda^2 I_n & * \\ \frac{\partial \alpha_i(\mathcal{T}\xi)}{\partial \mathcal{T}\xi}\mathcal{T} & -b^2 \end{bmatrix} \prec 0,$$

$$\forall \xi \in \{\mathcal{T}x \in \mathbb{R}^p : x \in \mathcal{L}\}, \forall i \in \mathcal{I}_N.$$

$$\Upsilon_{ij}(v) := \begin{bmatrix} \text{He}\left\{A_iP + B_iK_j + B_i\sum_{k=1}^{N}(K_k + M_{1j})v_k\right\} & * & * \\ P & -\kappa_{2ij}\lambda I_n & * \\ K_j^TB_i^T + \sum_{k=1}^{N}(K_k^T + M_{2j})v_kB_i^T & 0_{n\times n} & \kappa_{2ij}I_n - 2P \end{bmatrix} \quad (10)$$

Using the Schur complement, one gets

$$b^{-2}\mathcal{T}^T\frac{\partial\alpha_i(\mathcal{T}\xi)}{\partial\mathcal{T}\xi}^T\frac{\partial\alpha_i(\mathcal{T}\xi)}{\partial\mathcal{T}\xi}\mathcal{T} - \eta^{-1}\lambda^2 I_n \prec 0,$$
$$\forall\xi \in \{\mathcal{T}x \in \mathbb{R}^p : x \in \mathcal{L}\},\, \forall i \in \mathcal{I}_N$$
$$\Rightarrow e(t)^T\mathcal{T}^T\frac{\partial\alpha_i(\mathcal{T}\xi)}{\partial\mathcal{T}\xi}^T\frac{\partial\alpha_i(\mathcal{T}\xi)}{\partial\mathcal{T}\xi}\mathcal{T}e(t)$$
$$\le \eta^{-1}\lambda^2 b^2 e(t)^T e(t),$$
$$\forall\xi \in \{\mathcal{T}x \in \mathbb{R}^p : x \in \mathcal{L}\},\, \forall i \in \mathcal{I}_N,\, \forall e(t) \in \mathbb{R}^n$$
$$\Rightarrow (\mathcal{T}x(t) - \mathcal{T}\tilde{x}(t))^T\frac{\partial\alpha_i(\mathcal{T}\xi)}{\partial\mathcal{T}\xi}^T\frac{\partial\alpha_i(\mathcal{T}\xi)}{\partial\mathcal{T}\xi}(\mathcal{T}x(t) - \mathcal{T}\tilde{x}(t))$$
$$\le \eta^{-1}\lambda^2 b^2(x(t) - \tilde{x}(t))^T(x(t) - \tilde{x}(t)),$$
$$\forall\xi \in \{\mathcal{T}x \in \mathbb{R}^p : x \in \mathcal{L}\},\, \forall i \in \mathcal{I}_N,\, \forall x(t) \in \Omega(1).$$

In the last inequality, if $x(t) \in \Omega(1)$, then since $\Omega(1) \subseteq \mathcal{R}$ from the statement b), we have $x(t) \in \mathcal{R}$. This guarantees $x(t) \in \mathcal{L}$ and $\tilde{x}(t) \in \mathcal{L}$. Note also that, since $\mathcal{L}$ is a convex set in $\mathbb{R}^n$, $(1-c)x(t) + c\tilde{x}(t) \in \mathcal{L}$ holds for any $c \in [0, 1]$. In addition, we have

$$(1-c)\mathcal{T}x(t) + c\mathcal{T}\tilde{x}(t) = (1-c)z(t) + c\tilde{z}(t)$$
$$\in [-z_{1,\max},\, z_{1,\max}] \times \cdots \times [-z_{p,\max},\, z_{p,\max}],$$

which is also inside a convex set. Since the last inequality holds for all $\xi \in \{\mathcal{T}x \in \mathbb{R}^p : x \in \mathcal{L}\}$, ξ can be replaced by $(1-c)\mathcal{T}x(t) + c\mathcal{T}\tilde{x}(t)$. Let us rewrite the last inequality as follows:

$$(\mathcal{T}x(t) - \mathcal{T}\tilde{x}(t))^T\nabla\alpha_i((1-c_i)\mathcal{T}x(t) + c_i\mathcal{T}\tilde{x}(t))^T$$
$$\times \nabla\alpha_i((1-c_i)\mathcal{T}x(t) + c_i\mathcal{T}\tilde{x}(t))(\mathcal{T}x(t) - \mathcal{T}\tilde{x}(t))$$
$$\le \eta^{-1}\lambda^2 b^2(x(t) - \tilde{x}(t))^T(x(t) - \tilde{x}(t)),$$
$$\forall i \in \mathcal{I}_N,\, \forall x(t) \in \Omega(1). \quad (11)$$

Now, we can use the mean value theorem in several variables (Lemma 3), and conclude that there are some real numbers $(c_1, c_2, \ldots, c_N) \in [0, 1]^N$ for every time $t \in \mathbb{R}_{\ge 0}$ such that

$$\nabla\alpha_i((1-c_i)\mathcal{T}x(t) + c_i\mathcal{T}\tilde{x}(t))(\mathcal{T}x(t) - \mathcal{T}\tilde{x}(t))$$
$$= \alpha_i(\mathcal{T}x(t)) - \alpha_i(\mathcal{T}\tilde{x}(t)), \quad \forall(x(t), i) \in \Omega(1) \times \mathcal{I}_N.$$

Plugging the above equation into (11) yields

$$(\alpha_i(z(t)) - \alpha_i(\tilde{z}(t)))^2 \le \eta^{-1}\lambda^2 b^2 e(t)^T e(t),$$
$$\forall(x(t), i) \in \Omega(1) \times \mathcal{I}_N.$$

From statement d), we know $\Omega(1) \subseteq \{x \in \mathbb{R}^n : e^Te \le \eta\lambda^{-2},\, e = \lambda^{-1}\Delta x,\, \Delta \in \mathcal{D}\}$, which means

$$(\alpha_i(z(t)) - \alpha_i(\tilde{z}(t)))^2 \le \eta^{-1}\lambda^2 b^2 e(t)^T e(t) \le b^2,$$
$$\forall(x(t), i) \in \Omega(1) \times \mathcal{I}_N$$
$$\Rightarrow \begin{cases} |\alpha_1(z(t)) - \alpha_1(\tilde{z}(t))| \le b \\ \vdots \\ |\alpha_N(z(t)) - \alpha_N(\tilde{z}(t))| \le b \end{cases}$$
$$\forall x(t) \in \Omega(1)$$
$$\Rightarrow \Omega(1) \subseteq \mathcal{H}(b).$$

□

Proof for statement g): Applying Lemma 1 to (9), one can obtain (12) at the top of the next page. Since $\mathcal{H}(b) \subseteq \mathcal{L}$, we an replace $\mathcal{L}$ in the above inequality by $\mathcal{H}(b)$. In this case, we have

$$\alpha(\mathcal{T}\tilde{x}) - \alpha(\mathcal{T}x) = \begin{bmatrix} \alpha_1(\mathcal{T}\tilde{x}) - \alpha_1(\mathcal{T}x) \\ \vdots \\ \alpha_N(\mathcal{T}\tilde{x}) - \alpha_N(Tx) \end{bmatrix} \in \mathcal{V}(b),$$
$$\forall x(t) \in \mathcal{H}(b).$$

Letting $v \to \alpha(\mathcal{T}\tilde{x}) - \alpha(\mathcal{T}x)$ and using relation $\sum_{i=1}^{N}(\alpha(\mathcal{T}\tilde{x}) - \alpha(\mathcal{T}x)) = 0$, one has the matrix inequality in (13) shown at the top of the next page. Since $\Omega(1) \subseteq \mathcal{H}(b)$, the matrix inequality in (13) holds for all $x(t) \in \Omega(1)$ as in (14) at the top of the next page.

Applying the Schur complement and the extended Schur complement in [44, Theorem 1] yield

$$\text{He}\{A(\alpha)P + B(\alpha)K(\tilde{\alpha})\} + \kappa_2(\alpha^2)^{-1}\lambda^{-1}PP$$
$$+ \kappa_2(\alpha^2)B(\alpha)K(\tilde{\alpha})P^{-1}P^{-1}K(\tilde{\alpha})^TB(\alpha)^T \prec 0$$
$$\forall x(t) \in \Omega(1)$$

Pre- and post-multiplying the left-hand side of the above inequality by P^{-1} leads to

$$\text{He}\{P^{-1}(A(\alpha) + B(\alpha)K(\tilde{\alpha})P^{-1})\} + \kappa_2(\alpha^2)^{-1}\lambda^{-1}I_n$$
$$+ \kappa_2(\alpha^2)P^{-1}B(\alpha)K(\tilde{\alpha})P^{-1}P^{-1}K(\tilde{\alpha})^TB(\alpha)^TP^{-1} \prec 0$$
$$\forall x(t) \in \Omega(1)$$

and by using Lemma 2, one has

$$\text{He}\{P^{-1}(A(\alpha) + B(\alpha)(F(\tilde{\alpha}) + F(\tilde{\alpha})\lambda^{-1}\Delta))\} \prec 0$$
$$\forall x(t) \in \Omega(1), \quad \Delta \in \mathcal{D}$$

where $F(\tilde{\alpha}) = K(\tilde{\alpha})P^{-1}$, which implies

$$0 > 2x(t)^TP^{-1}(A(\alpha)x(t) + B(\alpha)F(\tilde{\alpha})\tilde{x}(t))$$
$$= \dot{V}(x(t)), \quad \forall x(t) \in \Omega(1)\backslash\{0_n\}$$
$$\Rightarrow \Omega(1)\backslash\{0_n\} \subseteq \{x \in \mathbb{R}^n : \dot{V}(x) < 0\}$$

□

$$\begin{bmatrix} \mathrm{He}\left\{A(\alpha)P + B(\alpha)K(\alpha) + B(\alpha)\sum_{k=1}^{N}(K_k + M_1(\alpha))v_k\right\} & * & * \\ P & -\kappa_2(\alpha^2)\lambda I_n & * \\ K(\alpha)^T B(\alpha)^T + \sum_{k=1}^{N}(K_k^T + M_2(\alpha))v_k B(\alpha)^T & 0_{n\times n} & \kappa_2(\alpha^2)I_n - 2P \end{bmatrix} \prec 0$$

$$\forall x(t) \in \mathcal{L}, \quad \forall v = \begin{bmatrix} v_1 \\ \vdots \\ v_N \end{bmatrix} \in \mathcal{V}(b). \tag{12}$$

$$\begin{bmatrix} \mathrm{He}\left\{A(\alpha)P + B(\alpha)K(\alpha) + B(\alpha)\sum_{k=1}^{N}(\alpha_i(\mathcal{T}\tilde{x}) - \alpha_i(\mathcal{T}x))K_k\right\} & * & * \\ P & -\kappa_2(\alpha^2)\lambda I_n & * \\ K(\alpha)^T B(\alpha)^T + \sum_{k=1}^{N}(\alpha_i(\mathcal{T}\tilde{x}) - \alpha_i(\mathcal{T}x))K_k^T B(\alpha)^T & 0_{n\times n} & \kappa_2(\alpha^2)I_n - 2P \end{bmatrix} \prec 0, \quad \forall x(t) \in \mathcal{H}(b) \tag{13}$$

$$\begin{bmatrix} \mathrm{He}\{A(\alpha)P + B(\alpha)K(\tilde{\alpha})\} & * & * \\ P & -\kappa_2(\alpha^2)\lambda I_n & * \\ K(\tilde{\alpha})^T B(\alpha)^T & 0_{n\times n} & \kappa_2(\alpha^2)I_n - 2P \end{bmatrix} \prec 0, \quad \forall x(t) \in \Omega(1) \tag{14}$$

Now, the proof of Theorem 1 is given below.

Proof of Theorem 1: From the above results, we know that $\Omega(1)\backslash\{0_n\} \subseteq \{x \in \mathbb{R}^n : \dot{V}(x) < 0\}$ holds. By the Lyapunov stability theory [40], the closed-loop system (2) is locally asymptotically stable and $\Omega(1)$ is an invariant subset of the DA [40]. □

A brief outline of the design procedure is presented below.

Step 1. Given continuous-time nonlinear system $\dot{x}(t) = f(x(t), u(t))$, obtain (A_i, B_i), $\alpha_i(z(t))$, $\forall i \in \mathcal{I}_N$, $z(t) = \mathcal{T}x(t)$ and $\mathcal{L}$ for the fuzzy model (2).

Step 2. Calculate $\partial\alpha_i(z)/\partial z$ and its vertices $\mathcal{G}_i$ for all $i \in \mathcal{I}_N$.

Step 3. Set b and η, and solve Problem 2.

Step 4. If infeasible, then (2) cannot be identified as locally asymptotically stabilizable via the Problem 2. Otherwise, if feasible, then (2) is locally asymptotically stable, and $\Omega(1)$ is an invariant subset of the DA.

Remark 1: In Theorem 1, there exist some design parameters that should be chosen by the designer. The constant $\lambda \in \mathbb{R}_{>0}$ can be determined based on the expected error bound. For $b \in (0, 1]$, there is no general guideline on how to select it. At the current stage, it should be determined by a combination of previous expertise and trial and error. Typically, the smaller the constant b, the less conservative the condition of Theorem 1. On the other hand, if b is too small, due to the constraint $\Omega(1) \subseteq \mathcal{H}(b)$ in the statement f), the volume of the level set $\Omega(1)$ can also be reduced. Therefore, one of strategies to determine b is to initially select b small enough and if the condition of Theorem 1 is feasible for the value of b, then increase b until it becomes infeasible.

Remark 2: In [45], a filter design problem was addressed under the assumption that the premise variables are unknown. Compared to the previous work, a distinguished feature of the proposed method is that we consider the variation rate of the membership functions to reduce the conservatism. In addition, the proposed approach provides an efficient method to analyze the local stability and estimate the domain of attraction.

IV. Examples

In this section, an example is given to show the validity of the proposed method. The numerical example is treated with the help of MATLAB R2012b. The LMI problems are solved with SeDuMi [1] and Yalmip [2].

Example 1: Consider the system (2) with

$$A_1 = \begin{bmatrix} 0 & -4 \\ 0 & -2 \end{bmatrix}, A_2 = \begin{bmatrix} -1 & -4 \\ 0 & -2 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix},$$
$$\alpha_1(z) = (1 + \sin z)/2, \quad \alpha_2(z) = 1 - \alpha_1(z),$$
$$z = \mathcal{T}x, \quad \mathcal{T} = [1, 0],$$
$$\mathcal{L} = \{x \in \mathbb{R}^n : \mathcal{T}x \in [-z_{\max}, z_{\max}]\}, \quad z_{\max} = \pi/2.$$

Since

$$\frac{\partial\alpha_1(z)}{\partial z} = 0.5\cos z \in \mathrm{co}\{0, 0.5\},$$
$$\frac{\partial\alpha_2(z)}{\partial z} = -0.5\cos z \in \mathrm{co}\{0, -0.5\}$$

for all $x \in \mathcal{L}$, we have $\mathcal{G}_1 = \{0, 0.5\}$ and $\mathcal{G}_2 = \{0, -0.5\}$. Problem 2 is solved with $\lambda = 3$ and $b = 9.9$, and the

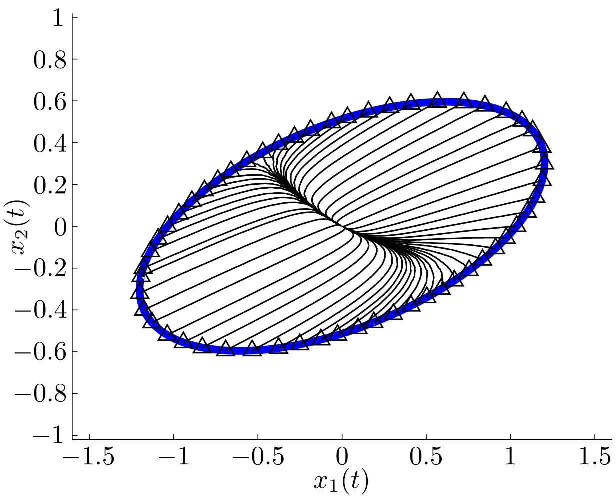

Fig. 1. Converging trajectories (solid lines) initialized at the "$\triangle$" marks and the level set $\Omega(1)$ (blue solid line) obtained by solving Problem 2.

state-feedback gain and the quadratic Lyapunov function are obtained as follows:

$$F_1 = \begin{bmatrix} -1.0449 & 0.4548 \end{bmatrix}, \quad F_2 = \begin{bmatrix} -1.0442 & 0.4539 \end{bmatrix}$$
$$V(x) = x^T \begin{bmatrix} 0.9318 & 0.9542 \\ 0.9542 & 3.7948 \end{bmatrix} x$$

With the random matrix $\Delta(t) \in \mathcal{D}$, the simulation result is depicted in Fig. 1, which shows converging trajectories (solid lines) initialized at the "$\triangle$" marks and the level sets $\Omega(1)$ (blue solid line) estimated by solving Problem 2.

Example 2: Consider the simplified continuous-time nonlinear Moore-Greitzer model of a jet engine with the assumption of no stall

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} -\frac{3}{2}x_1(t) - \frac{1}{2}x_1(t)^2 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u(t), \quad t \in \mathbb{R}, \quad t \geq 0,$$

where $x_1(t)$ and $x_2(t)$ are related to the mass flow and the pressure rise through the engine after an appropriate change of coordinate, respectively. In order to cast the above system into a T–S fuzzy system under consideration, let us define

$$f(x_1) := -\frac{3}{2}x_1 - \frac{1}{2}x_1^2$$

and

$$f_{\max} := \max_{x_1 \in [-x_{1,\max}, x_{1,\max}]} f(x_1),$$
$$f_{\min} := \min_{x_1 \in [-x_{1,\max}, x_{1,\max}]} f(x_1).$$

Letting $z_1(t) = x_1(t)$, $\mathcal{T} = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $z_{1,\max} = x_{1,\max}$, the model can be expressed as a continuous-time T–S fuzzy model with

$$A_1 = \begin{bmatrix} f_{\max} & -1 \\ 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} f_{\min} & -1 \\ 0 & 0 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix},$$
$$\alpha_1(z_1(t)) = \frac{f(z_1(t)) - f_{\min}}{f_{\max} - f_{\min}},$$
$$\alpha_2(z_1(t)) = \frac{f_{\max} - f(z_1(t))}{f_{\max} - f_{\min}},$$

which is modeled within $\mathcal{X} = \{\xi \in \mathbb{R}^n : \begin{bmatrix} 1 & 0 \end{bmatrix}\xi \in [-z_{1,\max}, z_{1,\max}]\}$. To compute polytopic bounds on the derivatives of the MFs with respect to the premise variable $z_1(k)$, consider

$$\begin{aligned}
\frac{\partial \alpha_1(z_1(k))}{\partial z_1(k)} &= \frac{1}{f_{\max} - f_{\min}} \frac{\partial f(z_1(k))}{\partial z_1(k)} \\
&= \frac{1}{f_{\max} - f_{\min}} \frac{\partial}{\partial z_1(k)} \left(-\frac{3}{2}z_1(k) - \frac{1}{2}z_1(k)^2 \right) \\
&= -\frac{3}{2}\frac{1}{f_{\max} - f_{\min}} - \frac{1}{f_{\max} - f_{\min}} z_1(k) \\
&=: g(z_1(k)), \\
\frac{\partial \alpha_2(z_1(k))}{\partial z_1(k)} &= -\frac{1}{f_{\max} - f_{\min}} \frac{\partial f(z_1(k))}{\partial z_1(k)} \\
&= -g(z_1(k)).
\end{aligned}$$

Since

$$g_{\max} = \max_{z_1 \in [-z_{1,\max}, z_{1,\max}]} g(z_1),$$
$$g_{\min} = \min_{x_1 \in [-z_{1,\max}, z_{1,\max}]} g(z_1),$$

one concludes

$$\frac{\partial \alpha_1(z_1(k))}{\partial z_1(k)} \in \mathrm{co}\{g_{\min}, g_{\max}\},$$
$$\frac{\partial \alpha_2(z_1(k))}{\partial z_1(k)} \in \mathrm{co}\{g_{\min}, g_{\max}\}.$$

With $z_{1,\max} = 0.1$, we obtain $f_{\max} = 0.1450$, $f_{\min} = -0.1550$. Moreover, since $g_{\max} = -4.6667$ and $g_{\min} = -5.3333$, the sets $\mathcal{G}_1$ and $\mathcal{G}_2$ are set to be

$$\mathcal{G}_1 = \mathcal{G}_2 = \mathrm{co}\{-5.3333, -4.6667\}.$$

Problem 2 is solved with $\lambda = 30$ and $b = 0.0999$, and the state-feedback gain and the quadratic Lyapunov function are obtained as follows:

$$F_1 = \begin{bmatrix} -1.8080 & 1.7207 \end{bmatrix}, \quad F_2 = \begin{bmatrix} -1.3464 & 1.6469 \end{bmatrix}$$
$$V(x) = x^T \begin{bmatrix} 122.5946 & 21.3823 \\ 21.3823 & 74.2944 \end{bmatrix} x$$

V. Conclusion

In this paper, a local state-feedback stabilization problem is solved based on LMI optimizations. It is assumed that the measured or estimated state for the state-feedback control law involves errors, thereby existing control design methods can fail to find a state-feedback controller which guarantees the stability. Examples illustrate the proposed method.

References

[1] J. F. Sturm, "Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cons," *Optim. Meth. Software,* vol. 11-12, pp. 625-653, 1999.

[2] J. Löfberg, "YALMIP: A toolbox for modeling and optimization in MATLAB," in *Proc. IEEE CCA/ISIC/CACSD Multiconf.,* 2004, pp. 284-289 [Online]. Available: http://control.ee.ethz.ch/ joloef/yalmip.php

[3] K. Tanaka, T. Ikeda, and H. O. Wang, “Fuzzy regulators and fuzzy observers: relaxed stability conditions and LMI-based designs,” *IEEE Trans. Fuzzy Syst.*, vol. 6, no. 2, pp. 250-265, 1998.

[4] K. Tanaka and H. O. Wang, *Fuzzy Control Systems Design and Analysis: A Liinear Matrix Inequality Approach*. New York: Wiley, 2001.

[5] H. Li, X. Jing, H. Lam, and P. Shi, “Fuzzy sampled-data control for uncertain vehicle suspension systems,” *IEEE Trans. Cybern.*, vol. 44, no. 7, pp. 1111-1126, 2014.

[6] E. Kim and H. Lee, “New approaches to relaxed quadratic stability condition of fuzzy control systems,” *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 5, pp. 523-534, 2000.

[7] H. D. Tuan, P. Apkarian, T. Narikiyo, and Y. Yamamoto, “Parameterized linear matrix inequality techniques in fuzzy control system deisgn,” *IEEE Trans. Fuzzy Syst.*, vol. 9, no. 2, pp. 324-332, 2001.

[8] A. Sala and C. Ariño, “Relaxed stability and performance conditions for Takagi–Sugeno fuzzy systems with knowledge on membership function overlap,” *IEEE Trans. Syst., Man Cybern.*, vol. 37, no. 3, pp. 727-732, 2007.

[9] M. Narimani and H. K. Lam, “Relaxed LMI-based stability conditions for Takagi–Sugeno fuzzy control systems using regional-membership-function-shape-dependent analysis approach,” *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 5, pp. 1221-1228, 2009.

[10] A. Sala and C. Ariño, “Asymptotically necessary and sufficient conditions for stability and performance in fuzzy control: Applications of Pólya’s theorem,” *Fuzzy Sets Syst.*, vol. 158, no. 24, pp. 2671-2686, 2007.

[11] M. Bernal, T. M. Guerra, A. Kruszewski, “A membership-function-dependent approach for stability analysis and controller synthesis of Takagi–Sugeno models,” *Fuzzy Sets Syst.*, vol. 160, no. 19, pp. 2776-2795, 2009.

[12] Y. J. Chen, H. Ohtake, K. Tanaka, W. J. Wang, and H. O. Wang, “Relaxed stabilization criterion for T–S fuzzy systems by minimum-type piecewise Lyapunov function based switching fuzzy controller,” *IEEE Trans. Fuzzy Syst.*,, vol 20, no. 6, pp. 1166-1173, 2012.

[13] V. C. S. Campos, F. O. Souza, L. A. B. Tôrres, and R. M. Palhares, “New stability conditions based on piecewise fuzzy Lyapunov functions and tensor product transformations,” *IEEE Trans. Fuzzy Syst.*,, vol 21, no. 4, pp. 748-760, 2013.

[14] K. Tanaka, T. Hori, and H. O. Wang, “A fuzzy Lyapunov approach to fuzzy control system design,” in *Proc. Amer. Control Conf.*, Arlington, VA, 2001, pp. 4790-4795.

[15] K. Tanaka, T. Hori, and H. O. Wang, “A multiple Lyapunov function approach to stabilization of fuzzy control systems,” *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 4, pp. 582-589, 2003.

[16] T. M. Guerra and L. Vermeiren, “LMI-based relaxed nonquadratic stabilization conditions for nonlinear systems in the Takagi–Sugeno’s form,” *Automatica*, vol. 40, no. 5, pp. 823-829, 2004.

[17] B. C. Ding, H. X. Sun, and P. Yang, “Further studies on LMI-based relaxed stabilization conditions for nonlinear systems in Takagi–Sugeno’s form,” *Automatica*, vol. 42, no. 3, pp. 503-508, 2006.

[18] L. A. Mozelli, R. M. Palhares, F. O. Souza, and E. M. A. M. Mendes, “Reducing conservativeness in recent stability conditions of T–S fuzzy systems,” *Automatica*, vol. 45, no. 6, pp. 1580-1583, 2009.

[19] B. -J. Rhee and S. Won, “A new fuzzy Lyapunov function approach for a Takagi–Sugeno fuzzy control system design,” *Fuzzy Sets Syst.*, vol. 157, no. 9, pp. 1211-1228, 2006.

[20] A. Sala and C. Ariño, “Polynomial fuzzy models for nonlinear control: A Taylor series approach,” *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 6, pp. 1284-1295, 2009.

[21] K. Tanaka, H. Yoshida, H. Ohtake, and H. O. Wang, “A sum-of-squares approach to modeling and control of nonlinear dynamical systems with polynomial fuzzy systems,” *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 4, pp. 911-922, 2009.

[22] M. Bernal, A. Sala, A. Jaadari, and T. M. Guerra, “Stability analysis of polynomial fuzzy models via polynomial fuzzy Lyapunov functions”, *Fuzzy Sets Syst.*, vol. 185, no. 1, pp. 5-14, 2011.

[23] H. K. Lam, “Polynomial fuzzy-model-based control systems: Stability analysis via piecewise-linear membership functions,” *IEEE Trans. Fuzzy Syst.*, vol. 19, no. 3, pp. 588-593, 2011.

[24] H. K. Lam, M. Narimani, H. Li, and H. Liu “Stability analysis of polynomial-fuzzy-model-based control systems using switching polynomial Lyapunov function,” *IEEE Trans. Fuzzy Syst.*, vol. 21, no. 5, pp. 800-813, 2013.

[25] B. Ding, “Homogeneous polynomially nonquadratic stabilization of discrete-time Takagi–Sugeno systems via nonparallel distributed compensation law,” *IEEE Trans. Fuzzy Syst.*, vol. 18, no. 5, pp. 994-1000, 2010.

[26] H. Zhang and X. Xie, “Relaxed stability conditions for continuous-time T–S fuzzy-control systems via augmented multi-indexed matrix approach,” *IEEE Trans. Fuzzy Syst.*, vol. 19, no. 3, pp 478-492, 2011.

[27] E. S. Tognetti, R. C. L. F. Oliveira, and P. L. D. Peres, “Selective $\mathcal{H}_2$ and $\mathcal{H}_\infty$ Stabilization of Takagi–Sugeno Fuzzy Systems,” *IEEE Trans. Fuzzy Syst.*, vol. 19, no. 5, pp. 890-900, 2011.

[28] X. Xie, H. Ma, Y. Zhao, D. W. Ding, and Y. Wang, “Control synthesis of discrete-time T–S fuzzy systems based on a novel non-PDC control scheme,” *IEEE Trans. Fuzzy Syst.*, vol. 21, no. 1, pp. 147-157, 2013.

[29] X. Xie, D. Yue, T. Ma, and X. Zhu, “Further studies on control synthesis of discrete-time TS fuzzy systems via augmented multi-indexed matrix approach,” *IEEE Trans. Cybern.*, vol. 44, no. 12, pp. 2784-2791, 2014.

[30] A. Kruszewski, R. Wang, and T. M. Guerra, “Nonquadratic stabilization conditions for a class of uncertain nonlinear discrete time T–S fuzzy models: a new approach,” *IEEE Trans. Autom. Control*, vol. 53, no. 2, pp. 606-611, 2008.

[31] T. M. Guerra, A. Kruszewski, and M. Bernal, “Control law proposition for the stabilization of discrete Takagi-Sugeno models,” *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 3, pp. 724-731, 2009.

[32] H. K. Lam and F. H. F. Leung, “LMI-Based stability and performance conditions for continuous-time nonlinear systems in Takagi–Sugeno’s form,” *IEEE Trans. Syst., Man Cybern.*, vol. 37, no. 5, pp. 1396-1406, 2007.

[33] X.-H. Chang and G.-H. Yang, “Relaxed stabilization conditions for continuous-time Takagi–Sugeno fuzzy control systems,” *Inf. Sci.*, vol. 180, pp. 3273-3287, 2010.

[34] S. H. Kim, “Relaxation technique for a T–S fuzzy control design based on a continuous-time fuzzy weighting-dependent Lyapunov function,” *IEEE Trans. Fuzzy Syst.*, vol. 21, no. 4, pp. 761-766, 2013.

[35] M. Bernal and T. M. Guerra, “Generalized nonquadratic stability of continuous-time Takagi–Sugeno models,” *IEEE Trans. Fuzzy Syst.*, vol. 18, no. 4, pp. 815-822, 2010.

[36] J. T. Pan, T. M. Guerra, S. M. Fei, and A. Jaadari, “Nonquadratic stabilization of continuous T–S fuzzy models: LMI solution for a local approach,” *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 3, pp. 594-602, 2012.

[37] L. Wang and X. Liu, “Local analysis of continuous-time Takagi–Sugeno fuzzy system with disturbances bounded by magnitude or energy: a lagrange multiplier method,” *Inf. Sci.*, vol. 248, pp. 89-102, 2013.

[38] D. H. Lee, “Local stability and stabilization of discrete-time Takagi–Sugeno fuzzy systems using bounded variation rates of the membership functions,” in *Proc. IEEE Symp. Series Comput. Intell.*, Singapore, 2013, pp. 57-64.

[39] A. Jaadari, T. M. Guerra, A. Sala, and M. Bernal, “Finsler’s relaxation for local H-infinity controller design of continuous-time Takagi–Sugeno models via non-quadratic Lyapunov functions,” in *Proc. Amer. Control Conf.*, Washington, DC, 2013, pp. 5648-5653.

[40] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Upper Saddle River, NJ: Prentice-Hall, 2001.

[41] S. Boyd, L. EI Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in Systems and Control Theory.* Philadelphia, PA: SIAM, 1994.

[42] L. Xie, M. Fu, C. E. de Souza, “H_∞ control and quadratic stabilization of systems with parameter uncertainty via output feedback,” *IEEE Trans. Autom. Control*, vol. 37, no. 8, pp. 1253-1256, 1992.

[43] R. C. Buck, *Advanced Calculus*, 3ed ed., McGraw-Hill, 1978.

[44] M. C. de Oliveira, J. Bernussou, and J. C. Geromel, “A new discrete-time 416 robust stability condition,” *Syst. Contr. Lett.*, vol. 37, no. 4, pp. 261-265, 1999.

[45] J. Yoneyama, “H_∞ filtering for fuzzy systems with immeasurable premise variables: an uncertain system approach,” *Fuzzy Sets Syst.*, vol. 160, no. 12, pp. 1738–1748, 2009.