Approaches for Stochastic Safety Analysis Arising in ATM Application

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Abstract— **This paper studies the stochastic safety analysis and resolution problems for Air Traffic Management (ATM) systems. Two approaches to estimating the aircraft safety are introduced: a Multi- level Markov Chain (MMC) approximation method for probabilistic prediction of the aircraft conflict, and an Air Traffic Density Estimation (ATDE) approach, which is a simulation-based method for predicting congestions in the airspace within a period of time. Using the estimated aircraft safety, we formulate an optimal trajectory planning problem and develop an efficient solution algorithm for resolving the conflicts. Numerical examples arising in ATM systems are presented to evaluate and compare the two conflict detection approaches along with the developed conflict resolution algorithm.**

I. INTRODUCTION

Due to the rapidly increasing air traffic, guaranteeing the safety of air travel has become an important problem. Thus, the safety analysis of air traffic, in particular conflict detection and resolution (CDR), has received much attention by researchers. The challenging is to meet the safety requirements of the Air Traffic Management (ATM) systems. For instance, the aircraft must keep a minimum distance (safety zone) from other aircraft and from restricted or congested areas, such as severe weather zones or Special Use Airspaces (SUA), to avoid conflicts. To achieve this goal, two tasks are typically performed: conflict detection which is to detect the possible occurrence of conflicts, and conflict resolution which is to replan the trajectories of aircraft if necessary to avoid the conflicts [1], [2].

This paper introduces two approaches for safety analysis in ATM application. The first approach is the Multi-level Markov Chain approximation which first introduced in [3] as Two-level Markov Chain approximation for simplicity. This approach was inspired by the Multi-Level (ML) methods for solving PDEs. As shown in [4], [5], it can achieve a good compromise between computation time and solution accuracy compared to the traditional one-level method. One drawback of this approach is that it can only handle few number of aircraft. For multi-aircraft encounters, the computation time would become prohibitive. To this purpose, we introduce the second method, the Air Traffic Density Estimation (ATDE) method, which can be used to estimate a high-level metric of the congestion in the airspace. Dynamic density or traffic density has been previously introduced in the literature for general transportation systems, for example, freeway traffic density in [6], traffic networks in [7], and many other applications. The concept of traffic density adopted in this paper was first introduced in [8]. We develop a simulation-based method that can evaluate traffic density for encounters involving many aircraft. Another approach to multi-aircraft trajectory prediction can be found in [9].

The conflict resolution problem studies the necessary actions it takes to avoid a conflict predicted within a future time horizon. Its main goal is to ensure safety and smooth travel for the aircraft while reducing delay and fuel consumption. Many approaches to conflict resolution have been proposed by researchers, for example, using a tool table or manual [10], probabilistic estimation by analytical methods or using Monte Carlo simulation [11], and trajectory planning by solving an optimal control problem [12]. In this paper, an optimal trajectory planning method based on our results on conflict detection has been developed.

This paper is organized as follows. Section II introduces the dynamic model of the aircraft motion as a stochastic differential equation. Problem formulation of this work in presented in Section III. Two approaches for conflict detection are developed in Section IV. Section V presents the optimal conflict/congestion resolution method. Numerical examples are provided in Section VI. Finally, conclusion is drawn in Section VII.

II. AIRCRAFT DYNAMICS MODEL

Modeling the aircraft motions is an important problem in the study of ATM systems. According to ATM practice, the nominal aircraft motion is a piecewise constant speed (air speed, to be precise) linear motion specified through a sequence of *way points*. For a realistic model, the wind factor needs to be taken into consideration, which is one of the main contributors to uncertainty in the aircraft future positions. The wind factor, or more precisely, the wind speed, is composed of two terms: (i) a deterministic nominal wind speed term f, also called the wind field, that is known to the ATC through measurements or forecast, and (ii) a stochastic term representing the effect of air turbulence and/or errors in the wind speed measurements and forecast [13].

Let $X(t) \in \mathbb{R}^3$, $t \in T$, be the position of the aircraft during the look ahead time horizon T, and let the wind field f be uniform at any given time $t \in T$ in the region of interest in the airspace. For simplicity, we can absorb the deterministic wind field term into the aircraft nominal air velocity $u(t)$. Then the aircraft dynamics model is given by the following stochastic differential equation:

$$dX(t) = u(t)dt + \Sigma(X,t)dB(t). \quad (1)$$

Here, the second term on the right-side is the stochastic term modeling all the uncertainty, with $B(t)$ being a standard

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three-dimensional Brownian motion whose variance is modulated by $\Sigma : \mathbb{R}^3 \times T \rightarrow \mathbb{R}^{3\times 3}$ that in general depends on both the spatial locations and times. For ease of analysis, we assume that $\Sigma \equiv \text{diag}\,(\sigma_h, \sigma_h, \sigma_v)$ is a constant diagonal matrix for some given parameters σ_h and σ_v in this paper.

The simplified aircraft dynamic model will be used throughout the rest of this paper. However, our analysis can be easily generalized to more complicated aircraft dynamics (e.g. general spatially correlated wind field).

III. PROBLEM FORMULATION

The conflict prediction problem studies an aircraft flying near a forbidden region of the airspace (aircraft-to-airspace conflict problem) or trying to keep a minimum separation from other aircraft by a horizontal distance r and a vertical distance H (aircraft-to-aircraft conflict problem). Based on the minimum operational performance standards for the air traffic alert, the minimum horizontal separation is 5 nautical miles (nmi), whereas within TRACON area it is reduced to 3 nmi, and the minimum vertical separation is 2000 ft or 1000 ft if the aircraft flies at an altitude above 29,000 ft or below 29,000 ft, respectively ([14], [15]).

To study the aircraft-to-airspace conflict problem, consider an aircraft flying in an open bounded region U with the assigned nominal velocity $u(t),\ t \in T = [0, t_f]$, where 0 represents the current time instant and t_f represents the look-ahead time horizon. Then, the evolution of $X(t)$ over the time interval is given by equation (1). Let a compact set $D \subset U$ be a restricted region to avoid. For example, D can have the shape of a closed cylinder of radius r and height *2H* centered at the origin, denoted by C. To evaluate the safety of the aircraft path, we need to compute the probability of the aircraft's position entering the region D, namely, detecting the likelihood of the occurrence of conflict. Therefore, the conflict event is defined as follows:

$$\{X(t) \in D \text{ for } t \in T\}\,, \tag{2}$$

and the probability of conflict is formulated as follows:

$$P_c \triangleq P\{X(t) \in D \text{ for } t \in T\}\,. \tag{3}$$

The study of the aircraft-to-aircraft case can be considered as a special case of the aircraft-to-airspace conflict problem, though in a higher dimensional space. Let $X_1(t)$ and $X_2(t)$ be the positions of aircraft 1 and aircraft 2, respectively, where $X_1(t),\ X_2(t) \in \mathbb{R}^3$. According to equation (1), the dynamics model of both aircrafts over the time interval T are given by the following stochastic differential equations:

$$dX_1(t) = u_1(t)dt + \Sigma_1\ dB_1(t),$$
$$dX_2(t) = u_2(t)dt + \Sigma_2\ dB_2(t),$$

with initial condition $X_1(0)$ and $X_2(0)$, respectively.
Next, define the following:

$$Y(t) = \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix}, \qquad v(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix},$$
$$\hat{\Sigma} = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix}, \qquad \hat{B}(t) = \begin{bmatrix} B_1(t) \\ B_2(t) \end{bmatrix},$$

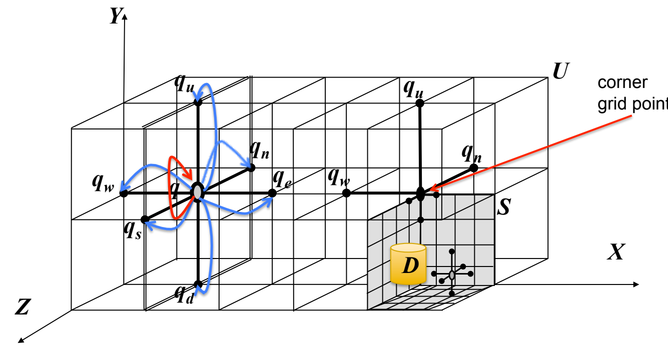

Fig. 1. Grid structure of the two-level Markov Chain Approximation

and

$$D = \left\{Y(t) \in \mathbb{R}^6 : \|X_1 - X_2\| \in C\right\},$$

where C is the closed cylinder defined above. Then, the dynamics of $Y(t)$ is given by:

$$dY(t) = v(t)dt + \hat{\Sigma}\ d\hat{B}(t).$$

Therefore, the aircraft-to-aircraft conflict problem becomes a aircraft-to-airspace conflict problem in $\mathbb{R}^6$. In this paper we focus on the aircraft-to-airspace conflict case as a general conflict problem.

IV. CONFLICT/CONGESTION DETECTION APPROACHES

In ATM literature as well as in this paper, aircraft dynamics are modeled by stochastic differential equations [15], and many solution methods of the stochastic differential equations have been proposed, for example, a statistical moments computation [16], a construction of barrier functions [17], a probabilistic testing method [18], and last but not least an approximated Markov Chain discretization and a simulation-based method which both are studied in this paper.

A. Multi-level Markov Chain Approximation

We first briefly describe the procedure for Markov chain approximation method. First, discretize the interested domain $U \backslash D$ into grid points with step size δ that form the state space of the Markov chain $\{Q_{k\Delta t}, k \geq 0\}$, where Δt represents the time step between two successive jumps. Then, choose transition probabilities carefully so that the solution of the Markov chain will converge weakly to the solution of the stochastic differential equation (1) as the grid size δ approaches zero. Notice that, the smaller the grid size is, the better estimates of P_c can be obtained, however the running time for the algorithm will be longer. Consequently, a new approach using Multi-Level Markov chains has been developed in this paper to achieve a better compromise between the accuracy and computation time of P_c.

For the analysis of the stochastic systems introduced in Section II, we propose a two-level Markov chain consisting of a Fine-Level Markov Chain (FLMC) and a Coarse-Level Markov Chain (CLMC) for simplicity. The FLMC will be used in a smaller subset $S \backslash D$ of the open domain U where we seek accurate estimates of P_c; and the CLMC will be used in the whole domain $U \backslash D$ (see Figure 1). Note that, the CLMC is used over $U \backslash D$ instead just over $U \backslash (D \cup S)$. This will speed up the algorithm since the common grid points of the CLMC and the FLMC will be updated more frequently than other grid points, and thus a faster solution is achieved in the interested region of the airspace.

Each of the CLMC and the FLMC is constructed in a similar way as described in [3] and thus is omitted. One new issue for the two-level method is that, since the CLMC and the FLMC will be defined on two different grid spaces, a question arises on how to merge one to the another along their common boundary. In other words, if some grid point q of either the CLMC or the FLMC lies on the common boundary, then q has at least one neighbor at a distance of δ which is inside $U \backslash D$ and at least one neighbor at a distance of the fine step size $\hat{\delta} < \delta$ inside $S \backslash D$. Therefore, every grid point lying on the boundary has neighbor points in different levels. Figure 1 shows one such corner point.

In the following, we describe in details how to deal with points on the common boundary of the Coarse- and Fine-Level Markov chains.

1) Boundary of Coarse- and Fine-Level Markov Chain Approximation: In this section, we discuss how to merge two different levels of Markov chain. According to our assumption on the location of the subset S, the eight corner grid points of the FL are also the CL grid points and the interior grid points among those points will have three neighbors inside the FL ($\hat{\mathcal{Q}}^o \backslash D$) and three neighbors inside the CL ($\mathcal{Q}^o \backslash D$). Take the example of one of the arbitrary corner state $\hat{q}$ shown in Figure 1, where $\hat{q}$ is the top-west-north corner grid point $\in \hat{\delta}\mathbb{Z}^3$ and has the following six neighbors: $\hat{q}_e = q + (\hat{\delta}, 0, 0), q_w = q + (-\delta, 0, 0), \hat{q}_s = q + (0, -\hat{\delta}, 0), q_n = q + (0, \delta, 0), \hat{q}_d = q + (0, 0, -\kappa\hat{\delta}), q_u = q + (0, 0, \kappa\delta)$, where $\hat{\delta} = s\ \delta$, with $s = 1/m$ for some $m \in \mathbb{Z}$ and $\kappa \triangleq \sigma_v/\sigma_h$. Starting from any interior corner point $\hat{q} \in \partial\hat{\mathcal{Q}}_S$, the Markov chain jumps to one of the six neighbors above or stays at the same state corresponding to the following transition probabilities: $P\{Q_{k+1} = q' | Q_k = q\} =$

$$\begin{cases} \hat{p}_e^{\hat{k}}(q) = s\ \exp(\hat{\delta}\xi_q^{\hat{k}})/C(q), & q' = \hat{q}_e; \\ p_w^k(q) = \exp(-\delta\xi_q^k)/C(q), & q' = q_w; \\ \hat{p}_s^{\hat{k}}(q) = s\ \exp(-\hat{\delta}\eta_q^{\hat{k}})/C(q), & q' = \hat{q}_s; \\ p_n^k(q) = \exp(\delta\eta_q^k)/C(q), & q' = q_n; \\ \hat{p}_d^{\hat{k}}(q) = s\ \exp(-\hat{\delta}\zeta_q^{\hat{k}})/C(q), & q' = \hat{q}_d; \\ p_u^k(q) = \exp(\delta\zeta_q^k)/C(q), & q' = q_u; \\ p_o^k(q) = \chi_{\hat{q}}^k/C(q), & q' = q. \end{cases} \tag{4}$$

The parameters in the above expression are similar to those defined in [3, eq (13)] except for the following: $\widehat{\Delta t} = \lambda\hat{\delta}^2 = \lambda s^2\delta^2$, $\chi_q^k = (s + 1/\lambda\sigma_h^2) - 3 - 3s$, and

$$\begin{aligned} C(q) \quad = \quad & \chi_q^k + \exp(-\delta\xi_q^k) + \exp(\delta\eta_q^k) \\ & + \exp(\delta\zeta_q^k) + s\ \exp(\hat{\delta}\xi_q^{\hat{k}}) \\ & + s\ \exp(-\hat{\delta}\eta_q^{\hat{k}}) + s\ \exp(-\hat{\delta}\zeta_q^{\hat{k}}). \end{aligned}$$

We have $0 < \lambda \leq (3\sigma_h)^{-1}$ to guarantee the positivity of χ_q^k for all k and q.

Finally, the estimated probability of conflict for the corner grid points is defined as follows: $\hat{P}_c^k(q) =$

$$\begin{cases} p_o^k(q)P_c^{k+1}(q) + \hat{p}_e^{\hat{k}}(q)\hat{P}_c^{\hat{k}+1}(\hat{q}_e) \\ + p_w^k(q)P_c^{k+1}(q_w) + \hat{p}_n^{\hat{k}}(q)P_c^{\hat{k}+1}(\hat{q}_n) \\ + p_s^k(q)P_c^{k+1}(q_s) + \hat{p}_u^{\hat{k}}(q)P_c^{\hat{k}+1}(\hat{q}_u) \\ + p_d^k(q)P_c^{k+1}(q_d), & q \in \partial\hat{\mathcal{Q}}_S \\ 1, & q \in \partial\mathcal{Q}_D \\ 0, & q \in \partial\mathcal{Q}_U, \end{cases} \tag{5}$$

where the initial condition is:

$$P_c^{\hat{k}_f}(q) = \begin{cases} 1, & \text{if } q \in \partial\hat{\mathcal{Q}}_D \\ 0, & \text{elsewhere.} \end{cases} \tag{6}$$

For the developed algorithm, the reader is referred to [3].

B. Air Traffic Density Estimation Approach

Traffic density is an effective tool for identifying the safety and the traffic flow efficiency of any transportation system. It is used in this paper to study the flow of multiple aircraft traveling within a given time horizon in a sector of the airspace. Air traffic density estimation not only applies to aircraft-to-aircraft conflict problem, but also to the much more complicated multiple aircraft conflict case.

Air Traffic Density (ATD) can be defined and formulated in many different ways. In this paper, we define it to be the expected number of aircraft occupying a given subregion of the airspace at a given time. In this case ATD can be formulated as follows. Suppose the airspace domain of interest $U \in \mathbb{R}^3$ is divided into subregions $U = \cup U_j$, $j = 1, 2, ..., n$ and consider M en-route aircraft traveling within the time interval $T = [0, t_f]$ in U. Then the ATD can be defined as follows:

$$D(U_j, t) = \frac{1}{M} E\left[N(U_j, t)\right], \tag{7}$$

$$\text{where} \qquad N(U_j, t) = \sum_{m=1}^{M} 1_{X_m(t) \in U_j}.$$

Here, $X_m(t) \in \mathbb{R}^3, m = 1, ..., M$, denote the aircraft positions at time $t \in T$.

In simulating the aircraft trajectory, the aircraft stochastic differential equation (1) can be solved by integration. Therefore, with the given aircraft's initial position and nominal speed along with the perturbation's power spectral, the different realizations of the aircraft trajectory can be simulated.

For demonstration, consider the following example, where six aircraft $X_i(t) \in \mathbb{R}^3$, $t \in T = [0, 30]$, travel at different constant speed with different noise variances σ_h and σ_v. Figure 2 plots the nominal trajectory in a dash line and a simulated trajectory in a solid line for each aircraft traveling from its initial position ($\circ$) to its destination ($\times$).

By repeating this process a large number of times, we can increase the accuracy of the estimates of the air traffic density, which is a functional of the aircraft trajectories, based on the Law of Large Number. Precisely, H experiments are

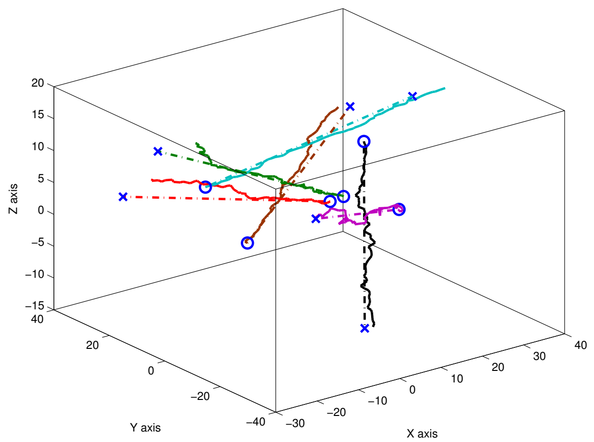

Fig. 2. Nominal trajectories and one possible simulated path of each aircraft

performed for each aircraft and an estimate of the ATD can be given as follows:

$$D(U_j,t) = \frac{1}{MH}\sum_{i=1}^{H} E\left[N^i(U_j,t)\right].$$

With the obtained estimated values of $D(U_j,t)$, consider a threshold value $\nu > 0$. Then, three different zones of the airspace, free zone, less congested zone, and congested zone, can be identified according to the following rules:

$$U_{j,\,t} = \begin{cases} \text{free zone}, & \text{if } D(U_j,t)=0,\\ \text{less congested zone}, & \text{if } 0<D(U_j,t)<\nu,\\ \text{congested zone}, & \text{if } D(U_j,t)\geq \nu. \end{cases}$$

In this way, ATD estimation enables us to specify varying level of danger at future times, so that the ATC and the pilot can take appropriate actions and immediate resolutions for more imminent threats.

V. CONFLICT RESOLUTION

To guarantee safety and smoothness of travel for the aircraft, the predicted conflicts need to be avoided from occurring by re-routing the involved aircraft trajectories away from the congested zones, while at the same time minimizing the deviations from the original assigned trajectories in order to meet the scheduled arrival time and reduce fuel consumption. Accordingly, we formulate the conflict resolution problem as an optimization problem, where the cost function to be minimized is a weighted sum of the congestion along the trajectory and the deviations. We iteratively update a sequence of way points parameterizing the aircraft trajectories, until some stopping criteria are met. For other existing papers on ATM systems that also formulate the conflict resolution as an optimization problem, see e.g. [19].

Let $X_i(t) = \left[x_i(t) y_i(t) z_i(t)\right]^T \in \mathbb{R}^3$ be the position of aircraft i at time $t = t_1, t_2, ..., t_K$. Then, the trajectory planning problem for aircraft i is to minimize the following cost function:

$$\begin{aligned} V(X_i,t) &= \sum_{k=1}^{K-1} \|X_i(t_k)-X_i(t_{k+1})\|^2 \\ &+ \lambda \sum_{k=1}^{K-1} \oint_{X_i(t_k)\to X_i(t_{k+1})} F(X_i,t)dt. \end{aligned}$$

In the above expression of the cost function, minimizing the first summation tends to result in a smooth trajectory with small deviation from a straight line motion. The second term characterizes the safety of flying along the different segments of the trajectory. The function $F(X_i,t)$ is the conflict function and could be chosen to be either the conflict probability P_c or the ATD $D(U_j,t)$. The parameter λ here is a weight parameter. A larger value of λ means drawing more focus on ensuring safety and less attention on smoothness of travel.

To solve the above optimization problem, we focus on three consecutive way points, $X_i(t_{k-1})$, $X_i(t_k)$, and $X_i(t_{k+1})$. Assume $X_i(t_{k-1})$ and $X_i(t_{k+1})$ are fixed and $X_i(t_k)$ could be freely allocated. This results in the following sub-problem:

$$\begin{aligned} &\min_{X_i(t_k)} \{V(X_i(t_k),\lambda)\} = \\ &= \min_{X_i(t_k)} \Big\{ \|X_i(t_{k-1})-X_i(t_k)\|^2 \\ &+ \|X_i(t_k)-X_i(t_{k+1})\|^2 \\ &+ \lambda \Big[\oint_{X_i(t_{k-1})\to X_i(t_k)} F(X_i,t)dt \\ &+ \oint_{X_i(t_k)\to X_i(t_{k+1})} F(X_i,t)dt \Big] \Big\}. \end{aligned} \tag{8}$$

We can define similar sub-problems with respect to other way points. An iterative solution to the original optimization problem can be obtained by repetitively solving the sub-problem for way points $X_i(t_k)$ cyclically.

VI. CDR NUMERICAL EXAMPLES

In this section we study two conflict problems: (1) aircraft-to-airspace conflict problem, which will be solved by approach A and (2) multiple aircraft conflict problem, which will be solved by approach B.

A. Example 1

Consider an aircraft flying in the open domain $U = (-40,5)\times(-20,7)\times(-5,5)$ of the airspace during the time interval $T=[0,30]$ and starting at $(-30,-10,-4)$ with a nominal velocity as follows:

$$u(t) = \begin{cases} (2,2,0), & 0\leq t<10;\\ (2,-0.5,0), & 10\leq t<20;\\ (1,-1.5,0), & 20\leq t\leq 30. \end{cases}$$

Let the forbidden region be a closed cylinder D of radius $r=3$ and hight $H=2$ centered at the zero in the opened domain U, and let the fine subset S to be $S=(-12,5)\times(-6,7)\times(-3,5)$. Set the coarse step size to be $\delta=1$, the fine step size $\hat{\delta} = 0.5$, and $\sigma_h = \sigma_v = 1$. Hence, we have $\lambda = 1/3$, $\Delta t = 1/3$, and $\Delta \hat{t} = 1/12$.

Next, assume another intruding aircraft that passes through the airspace domain U with a constant speed starting from $(-40, -15, 0)$ and ending at the destination position $(5, 5, 0)$ within the same time period $T = [0, 30]$.

1) Approach A: : We first use the algorithm of two-level Markov Chain in [3] to compute the probability of conflict, then the optimal resolution to replan the intruding aircraft. In Figure 3, the top row shows the top view of the three dimensional plot of $\hat{P}_c$ with isosurface of value 0.1; and the lower is the full view of the same isosurface. Figure 4 shows the computed P_c contours varying over time T and the intruding aircraft's nominal trajectory, where the $\star$ represents the intruding aircraft's position. We can see that at time $t = 20$ min, the intruding aircraft is approaching a high value of conflict so an alert to the pilots is sufficient. However, at times 25 and till before its destination at time 30 min the intruding aircraft would be in a conflict with the existing aircraft, thus resolution maneuvers need to be designed.

Next is to perform the resolution maneuvering by applying the optimization trajectory planning algorithm above, we search for sequence of way points using equation (8) with $F(X_{A_i}, t)$ being the probability of conflict $P_c(q, t)$ and consider the two end points to be $X(t = 0) = (-40, -15, 0)$ and $X(t = 30) = (5, 5, 0)$. Then, we solve the optimization problem with different values of the weight parameter λ: λ=10, 15, and 25, to study its effects on the solution. Figure 5 shows the optimal trajectories for the different λ. Note that, the bigger λ used, the safer but less smooth resolution trajectory is obtained.

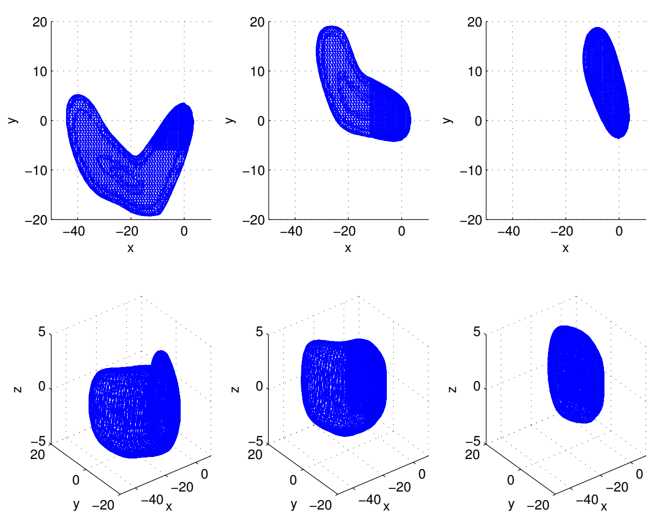

Fig. 3. Probability of conflict $\hat{P}_c$ over time horizon $T = [t, 30]$, where $t = 0, 10$, and 20 from left to right.

B. Example 2

Consider six aircrafts $X_i(t)$, $i = 1, 2, ...6$, traveling within the time period $T = [0, t_f]$, $t_f = 30$ min, in an open bounded domain of the airspace $U = (-30, 40, -10) \times (-40, 60, 10)$. Each aircraft has a constant nominal velocity from its starting

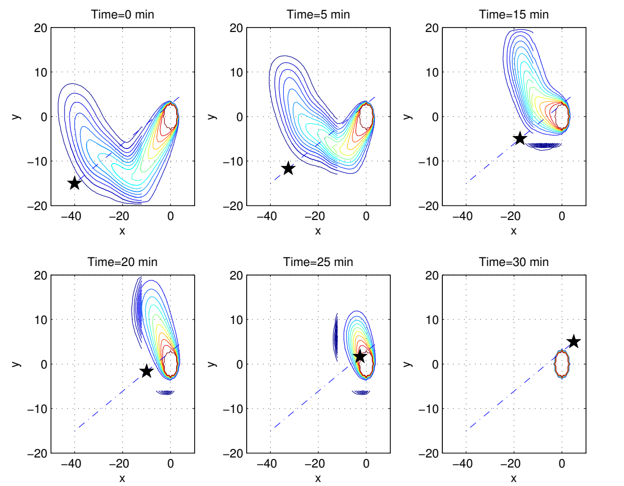

Fig. 4. Probability of conflict along with intruded aircraft within time period T=[0,30]

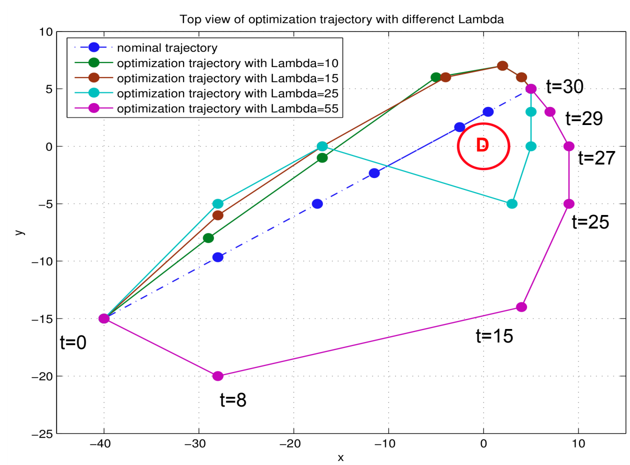

Fig. 5. Optimization trajectory replanning

position to its destination, but with different noise variance ($\sigma_h \& \sigma_v$). Let the minimum horizontal and vertical separation between aircraft to be $r = 5$ nmi and $h = 2$ nmi, respectively, and the number of simulation trials to be $H = 10$.

Next, assume another intruding aircraft that passes through the airspace domain U with a constant speed starting from $(-20, -20, 0)$ and ending at the destination position $(40, 60, 0)$ within the same time period $T = [0, 30]$.

1) Approach B: :

Figure 6 shows at zero altitude the ATD map overlapped with the intruding aircraft's nominal trajectory at various future times, where the $\star$ represents the intruding aircraft's position $X_{A_7}(t)$. Note that, at time $t = 7.5$ min, the intruding aircraft is approaching a congested zone so an alert to the pilots to be caution is sufficient. However, at times 15, 18.5, and 22.5 min, the intruding aircraft would be inside congested zones.

To avoid these potential conflicts, we apply the optimal trajectory planning algorithm above. Here, $F(X_{A_i}, t)$ is the estimated ATD $D(U_j, t)$, and the two end points are $X_{A_7}(t = 0) = (-20, -20, 0)$ and $X_{A_7}(t = 30) = (40, 60, 0)$. Figure 7 shows the optimal trajectory for λ:

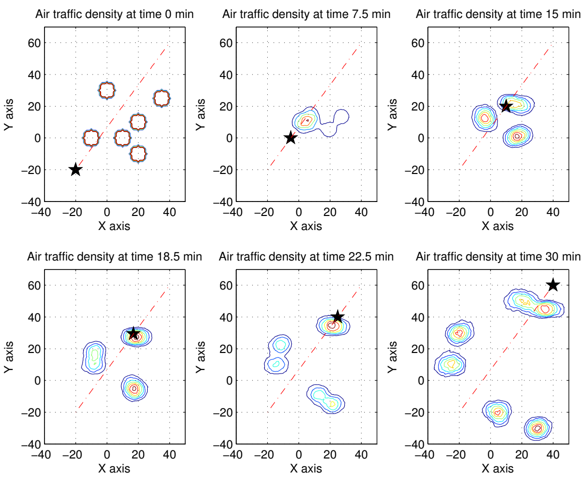

Fig. 6. Air traffic density varying with time period T

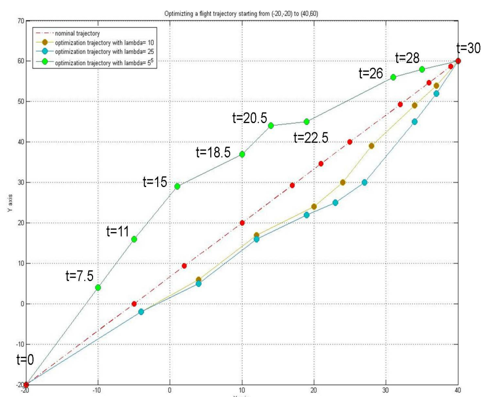

Fig. 7. Optimization trajectory with different Lambdas

λ=10, 25, and 5^5. Again, the bigger λ used, the safer but less smooth the optimal resolution trajectory.

TABLE I
COMPARISON TABLE BETWEEN TWO-LEVEL MARKOV CHAINS & AIR TRAFFIC DENSITY ESTIMATION APPROACH.

Features	**Two-level MC approach**	**ATD estimation approach**
Conflict problems aircraft-to-airspace aircraft-to-aircraft multiple aircraft	 yes yes no	 yes yes yes
Dimension	2-D and 3-D	2-D and 3-D
Computation time example 1 example 2	 7.2 min (3-D) n/a	 19.6 min (2-D), 16 hr (3-D) 22 min (2-D), 20 hr (3-D)

VII. CONCLUSIONS

In this paper we introduced and compared two approaches of stochastic safety analysis; the Multi-level Markov Chains approximation and the Air Traffic Density estimation approaches. Both approaches were employed to solve the simplified stochastic model of aircraft dynamics and detect conflicts in future time. Both approaches can be used in the conflict resolution problem, where the aircraft trajectories are re-routed by solving an optimization problem. A comparison table was provide for the two approaches. Numerical experiments showed that the proposed CDR algorithm is effective in resolving conflicts involving multiple aircraft.

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