Air traffic complexity in future Air Traffic Management systems

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Abstract

In this paper we study the issue of characterizing the complexity of air traffic to support Air Traffic Management (ATM) operations. We discuss, in particular, the features of a complexity metric that are relevant for application to future ATM systems where part of the responsibility for separation assurance and trajectory management operations is distributed on board of the aircraft. We then describe a probabilistic complexity metric that meets all those features and is amenable for supporting onboard conflict detection and resolution and trajectory management operations. A numerical example illustrates its possible use in a fully automated self-separation context.

Keywords: Air traffic complexity; air traffic management; multi-agent system.

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1 Introduction

The growth in air traffic demand is causing a disproportional increase of delay in the current centralized and ground-based Air Traffic Management (ATM) system. For example, in 2007, there was a 5.3% growth in the air traffic over Europe over 2006, with an increase of 17.4 % in the delay [4]. The traffic grew by 0.5% in 2008 compared to 2007 while the total delay increased by 10.4% over 2007 [5].

In the current ATM system, aircraft are forced to fly along predefined routes and the airspace is structured in sectors with a team of Air Traffic Controllers (ATCs) per sector in charge of maintaining the appropriate separation between aircraft. Most of the strategic tasks (flight planning and slots allocation) are performed prior to take off, and tactical intervention with dynamic re-planning of the flights is delegated to the ATCs. Limits on the capacity of the ATM system are imposed by the maximum amount of workload that ATCs are able to sustain. A radical change, more substantial than simply reconfigurations sectors and reassignment human resources, is required to adapt the system capacity to the growth in air traffic demand.

This has fostered the development of new operational concepts in ATM, as witnessed by the SESAR (Single European Sky ATM Research, [1]) project in Europe and the NextGen (Next Generation Air Transportation System, [2]) project in the United States. Both projects envision an enhancement of strategic ATM, moving from a rigidly-structured airspace to trajectory-based operations with dynamic sharing of 4D trajectories flown using advanced airborne navigation systems. This should allow each aircraft to achieve its preferred routes and time of arrival while reducing the need of tactical intervention.

Tactical ATM will be provided only in part by the ATCs. For a large part of airspace, new separation modes are envisioned with tactical actions either fully or only partially delegated to aircrews assisted by Airborne Separation Assistance Systems (ASASs) in performing their navigation and self-separation tasks. The ATM function will be realized through a distributed control scheme, where each aircraft evaluates the criticality of forthcoming encounters based on the information on the current position and intended destination of neighboring aircraft and eventually coordinates with them to avoid the actual occurrence of conflict (intent-based conflict detection and resolution).

This shift towards a distributed ATM function will be enabled by the introduction of novel information-sharing systems (the System Wide Information Management system developed within SESAR and the Netcentric Infrastructure System developed within NextGen), together with new data-link technologies (e.g., the Automatic Dependent Surveillance - Broadcast service) that allow the aircraft to broadcast their own position and velocity and to get up-to-date information on the other aircraft position, velocity, and intent, on locally sensed weather data, on global weather conditions and forecast, and on areas to avoid.

In the SESAR and NextGen operational concepts, humans will maintain a central decision role, eventually assisted by decision support tools. Longer term innovative research activities are currently ongoing within the SESAR project itself, aiming at relaxing the constraint on the central role of humans and opening up new possibilities related to the introduction of high degrees of automatic, possibly up to full automation, with application in a time-frame beyond the nominal SESAR time horizon of 2020.

Air traffic complexity is a concept introduced to measure the difficulty and effort required to safely manage air traffic. Most of the complexity metrics developed in the literature address ground-based ATM and are conceived so as to assess the impact of a given air traffic configuration on the workload of the ATCs in charge of safely handling it. Complexity measures are currently employed to redistribute and reassign human resources and to reconfigure sectors in order to avoid excessive ATC workload, which could eventually compromise air travel safety and efficiency.

In future distributed ATM systems, suitable notions of complexity should be defined to assess and predict those air traffic conditions that may be overdemanding to the autonomous aircraft design. This is a crucial task for avoiding encounters that appear safe from the individual aircraft perspective, but are actually safety-critical from a global perspective.

In high density airspace, the achievement of the performance and safety objectives of an aircraft is hampered by the presence of other aircraft flying in the same region. More specifically, performance is deteriorated when the aircraft is flying through an area with highly congested traffic, since this will typically require to perform many tactical maneuvers; whereas safety might be compromised when an aircraft is involved in a multi-aircraft conflict situation that exceeds the capabilities of the onboard conflict resolution system.

Complexity evaluation on a long term prediction horizon can help to identify congested areas and support strategic flight plan optimization, whereas complexity evaluation on a mid/short term horizon can help to identify encounter situations that are critical for distributed conflict resolution operations.

In this paper, we first briefly discuss characteristics relevant to future ATM and review existing approaches to complexity evaluation based on such characteristics. We then focus on a method for complexity evaluation that meets the requirements posed by the future ATM systems and accounts for possible deviations of the aircraft from their nominal trajectories. In particular, the complexity as experienced by a single aircraft is evaluated through some measure of robustness of the aircraft trajectory to possible deviations of the other aircraft from their nominal trajectories. Given that these possible deviations are described through a stochastic process, the introduced measure of robustness can be interpreted as some stochastic version of the notion of flexibility of a trajectory (i.e., the extent to which a trajectory can be modified without causing a conflict with neighboring aircraft or entering a forbidden airspace area) in [18, 19].

The method was introduced in [31] for level flight and then extended to 3D in [34]. Its performance has been studied recently in [30] with reference to the problem of identifying those air traffic configurations that are difficult to control in a decentralized way. A correlation analysis with collision risk was performed based on experiments on the hypothetical Autonomous Mediterranean Free Flight high density air traffic scenario considered in [7] for collision risk estimation by the Interacting Particle System method. The obtained results look promising, especially since no specific tuning of the model describing the possible deviations of the aircraft from their nominal trajectories was made.

Here, we only provide some simple 2D numerical example to show the efficacy of the method.

2 Characteristics of complexity relevant to future ATM systems

Future ATM poses novel requirements to complexity metrics, which ultimately result in certain features that the metrics should possess. A list with a brief explanation of these features is reported next.

Accounting for traffic dynamics besides density. Traffic density is the most important factor determining the complexity of air traffic, irrespectively of the specific application. However, traffic density alone provides only a partial information on complexity as already acknowledged in the context of ground-based ATM, where ATCs accept different level of traffic depending on the traffic configuration, [3]. The dynamics of the traffic plays a major role in this and density on its own is a crude estimator of complexity.

Independent of the airspace structure. Self separation airspace is a sector-free context where aircraft are allowed to select their preferential routes subject to some constraints. As such, complexity metrics should not present any structural dependence on the sector characteristics. Given that the traffic density is a relevant factor for complexity characterization, [16, 21], the identification of aircraft clusters (i.e., groups of closely spaced aircraft) can complement and accelerate complexity assessment by isolating those airspace areas where to concentrate the attention. To some extent, aircraft clusters can play within a self separation context a role similar to sectors within the centralized human-operated ATM system.

Tailored to the look-ahead time horizon. Approaches for air traffic complexity assessment should be tailored to the look-ahead time horizon of the intended application. For instance, complexity metrics for onboard trajectory management should be computed based on the aircraft nominal trajectory over the reference (long term) time horizon for onboard trajectory optimization. Complexity should be recomputed from time to time to take care of possible modifications of the aircraft planned trajectories. Unexpected deviations on a finer time scale shall be accounted for by complexity metrics tailored to a mid/short term time horizon.
Complexity metrics for supporting conflict detection and resolution functions should be computed based on the trajectories reconstructed from the state and intent information on a mid/short term time horizon.

Dependent on the way traffic is controlled only indirectly, through the traffic organization. A complexity metric should measure the effort involved is safely handling air traffic and reveal critical encounter situations. In the current ATM systems, where ATCs are in charge of aircraft separation assurance complexity ultimately relates to the ATC workload. In fully automated ATM systems with the ATC replaced by an automatic solver, it should reflect the computational effort required to find an effective resolution strategy by the automatic solver. Different levels of automation could be realized and co-exist in future ATM systems: the pilot could be provided with a set of possible options to choose from or only be informed of the decision taken by some automated system, whose characteristics would depend on the adopted optimization and resolution strategy. As a result, the controller structure is difficult to characterize and to be explicitly accounted for in complexity evaluation, and involves also pilots as human-in-the-loop component, with the related problems of workload evaluation. All these considerations make a measure of complexity that dependent on the way traffic is controlled only indirectly, through the effect of the controller on the traffic organization, better suited for future ATM systems.

With a goal-oriented output form. Air traffic complexity is both a time- and space-dependent feature, that is typically expressed in aggregate form by condensing either the space or the time information (or both) of the traffic situation under consideration. Output forms range from a scalar value, describing the traffic complexity in a certain region at a specific time instant, to a spatial complexity map.

Regarding the prospective applications, scalar-valued metrics (possibly projected over some look-ahead time horizon) could be better suited to the mid term conflict detection and resolution function, providing a synthetic information on the level of complexity encountered by the aircraft along its current trajectory, which should be easier to interpret. On the other hand, complexity maps can be used to identify critical areas of the airspace that the aircraft should better avoid, and, hence, are more suitable for the long term trajectory management task.

With sustainable computational load. The computational effort involved in complexity evaluation is a critical feature for ATM operations, especially in on-board applications. The effort is related to the output form: those approaches providing a complexity map are typically computationally more intensive than those computing a scalar value of complexity, and for them the memory requirements and the need for an efficient (compact and easy to interpret) representation of the information are an issue.

A property that can reduce the computational load in trajectory management operations is that each aircraft contribution to complexity can be computed separately and then combined with that of the other aircraft to provide the overall air traffic complexity. This property in fact allows a simple evaluation of the impact of possible trajectory changes by removing the original contribution of the aircraft and replacing it with the new one based on the updated trajectory.

As the computational resources available on the ground are higher than those available on-board, the complexity/congestion prediction functionality for trajectory management could be implemented in the form of a ground automation tool and only relevant information on the ares-to-avoid be uploaded to the airborne system.

3 Brief survey on air traffic complexity

The concept of air traffic complexity was originally introduced with the purpose of assessing whether an air traffic configuration may cause unsustainable workload to ATCs and providing guidelines on how to obtain more manageable sectors by reconfiguring the airspace and by modifying traffic patterns, see e.g. [36, 37, 15]. The work [10] was perhaps the first one to systematically examine the relationship between air traffic characteristics and ATC workload.

Most studies on air traffic complexity have been developed with reference to ground-based ATM, as it clearly appears from the literature reviews [27] and [16], and the discussion paper [33].

Here, we provide a brief description of selected approaches to complexity metrics that have been proposed in the current ATM system and discuss their characteristics in view of a possible application to future ATM.

In the current practice, complexity of air traffic is accounted for in terms of number of aircraft and on a per-sector basis, [36, 16]. The number of aircraft in a sector is compared with some established threshold value representing the maximum number of aircraft that ATCs are willing to accept in that sector. Aircraft density is easy to compute and interpret.

Air traffic indicators other than the number of aircraft per sector affect the ATC's workload. These indicators are related to both structural (e.g., terrain configuration, number of airways, airway crossings and navigation aids) and flow (e.g., number of aircraft, weather, aircraft separation, closing rates, aircraft speeds, mix of aircraft and flow restrictions) characteristics of air traffic, [27, 16]. Both these static and dynamic factors interact in a nonlinear complex way to produce air traffic complexity, [6, 26, 8].

Dynamic density is a single aggregate indicator of complexity where traffic density and other controller workload contributors (such as the number of aircraft undergoing trajectory change and requiring close monitoring due to reduced separation) are combined linearly or though a neural network. In both cases, weights are determined based on subjective ratings obtained through interviews to qualified ATCs or by regression analysis of their physical activity data. Different dynamic density measures have been proposed in the literature, depending on the complexity factors that they include. The choice of the complexity factors often relates to the specific Air Traffic Control Center (ATCC), which makes dynamic density a sector-dependent metric.

Dynamic density metrics directly incorporate some measure of the ATC workload. They are computationally intensive only in the tuning phase, when weights are determines, and not in their on-line use.

A time-smoothed version of dynamic density is the so-called integral complexity, introduced in [15] as an estimate of the ATC workload in a sector and defined as the average over a 5 to 10 minutes time window of the linear combination of certain complexity factors (number of aircraft flying within the sector, number of aircraft flying on nonlevel segments, and number of aircraft flying close to the border of the sector).

Both aircraft density and dynamic density are instantaneous scalar metrics that can can be projected over some look-ahead time horizon through aircraft trajectory prediction.

The difficulty in obtaining reliable workload measures has been one of the strongest motivations for developing approaches for complexity evaluation that are independent of the way traffic is controlled, such as the *fractal dimension* in [28], the *input-output approach* in [22], and the *intrinsic complexity* measures in [11, 12] and [20].

Fractal dimension is a scalar measure of the geometrical complexity of a traffic pattern, which evaluates the number of degrees of freedom used in the airspace by the existing air routes, [28]. In the current ATM system, aircraft cruise on linear routes at specified altitudes, corresponding to a geometrical dimension of 1. In future ATM, flights will be allowed to move from these linear routes. If all of the airspace were covered by routes, the FD would be 3. However, there will still be preferred routes (due to the position of connected airports, or to wind currents, etc.), thereby decreasing the actual dimension of the route structure. Fractal dimension is sector-independent, determined only based on the aircraft trajectories, and not accounting for timing information.

In [22, 23, 24], air traffic complexity is defined in terms of the control effort needed to avoid the occurrence of conflicts along some reference time horizon when an additional aircraft enters the traffic. To this purpose a feedback control scheme is introduced, where the air traffic within the region of the airspace under consideration is the system to be controlled, and an automatic conflict solver is the feedback controller. The input to the closed-loop system is represented by a (fictitious) additional aircraft entering the traffic, whereas the output is given by the deviation of the aircraft already present in the traffic from their original flight plans as issued by the feedback controller to safely accommodate the incoming aircraft. The overall amount of corrective action needed to recover a conflict-free condition is taken as a measure of the air traffic complexity. A "complexity map" is constructed as a function of the entering position and bearing of the incoming aircraft. The operational interpretation of this complexity map is difficult. A scalar value can be extracted from the map taking, *e.g.*, the "worst-case" value for the corrective action needed to safely accommodate the additional aircraft. Note that different measures of the control effort and different solvers could be used, and that the choice of the conflict solver has a large impact on complexity evaluation.

Finally, intrinsic complexity metrics were introduced with the purpose of capturing the level of disorder as well the organization structure of the air traffic distribution, irrespectively of its effect on the ATC workload.

In [35], in particular, to capture the complexity associated to a lack of organization, an air traffic situation is modelled by a dynamical system, with the aircraft trajectories interpreted as the system trajectories obtained for different initial conditions. The Lyapunov exponents of the dynamical system provide an indicator of the air traffic complexity, since Lyapunov exponents measure the rate of exponential convergence or divergence of nearby trajectories, and can be taken as indicators of the level of order/disorder of a system. The idea is that the larger is a positive Lyapunov exponent, the higher the rate at which one loses the ability to predict the system behavior. Areas characterized by high air traffic complexity are then easily identified by plotting the largest Lyapunov exponent as a function of the airspace position, thus obtaining a complexity map over the considered airspace area. This metric presents all the characteristics in Section 2 except for the last one since determining the interpolating vector field of the dynamical system describing air traffic is computationally quite intensive.

In all approaches, evaluating complexity over some look-ahead time horizon involves predicting the aircraft future positions. Despite the extensive studies on uncertainty in the modeling and analysis of ATM systems by various researchers (see e.g. [14], [29], [25], [9] and [32]) its effect on air traffic complexity evaluation has not received adequate attention. This is a critical aspect since the reliability of complexity prediction depends on the aircraft motion prediction which is affected by different sources of uncertainty, primarily by wind, but also errors in tracking, navigation and control.

In this paper, we focus on an approach to air traffic complexity evaluation that was first proposed in [31] for level flight and then extended to 3D in [34]. According to this approach, uncertainty affecting the aircraft future position is incorporated in the complexity measure to account for possible local deviations of the aircraft from their nominal trajectories.

4 Probabilistic approach to complexity

Complexity is evaluated in terms of proximity in time and space of the aircraft present in the traffic as determined by their intent and current state while accounting for possible local deviations of the aircraft from their nominal trajectory.

We adopt a probabilistic description of the uncertainty affecting the aircraft future position, which allows us to attribute different likelihood to different trajectories, and, in particular, a lower likelihood to trajectories that are farther away from the nominal one. We describe the uncertainty affecting the future aircraft position through a Brownian motion characterized by continuous trajectories as realizations, with a dispersion that grows linearly in time –at possibly different rates– along the cross-track and along-track directions. This model is motivated by quite similar models that have been adopted in the literature on aircraft conflict prediction, see, e.g., [14], [29] and [32]. Also, it presents the advantages of being easy to describe (one has only to set the parameters representing the variance growth rates in the cross-track and along track directions) and mathematically more tractable that alternative models like the Gaussian random field in [17].

Air traffic complexity at a point x in an airspace region $\mathcal{S}$ and at time t within some look-ahead time horizon T is evaluated as the probability that a certain buffer zone in the airspace surrounding x will be “congested” within $[t, t+\Delta]$, with $\Delta > 0$ (probabilistic occupancy). By defining congestion as the simultaneous occupancy of the buffer zone by a certain number of aircraft and evaluating this complexity measure at all possible points in $\mathcal{S}$, a complexity map can be built. Forming the complexity maps associated with different consecutive time intervals allows to predict when the aircraft will enter and leave a particular zone in the airspace, and to identify regions of the airspace $\mathcal{S}$ with a limited inter-aircraft maneuverability space that might cause an aircraft excessive tactical manoeuvering and deteriorate its flight performance.

From a single aircraft perspective, the information on the probabilistic occupancy along its own trajectory can be used to detect critical encounter situations that would be difficult for the aircraft to solve autonomously, and also to provide guidance for trajectory design in conflict resolution operations.

It is worth noticing that, although evaluating complexity according to the proposed approach does not require to evaluate the interaction (convergent/divergent behavior, relative orientation of the direction of flight, etc) between aircraft, this interaction strongly affects complexity. Indeed, if two aircraft are converging, they will both contribute to complexity at the same 4D (time cross space) points, whereas this will not be the case if they are diverging; a higher value of complexity will be associated with two aircraft converging along directions of flight with a smaller relative orientation since the aircraft will stay closer one to the other for a longer time interval along the look-ahead time horizon, etc. In other words, the proposed method for evaluating complexity actually accounts for the traffic dynamics through the traffic density evolution. This is a feature that the approach shares with the intrinsic complexity metrics.

Analytic –though approximate– formulas for the global and single-aircraft complexity metrics have been derived in [34] under the assumption that a piecewise linear approximation of the nominal trajectory can be adopted, with the aircraft flying at constant velocity between consecutive way-points. Here, we describe the 2D case, which is easier to explain. The reader is referred to [34] for details on the computational aspects in the 3D case.

4.1 Complexity from a global perspective

Consider N aircraft $A_i, i = 1, \dots, N$, flying at the same altitude in the 2-D airspace $\mathcal{S} \subset \mathbb{R}^2$ during the look-ahead time horizon $T = [0, t_f]$, with $t = 0$ representing the current time instant and $t_f > 0$ the time horizon length. Suppose that each aircraft is following a nominal trajectory with a velocity profile $u^{A_i} : T \to \mathbb{R}^2$, starting from the initial position $x_0^{A_i}$ at time $t = 0$. The aircraft future position during T is not known exactly, and we assume that it is given by

$$x^{A_i}(t) = x_0^{A_i} + \int_0^t u^{A_i}(s)ds + Q^{A_i}(t)\Sigma^{A_i}W^{A_i}(t), \tag{1}$$

where $W^{A_i}(t)$ is a standard 2-D Brownian Motion (BM) starting from the origin whose variance grows linearly and is modulated by the matrix $Q^{A_i}(t)\Sigma^{A_i} \in \mathbb{R}^{2\times 2}$. Matrix $\Sigma^{A_i} = \operatorname{diag}(\nu_a^{A_i}, \nu_c^{A_i})$ is a diagonal matrix whose entries $\nu_a^{A_i}$ and $\nu_c^{A_i}$ are the variance growth rates of the perturbation in the along-track direction (namely the direction of u^{A_i}) and in the cross-track direction (i.e., direction orthogonal to u^{A_i}) and satisfy $\nu_a^{A_i} > 0$ and $\nu_c^{A_i} > 0$, whereas $Q^{A_i}(t) = \begin{bmatrix} q_1^{A_i}(t) & q_2^{A_i}(t) \end{bmatrix} \in \mathbb{R}^{2\times 2}$ is an orthogonal matrix whose first column $q_1^{A_i}(t)$ is aligned with $u^{A_i}(t)$: $q_1^{A_i}(t) = \frac{u^{A_i}(t)}{\|u^{A_i}(t)\|}$. Note that different values can be assumed for the variance growth rates of the aircraft, e.g., to account for different trajectory precisions.

For each $x \in \mathcal{S}$, let us consider the ball of radius ρ centered at x:

$$\mathcal{M}_\rho(x) = \left\{\hat{x} \in \mathbb{R}^2 : \ \|\hat{x} - x\|^2 \leq \rho^2\right\}. \tag{2}$$

The complexity of air traffic within the airspace region $\mathcal{S}$ can be evaluated through the following first order and second order complexity measures.

The *first order complexity* $\mathrm{c}_1(x,t)$ at position $x \in \mathcal{S}$ within the time interval $[t, t+\Delta] \subseteq T$ is defined as

$$\mathrm{c}_1(x,t) := P\big(x^{A_i}(s) \in \mathcal{M}_\rho(x), \text{ for some } s \in [t, t+\Delta] \text{ and } i \in \{1, 2, \dots, N\}\big) \tag{3}$$

and represents the probability of at least one aircraft entering the ellipsoid $\mathcal{M}_\rho(x)$ within the time frame $[t, t+\Delta]$.

Note that $\mathrm{c}_1(x,t) = 0$ means that none of the existing aircraft will be inside the ellipsoid $\mathcal{M}_\rho(x)$ during the time interval $[t, t+\Delta]$. On the other hand, $\mathrm{c}_1(x,t) = 1$ implies that with certainty there will be at least one aircraft within $\mathcal{M}_\rho(x)$ at some time instant belonging to $[t, t+\Delta]$.

The *second order complexity* $\mathrm{c}_2(x,t)$ at position $x \in \mathcal{S}$ within the time interval $[t, t+\Delta] \subseteq T$ is defined as

$$\mathrm{c}_2(x,t) := P\big(x^{A_i}(s) \text{ and } x^{A_j}(s') \in \mathcal{M}(x) \text{ for some } s, s' \in [t, t+\Delta] \text{ and } i \neq j \in \{1, 2, \dots, N\}\big) \tag{4}$$

and represents the probability of at least two aircraft entering the ellipsoid $\mathcal{M}_\rho(x)$ within the time frame $[t, t+\Delta]$.

If $c_2(x,t) = 0$, then there will be at most a single aircraft inside the ellipsoid $\mathcal{M}_\rho(x)$ within the time interval $[t, t+\Delta]$. Hence, at any time $t \in [t, t+\Delta]$, an aircraft passing through $\mathcal{M}(x)$ will not be sharing $\mathcal{M}(x)$ with any of the other N aircraft. If $c_2(x,t) = 1$, then with probability 1, at least two aircraft will enter the ellipsoid $\mathcal{M}_\rho(x)$ during the time interval $[t, t+\Delta]$, though possibly not at exactly the same time.

By letting x vary over $\mathcal{S}$, one can define the *first order and second order complexity maps* of the airspace region $\mathcal{S}$ within the time frame $[t, t+\Delta]$ as follows:

$$\mathbb{C}_1(\cdot,t) : x \in \mathcal{S} \to c_1(x,t)$$
$$\mathbb{C}_2(\cdot,t) : x \in \mathcal{S} \to c_2(x,t).$$

Evidently, at any point $x \in \mathcal{S}$, the $\mathbb{C}_2$ map has a value smaller than or equal to the $\mathbb{C}_1$ map, since the corresponding events are nested. Higher order complexity measures and maps can also be defined according to a similar procedure.

Forming the complexity maps for different consecutive time intervals allows to detect congested areas (i.e., areas where multi-aircraft encounters with limited inter-aircraft spacing are likely to occur) in the time-space coordinates, and to identify surrounding areas where the traffic could be deviated. The presence of a region with a high value of the second order complexity implies a high likelihood that two or more aircraft will get close in time and space, hence having a conflict. Trajectories should be designed so as to reduce second order complexity.

By integrating with respect to time, we can reduce the space cross time map to a spatial map and identify the airspace region with high *percentage of occupancy* over the time horizon T.

Even more compact global information can be obtained according to the following procedure.

The complexity measures associated with region $\mathcal{M}_\rho$ depends on the radius ρ of the ball $\mathcal{M}_\rho(x)$. Let make this dependence explicit through the notations $c_1^\rho(x,t)$ and $c_2^\rho(x,t)$. Both $c_1^\rho(x,t)$ and $c_2^\rho(x,t)$ are increasing as a function of ρ. Let

$$\rho_{\max}(x,t) := \sup\{\rho \geq 0 : c_2^\rho(x,t) \leq p_T\},$$

where p_T is some threshold value for the probability that two aircraft come close one to the other, and define

$$\rho^\star_{\max} := \inf_{t \in T, x \in \mathcal{S}} \rho_{\max}(x,t).$$

Then, one can take

$$\xi := \frac{1}{\rho^{\star}_{\max}}$$

as a synthetic indicator of complexity of the traffic during the time horizon T. ξ is a measure of the extent of the maneuverability space within $\mathcal{S}$ and along the time horizon T. The smaller ξ is, the easier it is to accomodate an additional aircraft without causing conflicts. Note that the extent of the available maneuverability space as measured by ξ will depend on both the local aircraft density and the traffic dynamic through the aircraft intent. Since uncertainty in the predicted aircraft position models possible deviations of the aircraft from their intended trajectory, ξ can be interpreted as a measure of robustness of air traffic to perturbations of the nominal situation.

4.2 Complexity from a single aircraft perspective

According to Definitions 3 and 4, complexity is evaluated from a global perspective as the probability of occupancy of a buffer zone surrounding a point by a certain number of the aircraft A_i, $i = 1, 2, \ldots, N$, that are present in the airspace region $\mathcal{S}$. These complexity measures can be easily adapted to provide a measure of complexity from the perspective of an additional aircraft that is entering the airspace region $\mathcal{S}$ following some nominal trajectory. The resulting single-aircraft complexity measure can be interpreted as an indicator of the robustness of the aircraft nominal trajectory with respect to possible deviations of the other aircraft from their intended trajectory.

Suppose that an additional aircraft, say aircraft B, is entering $\mathcal{S}$ at time 0 following a nominal trajectory $\bar{x}^B : T \to \mathbb{R}^2$. The idea is to evaluate the complexity encountered by aircraft B along its nominal trajectory by making the buffer zone move along the trajectory of aircraft B and computing the probability that some of the other aircraft A_i, $i = 1, 2, \ldots, N$, will enter such moving zone. This leads to the following definition of *single-aircraft complexity.*

The complexity experienced by aircraft B along its nominal trajectory $\bar{x}^B : T \to \mathcal{S}$ within the time interval $[t, t + \Delta]$ is defined as:

$$\mathrm{c}_B(t) := P\big(x^{A_i}(s) \in \mathcal{M}_\rho(\bar{x}^B(s)) \text{ for some } s \in [t, t + \Delta] \text{ and } i \in \{1, 2, \ldots, N\}\big) \tag{5}$$

Interestingly, if the time window $[t, t + \Delta]$ extends to the whole look-ahead time horizon T

and the buffer zone reproduces the protection zone surrounding each aircraft, the single-aircraft complexity measure can as well be interpreted as the probability of aircraft B getting in conflict with another aircraft A_i within T. Conflict detection and resolution then becomes an integrable task in complexity evaluation.

According to a reasoning similar to that in Section 4.1, we can introduce function $\rho_{\max,B} : T \to \mathbb{R}_+$ given by

$$\rho_{\max,B}(t) := \sup\{\rho \geq 0 : \mathrm{c}_B^\rho(t) \leq p_T\},$$

and define

$$\rho^\star_{\max,B} := \inf_{t \in T} \rho_{\max,B}(t).$$

$\rho^\star_{\max,B}$ is an index of robustness of the nominal trajectory of aircraft B. The larger is $\rho^\star_{\max,B}$, the more aircraft B is far from the other aircraft, both in time and in space, with high ($> 1 - p_T$) probability, and, hence, the larger is the robustness of its trajectory to possible deviations of the other aircraft from their intent.

The quantity $\xi_B := \frac{1}{\rho^\star_{\max,B}}$ can then be taken as a synthetic indicator of the air traffic complexity from the perspective of aircraft B during the time horizon T. Let ρ_{safe} denote the value of ρ such that $\mathcal{M}_\rho(x)$ represents the protection zone surrounding an aircraft positioned at x. If $\xi_B > \frac{1}{\rho_{\mathrm{safe}}}$, then, some conflict can occur with probability $\geq p_T$ and the criticality of this conflict can be better assessed by computing, for instance, the earliest *conflict time*: $t^\star_B = \min\{t \geq 0 : \rho_{\max,B}(t) < \rho_{\mathrm{safe}}\}$.

The introduced single-aircraft complexity measure (5) can be used by aircraft B to evaluate the maneuverability space surrounding its nominal trajectory and to eventually redesign its trajectory so as to improve its robustness. According to a similar perspective, in the works on trajectory flexibility [19, 18] it is suggested that, to achieve the aggregate objective of avoiding excessive 'air traffic complexity' in autonomous aircraft ATM, aircraft should plan their trajectory so as to preserve maneuvering flexibility to accommodate possible disturbances stemming, for example, from other traffic.

4.3 Computational aspects

Consider N aircraft A_i, $i = 1, \dots, N$, that are flying at the same constant altitude in the airspace $\mathcal{S}$. For each aircraft A_i we can define as $\pi_i(x,t)$ the probability that aircraft A_i will enter the ball of radius ρ centered at $x \in \mathcal{S}$ within the time frame $[t, t+\Delta]$.

If the BMs affecting the future positions of the N aircraft are independent, then the global complexity metrics $\mathrm{c}_1(x,t)$ and $\mathrm{c}_2(x,t)$, as well as the single-aircraft complexity metric $\mathrm{c}_B(t)$ can be expressed in terms of $\pi_i(x,t)$, $i = 1, 2, \dots, N$, as follows

$$
\begin{aligned}
\mathrm{c}_1(x,t) &= 1 - \prod_{j=1}^{N} \big(1 - \pi_j(x,t)\big) \\
\mathrm{c}_2(x,t) &= \mathrm{c}_1(x,t) - \sum_{h=1}^{N} \Big(\pi_h(x,t) \prod_{j=1,\, j\neq h}^{N} \big(1 - \pi_j(x,t)\big)\Big) \\
\mathrm{c}_B(t) &= 1 - \prod_{j=1}^{N} \big(1 - \pi_j(\bar{x}^B(t), t)\big).
\end{aligned}
$$

This implies that the contribution to complexity of each aircraft can be computed in isolation and then combined to get the overall complexity. This decoupling is quite convenient since computations can be parallelized and updating the complexity metrics becomes easier.

Remark 1 *The independence assumption is actually reasonable if the N aircraft are not flying too close one to the other, so that the correlation introduced by the wind affecting the aircraft motion is negligible, [17, 9]. If this is not the case, the expressions above can be considered as estimates. Accounting for the correlation introduced by wind will require to use a more complex trajectory prediction model with a random field term affecting the future aircraft positions, instead of a Brownian motion. Computing the probability that one or more than one aircraft will reach some region within some time interval then becomes more challenging and calls for appropriate numerical solutions as those described in [17] with reference to the aircraft conflict prediction problem. In addition, the advantage of decoupling the computations is lost, due to the spatial random field correlation.*

We now address the problem of determining the probability $\pi_i(x,t)$. Analytic – though approximate – expressions for $\pi_i(x,t)$ as a function of $x \in \mathcal{S}$ and $t \in [0, t_f]$ will be derived in the case when each of the N aircraft A_i, $i = 1, \dots, N$, that are flying at the same constant altitude is following a flight plan given by a sequence of way-points with the associated arrival times $\{(O_h^{(i)}, t_h^{(i)}),\ h = 0, 1, \ldots, n_i\}$ with $O_0^{(i)}$ representing the current position of the aircraft at time $t_0^{(i)} = 0$. Similar expressions can be derived for $\pi_i(\bar{x}^B(t), t)$ as pointed out in Remark 2.

The flight plan of aircraft A_i determines a nominal, piecewise constant, velocity profile $u^{A_i} : [0, t_f] \to \mathbb{R}^2$ that the aircraft is trying to follow starting from $O_0^{(i)}$. The corresponding, piecewise constant, nominal heading function is denoted as $\gamma^{A_i} : [0, t_f] \to [0, 2\pi)$. Then, the initial position $x_0^{A_i}$ and matrix $Q^{A_i}(t)$ in equation (1) are given by $x_0^{A_i} = O_0^{(i)}$ and $Q^{A_i}(t) = R(\gamma^{A_i}(t))$, where $R(\gamma)$ is the rotation matrix associated with $\gamma \in [0, 2\pi)$,i.e.,

$$R(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{bmatrix}.$$

We shall start from the case when aircraft A_i is following a one-leg nominal trajectory and then extending the approach to the multi-legged case. The approximation scheme in the one-leg case is based on the approach in [32] for estimating the probability of conflict. For ease of notation, we shall refer to aircraft A_i as aircraft A, omitting the subscript i.

One-leg nominal trajectory Consider aircraft A flying with constant velocity $u^A \in \mathbb{R}^2$ and heading $\gamma^A \in [0, 2\pi)$ starting from $x_0^A \in \mathcal{S}$. We address the problem of evaluating the probability $\pi(x, t)$ that aircraft A enters a circle of radius ρ centered at $x \in \mathcal{S}$ within the time frame $[t, t+\Delta]$. By (1), the relative position of aircraft A with respect to x is governed by:

$$\Delta x(t) = \Delta x_0 + \Delta u t - n(t), \tag{6}$$

where we set $\Delta x(t) := x - x^A(t)$, $\Delta x_0 := x - x_0^A$, $\Delta u := -u^A$, and $n(t) := R(\gamma^A)\Sigma^A W^A(t)$ (recall that $Q^A(t) = R(\gamma^A)$).

Process $n(t)$ can be reduced to the standard 2-D BM $W^A(t)$ by using the coordinate transformation with matrix $T = (\Sigma^A)^{-1} R(\gamma^A)^{-1}$:

$$\Delta s(t) = \Delta s_0 + u\, t - W^A(t),$$

where $\Delta s(t) := T\Delta x(t)$ is the relative position of the aircraft in the new coordinates, $\Delta s_0 := T\Delta x_0$ and $u := T\Delta u$. In the new coordinate system, the circular zone of radius ρ centered at x is transformed into an ellipse with boundary described by:

$$(\nu_a^A)^2(x_1 - \alpha_1(t))^2 + (\nu_c^{A_i})^2(x_2 - \alpha_2(t))^2 = \rho^2, \tag{7}$$

whose center $\alpha(t) = (\alpha_1(t),\, \alpha_2(t))$ moves according to $\alpha(t) = \Delta s_0 + u\,t$ (see Figure 1).

Aircraft A then gets within a distance ρ from x within $[t, t+\Delta]$ if the 2-D standard BM $W^A(t)$ starting at the origin wanders into this moving ellipse within $[t, t+\Delta]$. Denote this event by F_t. Then, $\pi(x, t)$ is the probability of F_t.

Let x_d be the distance of the origin from the line h along which the center of the ellipse is moving, and a be the distance from the position Δs_0 of the center at $t = 0$ to the projection of the origin on h, as indicated in Figure 1 representing the new coordinate system. Then, x_d and a can be computed as follows:

$$x_d = \frac{|\Delta s_0^T R(\frac{\pi}{2}) u|}{\|u\|}, \qquad\qquad a = -\frac{\Delta s_0^T u}{\|u\|}. \tag{8}$$

Observe that a positive value for a indicates that aircraft A is approaching x, whereas a negative value for a indicates that it is flying away from x.

The probability $P(F_t)$ of F_t does not admit a closed-form formula. However, we can approximate it by a "decoupled" event. Let k be the line passing through the center of the ellipse and orthogonal to u which moves along with the ellipse at the velocity u (see Figure 1). The projected width $2L$ of the ellipse along line k can be computed as follows:

$$L = \frac{\rho}{\nu_a^A \nu_c^A} \frac{\sqrt{u_1^2(\nu_a^A)^2 + u_2^2(\nu_c^A)^2}}{\|u\|}. \tag{9}$$

Denote by τ the first time $W^A(t)$ hits k and define F_t' to be the event that the first time $W^A(t)$ hits line k during the time horizon $[t, t+\Delta]$, it is within a distance of L from the center of the ellipse.

We consider $P(F_t')$ as an estimate of $P(F_t)$. This approximation is actually fairly accurate if the aircraft velocity is much larger than the variance growth rate of the BM. The intuition for this is that when the velocity of the moving ellipse is high, the event F_t is largely determined by the width of ellipse viewed in the direction of u.

Without loss of generality, to compute $P(F_t')$ we assume that u is aligned with the positive x_1 axis. Indeed, the axes rotation eventually necessary to make u aligned with the positive x_1 axis can be incorporated into the transformation matrix T, and still $W^A(t)$ remains a standard BM, since BM is invariant with respect to rotations.

Ignore the noise temporarily. Then, in the new coordinate system, a positive value for a in equation (8) indicates that aircraft A is approaching x and that the minimal distance from x

is given by x_d in (8). Moreover, time τ for the BM $W^A(t)$ to first hit line k has evidently the distribution $p_\tau(\cdot)$ given by the following Lemma 1 with $\mu = \|u\|$.

Lemma 1 (Bachelier-Levy, [13]) *Let $b(t)$ be a standard one dimensional BM starting at the origin. Fix $\mu \in \mathbb{R}$ and define $\tau := \inf\{t \geq 0 : b(t) = a - \mu t\}$ to be the first time $b(t)$ reaches a point which is moving with speed μ towards the origin starting at position $a > 0$. Then, τ has probability density function:*

$$p_\tau(t) = \frac{a}{\sqrt{2\pi t^3}} \exp[-\frac{(a-\mu t)^2}{2t}], \quad t \geq 0.$$

The approximate probability $P(F'_t)$ can then be written as:

$$\begin{aligned} P(F'_t) &= \int_t^{t+\Delta} p_\tau(t) \int_{|y-x_d|<L} \frac{1}{2\pi t} \exp(-\frac{y^2}{2t})\, dy\, dt \\ &= \int_t^{t+\Delta} p_\tau(t) \big[Q(\frac{x_d - L}{\sqrt{t}}) - Q(\frac{x_d + L}{\sqrt{t}})\big]\, dt, \end{aligned} \tag{10}$$

where we set $Q(y) := \int_y^\infty \frac{1}{\sqrt{2\pi}} \exp(-z^2/2)\, dz$.
It can be shown that $E[\tau] = a/\|u\|$ and $var[\tau] = a/\|u\|^3$. If we use a 0-th order expansions of $Q(\frac{x_d-L}{\sqrt{t}})$ and $Q(\frac{x_d+L}{\sqrt{t}})$ in (10), we get the following result.

Proposition 1 *Suppose that aircraft A is approaching position $x \in \mathcal{S}$ ($a > 0$). Then, the probability $\pi(x,t)$ that aircraft A enters the ball of radius ρ centered at x within the time frame $[t, t+\Delta]$ can be approximated by:*

$$\hat{\pi}(x,t) := \big(V(t+\Delta) - V(t)\big)\Big(Q\big(\frac{x_d - L}{\sqrt{t_0}}\big) - Q\big(\frac{x_d + L}{\sqrt{t_0}}\big)\Big) \tag{11}$$

where

$$V(s) = Q\big(\frac{a - \mu s}{\sqrt{s}}\big) + e^{2a\mu} Q\big(\frac{a + \mu\, s}{\sqrt{s}}\big),$$

$$t_0 = \begin{cases} \frac{a}{\mu}, & \text{if } \frac{a}{\mu} \in [t, t+\Delta] \\ t + \frac{\Delta}{2}, & \text{otherwise,} \end{cases}$$

with $\mu = \|u\|$. □

For the purpose of complexity computations, we set $\hat{\pi}(x,t) = 0$ when aircraft A is flying away from x ($a < 0$).

Multi-legged nominal trajectory Consider aircraft A flying with piecewise constant velocity $u^A : [0, t_f] \to \mathbb{R}^2$ and heading $\gamma^A : [0, t_f] \to [0, 2\pi)$ starting from $x_0^A \in \mathcal{S}$. The relative position of aircraft A with respect to x evolves according to the equation

$$\Delta x(t) = \Delta x_0 + \int_0^t \Delta u(s)ds - R(\gamma^A(t))\Sigma^A W^A(t), \tag{12}$$

where $\Delta x_0 := x - x_0^A$ is the aircraft relative position at time $t = 0$ and $\Delta u(s) := -u^A(s)$.

At time t the aircraft is tracking some leg h of its nominal trajectory, associated with the deterministic time interval $[t_h, t_{h+1})$, the (constant) heading angle $\gamma_h^A = \gamma^A(t_h)$, and the (constant) relative velocity $\Delta u_h = -u^A(t_h)$. With reference to $[t_h, t_{h+1})$, equation (12) can then be rewritten as follows:

$$\Delta x(t) = \Delta x_{h,0} + \Delta u_h t - n(t), \qquad t \in [t_h, t_{h+1}), \tag{13}$$

where we set $\Delta x_{h,0} := \Delta x_0 + \int_0^{t_h} \Delta u(s)ds$ and $n(t) := R(\gamma_h^A)\Sigma^A W^A(t)$.

Consider first the case when $[t, t + \Delta] \subseteq [t_h, t_{h+1})$.

By (13), it is easily seen that to the purpose of computing $\pi(x, t)$, one can evaluate the probability that the perturbation $n(t)$ enters the ball of radius ρ whose center is moving at constant velocity Δu_h starting from $\Delta x_{h,0}$ at time $t = 0$. Similarly to the one-leg case, by applying the transformation matrix $T_h = (\Sigma^A)^{-1} R(\gamma_h^A)^{-1}$, equation (13) can be rewritten as

$$\Delta s(t) = \Delta s_{h,0} + u_h t - W^A(t), \qquad t \in [t_h, t_{h+1}),$$

where $\Delta s(t) := T_h \Delta x(t)$, $\Delta s_{h,0} := T_h \Delta x_{h,0}$ and $u_h := T_h \Delta u_h$. The problem then becomes that of evaluating the probability that during the time horizon $[t, t + \Delta]$ the standard BM $W^A(t)$ enters the ellipse with boundary given by (7) and center $\alpha(t) = (\alpha_1(t), \alpha_2(t))$ moving according to equation $\alpha(t) = \Delta s_{0,h} + u_h\, t$. An estimate of this probability can then be derived by the same approximation scheme as in the one-leg case.

If $[t, t + \Delta] \subseteq [t_h, t_{h+1})$ is not satisfied, we can partition $[t, t + \Delta]$ into sub-intervals, each one corresponding to a leg of the nominal trajectory. For each element of the partition we can apply the procedure above to determine an estimate of the corresponding probability $P(F_{t,h})$ and then over-approximate $\pi(x, t) = P(F_t)$ as follows:

$$\hat{\pi}(x, t) = \sum_{i=1}^{m_t} \hat{P}(F_{t,h_i}),$$

where $h_1, h_2, \ldots, h_{m_t}$ identify the legs of the trajectory of aircraft A with a nonempty intersection with the time interval $[t, t+\Delta]$.

Remark 2 *The above procedure to estimate $\pi(x,t)$ can be easily adapted to compute an estimate of $\pi(\bar{x}^B(t), t)$. One has just to consider the relative position of aircraft A with respect to the nominal position $\bar{x}^B(t)$ of aircraft B instead of a fixed point x. Suppose, for example, that we are in the one-leg case with aircraft B staring at x and flying at constant velocity u^B, then, this will lead to an equation of the same form of (6), the only different being that $\Delta u = u^B - u^A$.*

4.4 Computational complexity and scalability

As already mentioned, if the errors affecting the prediction of the position of different aircraft are independent, each single aircraft contribution to complexity can be computed in isolation and then incorporated in the overall complexity measure. This means that the computational effort scales linearly with the number N of aircraft and does not radically increases when an additional aircraft is introduced. When coming to the construction of the complexity maps, however, one has to evaluate the complexity metrics $\mathrm{c}_1(x,t)$ and $\mathrm{c}_2(x,t)$ across the airspace $\mathcal{S}$. If a spatial uniform gridding of step size δ along all axes is adopted, then the number of grid points will scale as δ^{-3} in the 3D case. One possible way to alleviate the exponential growth of computation time as the grid size decreases would be to use a variable sized grid. A coarser grid could be used to evaluate the complexity in regions sufficiently far from the nominal trajectories of the aircraft that do not require a significant accuracy, while a finer grid could be used in regions requiring higher accuracy.

5 Numerical example

Consider a rectangular airspace region $\mathcal{S}$ where 6 aircraft are flying at the same altitude following a one-leg nominal trajectory from some starting to some destination position during the look-ahead time horizon $T = [0, t_f]$ with $t_f = 15$ minutes (min), while trying to keep at a minimum safe distance of 3 nautical miles (nmi). The configuration of the aircraft nominal trajectories is shown in Figure 2, where starting positions are marked with $*$ and destination positions with $\diamond$.

The trajectories in this figure are obtained by implementing the decentralized resolution strategy introduced in [32], which accounts for the uncertainty affecting the aircraft motion according to a similar model for the aircraft predicted motion. According to this strategy, resolution maneuvers involve only heading changes.

The global complexity index corresponding to $p_T = 0.2$ for the considered air traffic system is $\xi \simeq 3$, which means that aircraft are only guaranteed to keep at a distance of about 0.33 nmi, with probability greater than 0.8.

Figure 3 represents the spatial complexity map $\Xi_2 : \mathcal{S} \to [0,1]$ obtained by condensing the timing information as follows:

$$\Xi_2(x) = \frac{1}{t_f}\int_0^{t_f} \mathrm{c}_2(x,t)dt, \tag{14}$$

where the second order global complexity $\mathrm{c}_2(x,t)$ is computed by (11) with $\rho = 3$.

This map reveals that there are two main regions with some significant percentage of occupancy (larger than 10%): one in the upper left-hand-side, and the other close to the center of the airspace area $\mathcal{S}$.

$\Xi_2(x) = 0$ means that there will be at most a single aircraft within the ball of radius 3 nmi centered at x during the whole interval T. Aircraft passing through x such that $\Xi_2(x) > 0$ will be possibly involved in a conflict and the likelihood of this event grows with $\Xi_2(x)$. If $\Xi_2(x) = 1$, in particular, there will be more than 2 aircraft within the ball of radius 3 nmi centered at x during the whole interval T.

The earliest conflict time for both the two aircraft in the upper left-hand-side of the airspace area $\mathcal{S}$ is $t_B^\star = 2$ min. Indeed, the snapshot of the resolution maneuvers taken at time $t = 2$ min shows that this is the earliest time that a significant deviation action is taken by the decentralized solver and that it involves the two aircraft in the upper left-hand-side (Figure 4).

In this example, the complexity map Ξ_2 has been evaluated at uniformly sampled grid points $x \in \mathcal{S} = [0,120] \times [0,120]$ with a grid size $\delta_{x_1} = \delta_{x_2} = 1$. Adopting a variable grid resolution, with a larger grid size far from the aircraft and a finer one close to the aircraft, would reduce the computational load.

In the numerical evaluation of the integral over $[0,t_f]$ involved in (14), $[0,t_f]$ has been uniformly sampled with $\delta_t = 1$. The short term look-ahead time horizon Δ has been set equal to 2 min, and $\nu_a^{A_i} = 0.25$ and $\nu_c^{A_i} = 0.2$ for all aircraft, with the power spectral densities $(\nu_a^{A_i})^2$ and $(\nu_c^{A_i})^2$ measured in nmi^2/min.

6 Conclusions

New generation ATM systems will have a decentralized and distributed control structure, with separation and management tasks shared between the ground and the flight-deck. This poses new and formidable challenges in the ATM system design. This work has addressed in particular the problem of assessing air traffic complexity in an autonomous aircraft context. Prospective applications have been described, such as onboard trajectory management and conflict detection and resolution. The corresponding requirements on complexity metrics have been discussed.

A method to evaluate air traffic complexity from a global perspective and from the perspective of a single aircraft has been described. A key feature of the method is that it uses a stochastic model to account for possible deviations of the aircraft from their nominal trajectories. Possible applications have been illustrated through a simple numerical example, which include using complexity maps and measures for detecting congested airspace areas and critical situations from the conflict detection and resolution perspective. Examples of applications to trajectory design are reported in [34].

Acknowledgment

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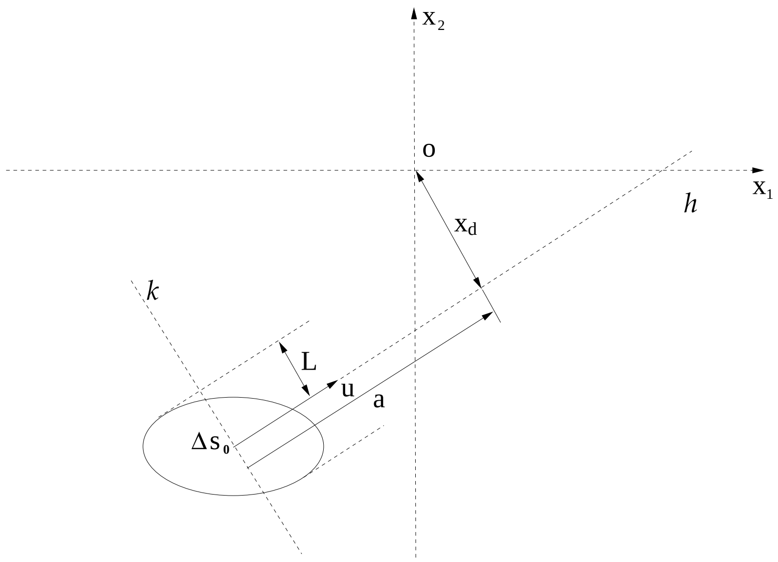

Figure 1: Transformed protection zone.

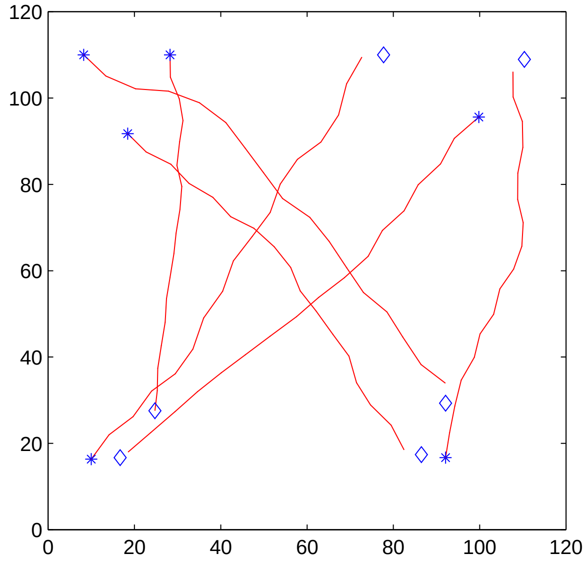

Figure 2: Sample paths of 6 aircraft moving from starting position ($*$) to destination position ($\diamond$), while trying to keep at a distance 3 nmi.

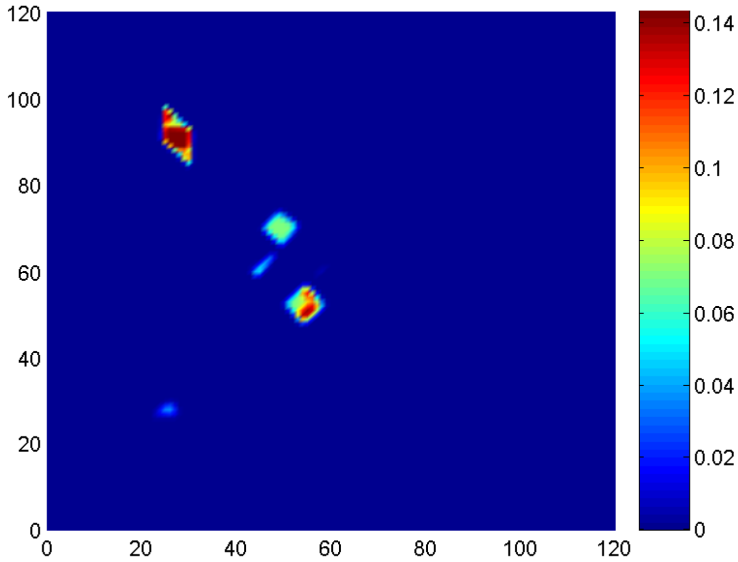

Figure 3: Complexity map $\Xi_2 : \mathcal{S} \to [0, 1]$ obtained for $\rho = 3$ nmi.

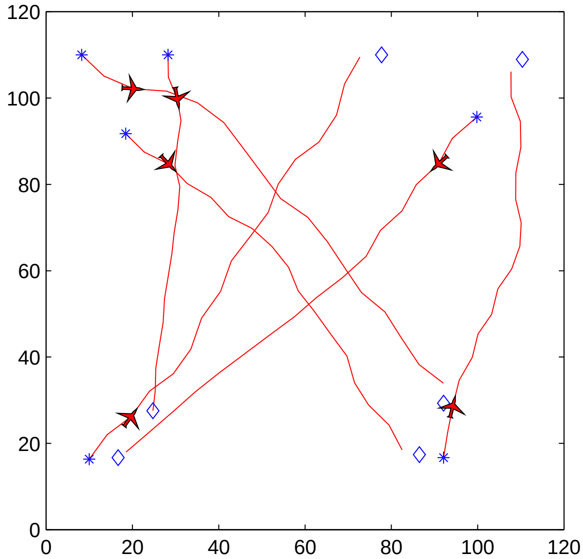

Figure 4: Snapshot of the resolution maneuvers for the 6 aircraft system in Figure 2 at time $t = 2$ min.

LIST OF CAPTIONS:

Figure 1: Transformed protection zone.

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