Energy Optimal Control in Mobile Sensor Networks Using Hybrid Systems Theory

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***Abstract*—This paper studies optimal control in a sensor network system consisting of mobile robots to minimize the overall energy consumption of the whole network. With communication energy cost and mobility energy cost taken into consideration, the problem is formulated as an optimal control of a hybrid system, which is solved by switched Linear Quadratic Regulator (LQR). Though switched LQR obtains globally optimal solution, the computational complexity is too high to implement the algorithm when the control horizon expands. For this reason, we resort to a switched system version of Receding Horizon Control (RHC), which is stable and provides a solution close to the optimal one. Finally, in order to attenuate the complexity due to the large networked system, the centralized RHC is modified into a distributed algorithm, which converges to a solution that can approximate the optimizer quite well as verified by simulations.**

***Index Terms*—Mobile ad hoc network, energy optimization, optimal control, hybrid control systems, distributed algorithm.**

I. INTRODUCTION

Advances in mechatronics have enabled mobile robots equipped with sensors to form a mobile sensor network [1], where every node has sensing, computation, wireless communication and mobility capabilities. It is known that the amount of energy consumed for wireless communication of one bit can be many orders of magnitude greater than the energy required for a single local computation [2], and energy required for locomotion can be even significantly higher than for communication [3]. Hence, this paper investigates computational algorithms to obtain an optimal control strategy to minimize the overall network energy cost including communication and mobility energy consumption in a mobile network system.

Goldenberg et al. in their seminal paper [4] proposed using localized mobility control to minimize overall communication power consumption in a sensor network consisting of mobile relay nodes; however, their work did not consider mobility energy cost in the problem formulation. Tang and McKinley [5] studied the energy optimization problem that considered for energy costs associated with both communication cost and physical node movement. Besides reducing total communication energy consumption, they considered increasing system lifetime as well. For the network lifetime problem, Yu and Lee [6], [7] proposed distributed algorithms to maximize the network lifetime by utilizing the mobility of robotic sensor nodes. Sharma et al. [8] improved lifetime and data quality by iteratively adjusting positions of mobile sensor nodes and routing strategy, however the cost of moving was not included.

While most of the previous work focused on obtaining the final optimal positions of the robotic nodes, what are the optimal moving trajectories for nodes and what are the optimal links during the transition period remains unanswered. In this paper, we consider a typical application scenario of sensor networks, in which one node, anchoring at some location, is collecting sensory data from the environment and transferring them back to a base station for processing through intermediate relaying nodes. The problems to be addressed are: 1) How should these intermediate mobile nodes serving as relays be controlled to move? And 2) how should the communication link be formed so that the energy cost of the whole network is minimized?

As the positions of robotic nodes change, the optimal network link is subject to change as well. As a result, we can model the network as a switched system, where every possible link configuration the system can have is a discrete state. The difficulty of optimal control of switched systems lies in the multiple choices of mode sequence and the switching instants. Discrete time switched linear systems with quadratic cost functions were studied by Zhang and Hu [9] and it was concluded that the value function is the pointwise minimum of a finite number of quadratic functions. Receding horizon control (a.k.a. model predictive control) for piecewise affine systems was investigated by Lazar et al. [10]; however, the discrete states were not directly controllable for the system studied. A decentralized version of RHC for large-scale systems was proposed by Keviczky et al. [11] by breaking the cost function into distinct local cost functions of smaller size. Dunbar [12] added a move suppression term into each local cost function to penalize the mismatch of states computed by different local controllers, and the stability of the algorithm was proved and guaranteed.

This paper adopts the ideas of RHC and decentralized RHC into hybrid system optimal control. The problem is first formulated into an LQR problem. Due to its inherently hybrid nature, a switched LQR method [9] is used to solve the problem. Then, RHC is introduced to tackle the computational complexity induced by the increasing of control horizon. Finally, a distributed RHC method of the networked system is proposed to attenuate the computational complexity of the RHC induced by the increasing of network scale. Computer simulations verify that the proposed distributed RHC method can obtain a solution close to the optimal one.

II. Preliminaries

A. Energy Models

Since communication and mobility are the two most energy-consuming functions in a mobile sensor system, we shall develop their energy models for our problem formulation.

For the communication model, we adopt a transmission power model similar to that used in [4]. The energy needed to transmit 1 bit of data across distance d is,

$$P_C(d) = a + b\, d^{\alpha}, \tag{1}$$

where α is the path loss exponent of the transmission medium with ranges $\alpha \in [2, 6]$, a and b are constants dependent on the characteristics of the electronic circuit of the transceiver. We assume in this paper that $\alpha = 2$. The energy cost to transmit data of r bits is

$$E_C(d, r) = r\, P_C(d). \tag{2}$$

For mobility, the energy cost is dependent on the parameters of locomotive module, mass of the mobile robot, acceleration, frictions, and distance traveled, etc. [3]. Since a longer distance a mobile robot traveled during a fixed time slot means a higher average speed, more acceleration and larger frictions. For simplicity, we approximate the energy cost for traveling d distance in one time step T (a.k.a. sampling time) as proportional to the square of the distance,

$$E_M(d) = w\, d^2, \tag{3}$$

where w is a constant representing the cost of mobility.

B. Problem Formulation

This paper considers a mobile sensor network, where the source and sink nodes are fixed, and the data-generation rate of the source node is given. A simple sensor network example is shown in Fig. 1. It can be proved that the optimal final network configuration is when all the sensor nodes are on the straight line and spaced equally (i.e., the blank circles in the figure). Intuitively, one might think that node-2 should move to the left and node-3 should moves to the right. However, if node-3 costs much more energy to move, then the reverse may be more energy conservative. Consequently, the communication link has to switch to make the communication energy costs lower.

This example demonstrates that not only the positions of the relaying nodes, which are continuous states, need to be considered, but also the network link structures, which are discrete states, need to be included. Hence, we shall first model the sensor network system using hybrid system theory.

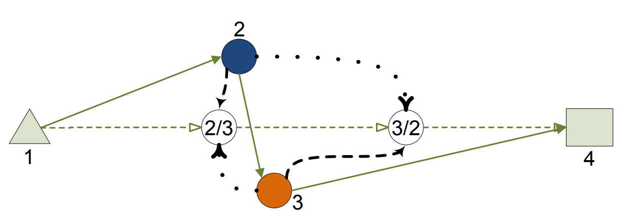


Fig. 1. A simple sensor-network example.

For simplicity, we start with a one-dimensional case and set the number of relaying nodes to two. Let x_i denote the i^{th} node position, so the positions of all the nodes in the network can be represented by a column vector $\mathbf{x} = [x_1, x_2, x_3, x_4]'$. Let $\mathbf{u} = [u_1, u_2, u_3, u_4]'$ be the input vector for position control. We have modeled the system dynamics as

$$\mathbf{x}(k+1) = A\mathbf{x}(k) + B\mathbf{u}(k), \tag{4}$$

where $\mathbf{x}(k)$ and $\mathbf{u}(k)$ are the states and control inputs at time step k, A is an identity matrix, and

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

which has 1's in all the diagonal elements except for the first and last ones, since the source node-1 and the sink node-4 are fixed.

The input $\mathbf{u}$ commands every nodes where to move to for the next time step, hence the energy consumption induced by the input is

$$E_M(k) = \mathbf{u}'(k) R \mathbf{u}(k), \tag{5}$$

where $R = \text{diag}(w_i)$ is a diagonal matrix with w_i at its i^{th} diagonal element. A large w_i means it will be costly for node-i to move.

For the above example of two relay nodes, we can have two possible communication links, which are link1: 1-2-3-4 and link2: 1-3-2-4. We define the discrete states $q \in \{q_1, q_2\}$, where q_1 for link1, and q_2 for link2.

Communication energy cost is dependent on the positions of the nodes, the amount of data transmitted and the link the network is using. Here we assume the data transmission rate is a constant. Once the sampling time T is given, the amount of data to be transmitted in every time step can also be known, denoted as r bits. When $q(k) = q_1$, by Eq. (2), the overall communication energy at time interval $[k, k+1]$ is,

$$\begin{aligned} E_C(k, q_1) &= \sum_{i=1}^{3} r P_C\left(|x_i(k) - x_{i+1}(k)|\right) \\ &= 4ra + rb \sum_{i=1}^{3} (x_i(k) - x_{i+1}(k))^2. \end{aligned} \tag{6}$$

Since E_C is going to be minimized, the first constant term in Eq. (6) can be omitted in computation. Moreover, rb in the second term of Eq. (6) is a constant; without loss of generality, it is assumed to be 1. Hence, we have $\bar{E}_C(k) = \sum_{i=1}^{3} (x_i(k) - x_{i+1}(k))^2$, which is a quadratic and can be rewritten in a matrix form as

$$\bar{E}_C(k, q_1) = \mathbf{x}'(k) Q_{q_1} \mathbf{x}(k),$$

where

$$Q_{q_1} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}. \tag{7}$$

For the same reason, we have

$$Q_{q_2} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & -1 & -1 \\ -1 & -1 & 2 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}. \tag{8}$$

So, when the network link is not fixed, the communication energy cost can be written as,

$$E_C(k) = \mathbf{x}'(k) Q_{q(k)} \mathbf{x}(k). \tag{9}$$

Using Eqs. (5) and (9), the cost function can be obtained as,

$$J = \sum_{k=1}^{N} \left(\mathbf{x}(k)^T Q_{q(k)} \mathbf{x}(k) + \mathbf{u}(k)^T R \mathbf{u}(k) \right) + \mathbf{x}(N+1)^T Q_{q(N+1)} \mathbf{x}(N+1), \tag{10}$$

where it is implicitly assumed that the communication energy cost rate is kept constant during one time step. The objective of the problem is to control positions of the relay nodes by using $\mathbf{u}$ and the network communication link structure q to minimize the overall energy cost; that is,

$$\min_{\substack{q(1),\cdots q(N+1) \\ \mathbf{u}(1),\cdots,\mathbf{u}(N)}} J. \tag{11}$$

The dynamics of this networked system given by Eq. (4) shows its nodes are independent of each other, hence it is a decoupled system. However, the cost function (Eq. (10)) couples the continuous states $\mathbf{x}$ through matrix $Q_{q(k)}$, which is related to the discrete state q and makes the system hybrid.

III. LQR Optimal Control

The problem formulation of hybrid system optimal control is rephrased as follows,

Discrete states: q,

Continuous states: $\mathbf{x}$,

Control input: $\mathbf{u}$, q,

System dynamics:

$$\mathbf{x}(k+1) = A\mathbf{x}(k) + B\mathbf{u}(k),$$

Cost function:

$$J = \sum_{k=1}^{N} \left(\mathbf{x}(k)^T Q_{q(k)} \mathbf{x}(k) + \mathbf{u}(k)^T R \mathbf{u}(k) \right) + \mathbf{x}(N+1)^T Q_{q(N+1)} \mathbf{x}(N+1),$$

Objective:

$$\min_{\substack{q(1),\cdots q(N+1) \\ \mathbf{u}(1),\cdots,\mathbf{u}(N)}} J.$$

This problem formulation fits directly into the switched LQR problem [9], so the problem is solved first using switched LQR to obtain the globally optimal solution, which is also used as a benchmark for the methods proposed in later sections.

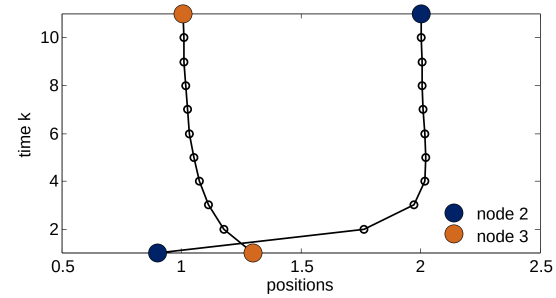


Fig. 2. Using switched LQR.

For the example system, let the initial positions of nodes be

$$\mathbf{x}(1) = [0,\ 0.9,\ 1.3,\ 3]',$$

and let the control cost matrix be

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

which means node-3 is costlier in moving than node-2.

Directly using the switched LQR for control horizon of 10 obtains the optimal solution as shown in Fig. 2, where the horizontal axis shows the positions of nodes, the vertical axis indicates different time steps, and the figure records evolution of node positions. It can be seen that node-3 moves to the left of node-2 after the first step, and hence the network link changes from 1-2-3-4 to 1-3-2-4 thereafter. Then node-2 moves a longer distance than node-3 to fill in the right final optimal position, while node-3 moves to the closer left position, and finally the optimal network configuration is achieved. The figure shows that node-2 goes beyond the right optimal position a little bit and then returns back, which is a strategy to reduce communication energy cooperating with node-3, since node-2 is cheaper to move. The minimal cost is 35.1767.

Switched LQR gives the globally optimal solution, but it requires complex computation of Switched Riccati Mapping to generate Switched Riccati Set for every time step (see Fig. 3). The Riccati set is calculated backwards from the final set, which is $\mathcal{H}_1 = \{Q_{q_1}, Q_{q_2}\}$ for the example, and evolves according to the Riccati Mapping for the length of control

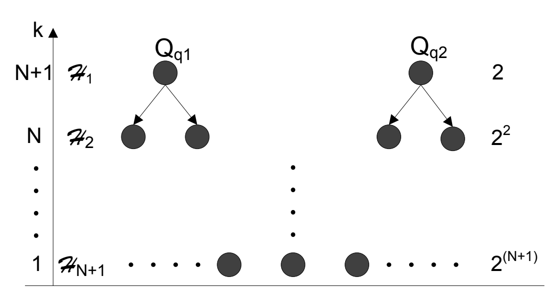


Fig. 3. Switched Riccati Mapping.

horizon. If horizon is N, then the final Riccati set $\mathcal{H}_{N+1}$ contains 2^{N+1} elements and the whole procedure requires about 2^{N+2} Riccati Mappings. When N becomes large, the computation will be too prohibitive to carry out. To solve this computational issue, we will resort to a method that divides the one time computation of long control horizon into many short horizon computations.

IV. Receding Horizon Control

Receding horizon control, also known as model predictive control, is a form of control in which the current control action is obtained by solving a finite optimal control problem on-line at each sampling instant, using the current state of the plant as the initial state. The optimization yields an optimal control sequence and the first control in this sequence is applied to the plant. In this section, we employ the method of RHC to the hybrid system model of the networked system. Different from RHC for traditional systems, discrete state q, which can be fully controlled, needs to be decided for each time instant as well.

At time k, the current state $\mathbf{x}(k)$ is known, and RHC minimizes the cost function

$$J(\mathbf{x}(k)) = \sum_{i=k}^{k+N} \left(\mathbf{x}(i)^T Q_{q(i)} \mathbf{x}(i) + \mathbf{u}(i)^T R \mathbf{u}(i)\right) + \mathbf{x}(k+N+1)^T Q_{q(k+N+1)} \mathbf{x}(k+N+1), \quad (12)$$

over the input sequences of horizon N; i.e., $\{q(1 \mid k), \cdots, q(N+1 \mid k)\}$ and $\{\mathbf{u}(1 \mid k), \cdots, \mathbf{u}(N \mid k)\}$. If there exists optimal sequences of controls $\{q^\star(1 \mid k), \cdots q^\star(N+1 \mid k)\}$ and $\{\mathbf{u}^\star(1 \mid k), \cdots, \mathbf{u}^\star(N \mid k)\}$ for the problem

$$V(\mathbf{x}(k)) = \min_{\substack{q(1|k),\cdots,q(N+1|k)\\ \mathbf{u}(1|k),\cdots,\mathbf{u}(N|k)}} J(\mathbf{x}(k)), \quad (13)$$

then the RHC control law is defined as

$$\begin{aligned} \mathbf{u}^{\mathrm{RHC}}(\mathbf{x}(k)) &= \mathbf{u}^\star(1 \mid k), \\ q^{\mathrm{RHC}}(\mathbf{x}(k)) &= q^\star(1 \mid k). \end{aligned} \quad (14)$$

Applying the RHC to the same system given in the LQR example, we first use the RHC with horizon 3, which is to calculate the optimal control for 3 further steps and use only the first one. Then repeat for 10 steps to make the whole procedure of the same length with the previous example. The result is shown in Fig. 4, from which we can see that the trajectories of relay nodes are almost the same with the LQR result. Furthermore the cost is 35.1811, meaning that the result is very close to the optimal. Compared with LQR, the result is almost the same, but the number of computations of Riccati mapping is reduced dramatically from 4094 to 30.

By reducing the horizon to 2, only 6 computations of Riccati mapping are needed, but the result is not as good. As shown in Fig. 4, using RHC with horizon 2 will not make node-2 and node-3 switch their relative positions, which is the case in the optimal solution. The resulted cost is 35.6189. It is obvious that the more horizon in RHC, the more optimal the result will be, but at the same time the more complicated computation is induced. Deciding the length of horizon used in RHC is a tradeoff of solution optimality and computational complexity.

Though RHC seems to be quite efficient, it is only true for networked systems of small size as in the previous example. When there exist m intermediate relay nodes, connecting them in one communication link will result in $m!$ different links. Even only for 1 step control, there will be $(m!)^2$ elements in Riccati set $\mathcal{H}_2$ (see Fig. 5). For RHC of horizon N, it requires about $(m!)^{N+1}$ Riccati mappings. To attenuate the computational complexity induced by the scale of the sensor network, we will resort to distributed algorithms as proposed in the next section.

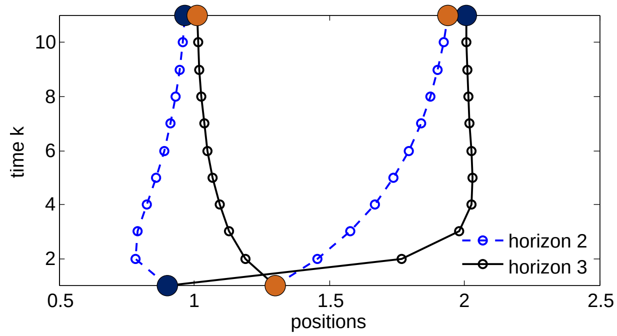


Fig. 4. Using RHC with different horizons.

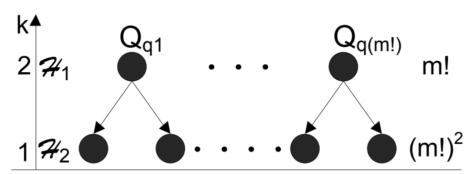


Fig. 5. Switched Riccati mapping with m relay nodes.

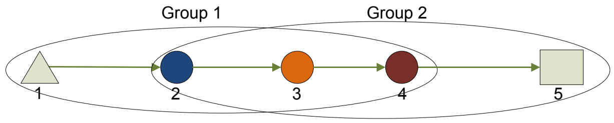
V. A Distributed Algorithm

For a networked system, distributed algorithms are desirable to utilize computational resources of individual nodes that are spread over the network. We propose a distributed algorithm to tackle the curse of dimensionality in networked-system optimal control by dividing the problem spatially, on top of dividing the problem in the dimension of time as RHC did.

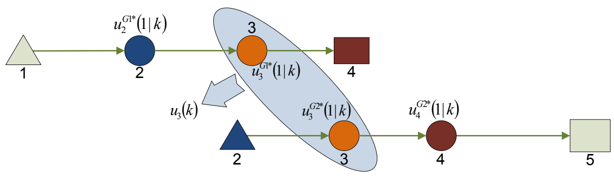
For simplicity of demonstration and discussion, the network is assumed to have 3 relay nodes. The method can be directly extended to networks consisting of more nodes. First, the networked nodes are divided into two groups of four nodes (Fig. 6a): group-1 contains nodes 1, 2, 3 and 4, and group-2 with nodes 2, 3, and 4, which are denoted as

$$\mathbf{x}_{G1} = [x_1, x_2, x_3, x_4]' \quad \text{and} \quad \mathbf{x}_{G2} = [x_2, x_3, x_4, x_5]'. \quad (15)$$

In every group, two end nodes are assumed to be fixed, then the system of each group is the same as the one previously studied. Their dynamics are given by Eq. (4) and the cost functions at time k are given by Eq. (12), which are denoted as $J_{G1}(\mathbf{x}_{G1}(k))$ and $J_{G2}(\mathbf{x}_{G2}(k))$, respectively. The optimization



(a) Grouping.



(b) Generate control inputs.

Fig. 6. Distributed algorithm.

of the overall cost is approximated by solving two smaller problems; i.e.,

$$V_{G1}(k) = \min_{\substack{q_{G1}(1|k),\cdots,q_{G1}(N+1|k)\\ \mathbf{u}_{G1}(1|k),\cdots,\mathbf{u}_{G1}(N|k)}} J(\mathbf{x}_{G1}(k)), \tag{16}$$

$$V_{G2}(k) = \min_{\substack{q_{G2}(1|k),\cdots,q_{G2}(N+1|k)\\ \mathbf{u}_{G2}(1|k),\cdots,\mathbf{u}_{G2}(N|k)}} J(\mathbf{x}_{G2}(k)), \tag{17}$$

which are solved by group-1 and group-2, respectively.

After solving Eqs. (16) and (17), node-2 applies $u_2(k) = u_2^{G1\star}(1 \mid k)$ received from group-1 and node-4 applies $u_4(k) = u_4^{G2\star}(1 \mid k)$ received from group-2. While node-3 receives control inputs from both groups, weighted average based on possible value function savings of each group, (ΔV_{G_i}: the decrease of the cost if optimal control input $\mathbf{u}_{Gi}^{\star}(1 \mid k)$ is applied), is used to come up with the input of node-3,

$$u_3(k) = \frac{u_3^{G1\star}(1 \mid k)\Delta V_{G1}(k) + u_3^{G2\star}(1 \mid k)\Delta V_{G2}(k)}{\Delta V_{G1}(k) + \Delta V_{G2}(k)}, \tag{18}$$

where the intuitive reasoning is that more weight should be given to the input that can possibly reduce the cost more.

After applying the control inputs to relay nodes at each time step, certain node may move to a position, where exchanging roles with a node of other group will save energy. For this purpose, when other node enters current group region, the group has to test whether it needs to be included and decides which node to be substituted for based on the communication cost. Methods such as distributed *minimum-weight path* algorithm can be used. When nodes of different groups exchange, the Riccati mapping of each group has to be recalculated again.

A computer simulation is carried out in a network with initial states $\mathbf{x}(1) = [0,\ 0.5,\ 1.1,\ 1.9,\ 4]'$, and the mobility cost weights are all 1's, except 10 for node-4. For every step the RHC control horizon is set to 4, and the control runs for 10 time steps. The result is shown in Fig. 7 in which the network is able to change the communication link and the cost is calculated to be 48.7615. For comparisons, centralized RHC of horizon 4 is used for the same system, whose result is shown in Fig. 7 and the cost is 48.3584. Though the result of the distributed algorithm is not as good as the centralized one, due to limited information when local groups are performing optimization, the distributed algorithm has reduced the number of Riccati mappings from 9330 to 124 for each group. An intractable problem is approximately solved with a reasonably good solution.

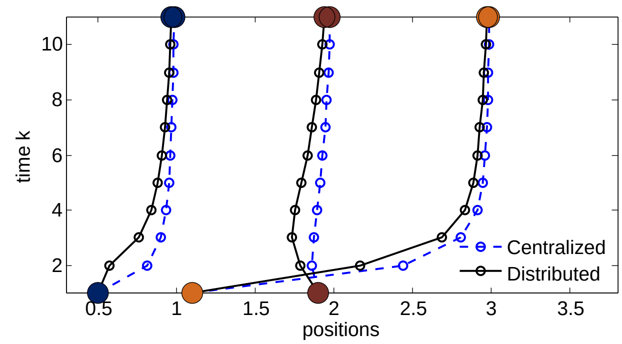


Fig. 7. An example of a sensor network with three relay nodes.

VI. Computer Simulations

Computer simulations have been performed on a planar network link using more realistic parameters to validate the effectiveness and correctness of the proposed distributed algorithm. Consider a mobile sensor network, which is deployed in a 100m$\times$100m area as shown in Fig. 8, consisting of one base station and 4 mobile robots, in which one of them is anchored at a certain location for acquiring data, and the rest of robots 2-4 are serving as relaying robots, which are able to adjust their positions for energy management. Using the proposed communication and mobility cost models, we adopt communication model parameters over an AWGN channel [4], where $\alpha = 2$, $a = 10^{-7}$ J/bit, and $b = 10^{-8}$ Jm^{-2}/bit. The mobility model parameter is set to $w = 0.1$ J/m^2. For the source robot that collects sensory information, we assume the data generation rate is at 100 Kbps. The sampling time for discrete control is set to $T = 100$ s.

To use the proposed distributed algorithm, the network is first divided into two groups, each of which consists of 4 nodes. For cases on a plane, 4 nodes with fixed source and sink can have 5 possible links rather than 2 in one-dimension cases. Because one-relay links or no-relay link may cost less energy than two-relay links, which is not the case in one-dimension link. Hence, we have 5 discrete states to be controlled for each group. This leads to high computational complexity when control horizon in RHC is long. In this simulation, we use horizon 3 and repeat for 5 sampling time steps. The trajectories of mobile relay nodes are depicted in Fig. 8. During the

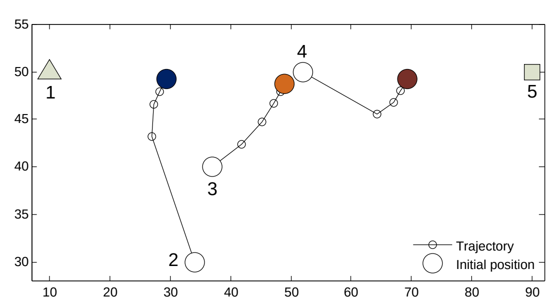
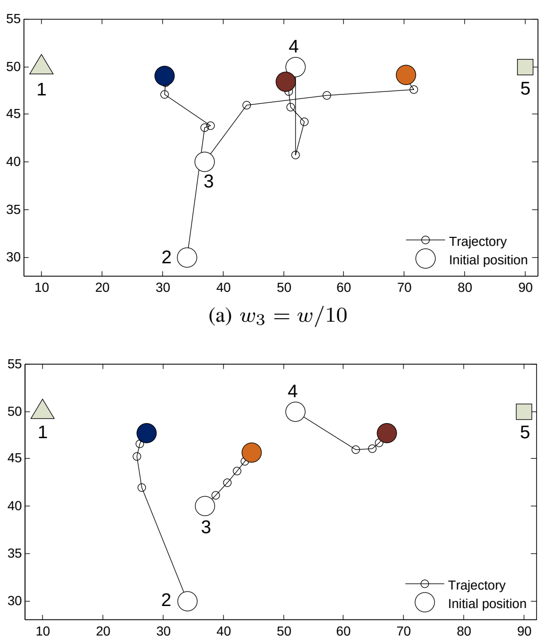


Fig. 8. Simulation of a planar sensor network.

position transition, the network link also changes from 1-3-4-5 to 1-2-3-4-5. The cost of energy for 5 time steps is reduced to 1.13×10^4 J from the original 1.71×10^4 J – a 30% reduction. The energy saving will increase more later on as the nodes are closer to their optimal positions and less distance needs to be moved.

On the other hand, when it costs less for node-3 to move, it is more likely to travel a long distance and consequently the network link is more prone to change as depicted in Fig. 9a, where we assume the mobility energy of node-3 costs $1/10$ as other nodes. Node-3 eventually moves to the rightmost position to assume the role of node-4, and the network link changes from 1-3-4-5 to 1-2-3-4-5, and finally to 1-2-4-3-5.

Though Figs. 8 and 9a show that within 5 steps, 3 mobile relay nodes approach to points that equally divide the source to sink link. This could happen much slower when node-3 is more expensive to move as indicated in Fig. 9b, where the mobility cost of node-3 is set to be ten times as much as other nodes. We can see that the position displacement of node-3 for every time step becomes smaller, since energy saving in data transmission during one time step can only afford shorter traveling distance due to higher mobility cost. This fact also explains why we choose sampling time $T = 100$ s, which is relatively large compared with other control systems. Because mobility, which is much costlier than data transmission, will only be paid off in the long run. However, the longer the horizon, the more the computation it will have to take. A simple way to deal with this situation is to enlarge the sampling time so that the same period of time is covered by less discrete time steps and the costs of mobility and data transmission during each step are of the same order. As a result, the optimal control method introduced in this paper is more suitable for sensor networks with intensive data flow, such as wireless video sensor networks.



(a) $w_3 = w/10$

(b) $w_3 = 10w$

Fig. 9. When node-2 has different mobility cost.

VII. Conclusions and Future Work

The main contributions of this paper are in 1) The study of optimizing both mobility and communication energy in a mobile sensor network, which was modeled as a hybrid system LQR; the optimization problem was solved by an existing switched LQR method; 2) Receding Horizon Control was proposed to be used in the modeled hybrid system, and the simulations showed that the RHC method was stable and provided a solution close to the optimal one; 3) Distributed RHC algorithm was proposed to attenuate the computational complexity due to the large scale of the networked system, and the algorithm converged to a solution that can approximate optimizer quite well as illustrated by computer simulations.

Although two algorithms were proposed for the hybrid system, they were verified only by computer simulations. Mathematical proofs of the stability of the RHC in a hybrid system, the convergence of the distributed algorithm and their optimality are for future work. We also believe that there exist other methods to decentralize the optimal control algorithm that can obtain better results.

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