An Approach to Air Traffic Density Estimation and Its Application in Aircraft Trajectory Planning

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Abstract: In this paper air traffic density is introduced as a new method for predicting and resolving aircraft conflicts. Based on a simplified stochastic differential equation model of aircraft dynamics, a simulation-based method is employed to predict the likelihood of aircraft presence in different part of the airspace. Air traffic density generated thus indicates the congestion zones to be avoided. The conflict resolution is then performed by solving an optimization trajectory planning problem. Numerical examples are provided to illustrate the efficacy of the proposed algorithms.

Key Words: Air Traffic Management, Conflict Detection and Resolution (CDR), Air Traffic Density, Trajectory Planning.

1 INTRODUCTION

Conflict detection and resolution (CDR) is a challenging task not only in Air Traffic Management (ATM) systems, but also in road transportation systems and robotics. Many different methods have been developed in the literature for CDR. The main challenges here is to maintain safety at all time. For instance, aircraft need to keep a minimum separation (safety zone) from one another and from obstacles to avoid conflicts. Due to the rapidly increase in air traffic, guaranteeing the safety of air travel has become even more important and challenging. The ATM systems typically consists of Air Traffic Control (ATC) and Traffic Flow Management (TFM). The ATC task is to meet the minimum separation requirement between aircraft, while TFM is responsible for ensuring smooth and efficient flow by organizing aircraft into flow patterns. To ensure safety and smooth flight, the airspace is divided into sectors and each sector has one to three controllers to keep the required minimum separation between all the aircraft within the same sector. Those sectors are well divided for the workload of the air traffic controllers. In case of over demand, such as changing routing or destination requested, the TFM function is used to maintain the flow efficiency. Therefore, TFM system has a valid input into the workload of the air traffic controllers [1].

Measuring sector complexity and controller workload instead of only studying the number of aircraft sharing the same airspace is a new approach for ensuring aircraft safety and efficient journey. This approach is called the *dynamic density*. Dynamic density or traffic density has been previously introduced in the literature for general transportation systems, for example, a freeway traffic tensity in [2], traffic networks in [3], and many other applications. In this paper we introduce air traffic density as an approach to study the air traffic flow and predict its behavior in future time horizons. However, the air traffic density approach in this paper differs from [1] in the following aspects: (1) our definition of air traffic density is relative to the aircraft's existence domain in the airspace; (2) we considered traffic density as a time-varying high-level view of travel; and (3) it is estimated by a simulation-based method.

The CDR is one of the essential problems to be considered when studying the ATM systems. Therefore, air traffic density generated here is applied into the CDR problems to demonstrate how efficient our approach is. The conflict prediction problem studies an aircraft flying near a forbidden region of the airspace (aircraft-to-airspace problem) or trying to keep a minimum separation from other aircraft by a horizontal distance r_h and a vertical distance H (aircraft-to-aircraft problem). Based on the minimum operational performance standard, the minimum horizontal separation is 5 nautical miles (nmi), whereas within TRACON area it is reduced to 3 nmi; and the minimum vertical separation is 2000 ft or 1000 ft depending on if the aircraft flies at an altitude above or below 29,000 ft [4]. The conflict detection analysis in this work differs from our previous work in [5] in that a simulation based method is used instead of a probabilistic approach and different degree of conflict is predicted. This enables us to study the conflict detection problem involving multiple aircraft instead of only two aircraft.

The conflict resolution problem studies the necessary actions it takes to avoid a conflict predicted to occur in a future time horizon. Predicted conflicts may have different urgency. For example; a conflict detected in the distant future is of low urgency and a simple alert to the pilots may suffice instead of mandatory corrective actions. On the other hand, an imminent conflict requires immediate resolution by the ATC and the pilots. The conflict resolution has been a challenging task in the ATM systems, with its main goal of ensuring safety and smooth travel for the aircraft while reducing delay and fuel consumption. Many approaches to conflict resolution have been proposed by researchers, for example, using a tool table or manual [6, 7], probabilistic estimation by analytical methods or using Monte Carlo simulation [8, 9], and trajectory planning by solving an optimal control problem [10, 11]. In this paper, the optimization trajectory planning method based on our results on conflict detection will be adopted.

The organization of this paper is as follows. Section 2 presents the aircraft dynamic model as a stochastic differential equation. Air traffic density approach using a simulation based method is introduced in Section 3. In Section 4, we discuss its application in the congestion/conflict detection and resolution. Numerical examples are presented in Section 5. Finally, Section 6 concludes our work.

2 Aircraft Dynamic Model

This paper adopts an approximated stochastic differential equation model of aircraft dynamics for predicting the aircraft's future positions during a given time horizon T. For more information on stochastic approximation, the reader can refer to [12,13]. According to ATM practice, the nominal aircraft motion is a piecewise constant speed (air speed, to be precise) linear motion specified through a sequence of *way points*. For a realistic model, the wind factor needs to be taken into consideration, which is one of the main contributors to uncertainty in the aircraft future positions. Thus, modeling the aircraft motion has been a challenging and interesting problem in the ATM systems. The wind factor, namely the wind contributed velocity called the *wind speed*, composes of two terms: (i) a deterministic nominal wind speed term that is known to the ATC through measurements or forecast, and (ii) a stochastic term representing the effect of air turbulence and errors in the wind speed measurements and forecast [14].

From the above discussions, we can formulate the approximated stochastic model of the aircraft dynamics. Consider an open bounded domain of the airspace $U \in \mathbb{R}^2$ and an aircraft A traveling during the time interval $T = [0, t_f]$. Let $X_A(t) \in \mathbb{R}^2$, $t \in T$, be the aircraft position. Then the aircraft dynamics model is given by the following stochastic differential equation:

$$dX(t) = u(t)dt + f(x,t)dt + \Sigma(x,t)dB_t, \tag{1}$$

where $u : T \to \mathbb{R}^2$ is the aircraft nominal velocity and $f : \mathbb{R}^2 \times T \to \mathbb{R}^2$ is a time-varying vector field on $\mathbb{R}^2$ representing the nominal wind speed at position x and time t, also called the *wind field*. The third term in the right-side of the above equation is the stochastic term, where $(B_t)_{t\geq 0}$ is a standard two-dimensional Brownian motion whose variance is modulated by the mapping $\Sigma : \mathbb{R}^2 \times T \to \mathbb{R}^{2\times 2}$ and characterized by the following properties:

1. $B_0 = 0$;
2. B_t is almost surely continuous;
3. For any $h \geq 0$, the process $(B_{t+h} - B_t)_{t\geq 0}$ is a standard Brownian motion;
4. For any $t \geq s \geq 0$, the random variable $B_t - B_s$ is independent increment with distribution $\sim \mathcal{N}(0, t-s)$;
5. For any $c \neq 0$, $\frac{1}{\sqrt{c}} B_{ct}$ is a standard Brownian motion.

For simplicity, we assume that Σ is a constant diagonal matrix: $\Sigma(x,t) = \Sigma = \mathrm{diag}(\sigma_c, \sigma_a)$, where σ_c, σ_a are the power spectral densities of the perturbations affecting the position in the along-track and the cross-track directions, respectively. Also assume that the nominal wind field f at any given $t \in T$ is uniform in the region of interest in the airspace, and acts additively on the aircraft velocity. In this case, equation (1) can be rewritten as follows:

$$dX(t) = u(t)dt + \Sigma\, dB_t. \tag{2}$$

Although the above simplified aircraft dynamics model is used in the rest of the paper, our analysis can be extended easily to the general case of equation (1).

3 Air Traffic Density Estimation

Traffic density is an effective tool for identifying the safety and the traffic flow efficiency of any transportation system. Air traffic density is used in this paper to study the flow of multiple aircraft traveling within a given time horizon in a sector of the airspace.

There are many different ways of defining air traffic density. In this paper, we define it to be the expected number of aircraft occupying a given subregion of the airspace at a given time. Suppose the airspace domain of interest $U \in \mathbb{R}^2$ is divided into subregions $U = \cup U_j$, $j = 1, 2, ..., n$ and consider M en-route aircraft traveling within the time interval $T = [0, t_f]$ in U. Then the air traffic density can be defined as follows:

$$D(U_j, t) = \frac{1}{M} E\left[N(U_j, t)\right], \tag{3}$$

$$\text{where} \quad N(U_j, t) = \sum_{m=1}^{M} 1_{X_{A_m}(t) \in U_j}.$$

Here, $X_{A_m}(t)$, $m = 1, 2, ..., M$, denote the aircraft positions at time $t \in T$.

In most ATM literature as well as in this paper, aircraft dynamics are modeled by stochastic differential equations [15, 16]. Many solution methods of the stochastic differential equations have been proposed, for example, approximated Markov Chain discretization [5], statistical moments computation [17], construction of barrier functions [18], and probabilistic testing method [19]. The approach in this paper is a simulation-based one with some additional assumptions [20].

In simulating the aircraft trajectory, the aircraft motion can be thought of as a deterministic nominal trajectory $u(t)$, $t \in T$, plus a stochastic perturbation $\frac{dB_t}{dt}$. Thus, we could simulate the different realizations of the stochastic process $\frac{dB_t}{dt}$ and then add them to the deterministic nominal trajectories. Given the sequence of way points specifying the nominal motion, the algorithm for simulating the aircraft trajectories is summarized as follows:

1. Let $X_A(0)$ be the initial aircraft position and $t_1, t_2, ..., t_K$ be the times corresponding to way points $1, 2, \dots, K$.

2. For $k = 1, 2, ...K$, let $\Delta t_k = t_{k+1} - t_k$ and solve equation (2) by integrating both sides as

$$\int_{t_k}^{t_{k+1}} X_A(t)dt = u\Delta t_k + \Sigma \int_{t_k}^{t_{k+1}} dB_s.$$

3. Scale the Brownian motion by $\sqrt{\Delta t_k}$ in the interval $\Delta t_k = [t_k, t_{k+1}]$ and compute

$$X_A(t_{k+1}) = X_A(t_k) + u\Delta t_k + \Sigma\, \sqrt{\Delta t_k}(B(t_{k+1}) - B(t_k)).$$

4. According to Property 4 of Brownian motion, $B(t_{k+1}) - B(t_k)$ is Gaussian random variable with zero mean and variance Δt_k.

5. Repeat for every $k = 1, 2, ...K$ to find the aircraft trajectory $X_A(t)$.

For a quick demonstration, consider six aircraft $X_{A_i}(t)$, $t \in T = [0, t_f]$ traveling with different constant speed and different along track variances σ_a and let $t_f = 30$. Figure 1 plots the nominal trajectory in a dash line and the simulated trajectory in a solid line for each aircraft traveling from its initial position ($\circ$) to its destination ($\times$). Notice the deviation of the simulated path from the nominal trajectory.

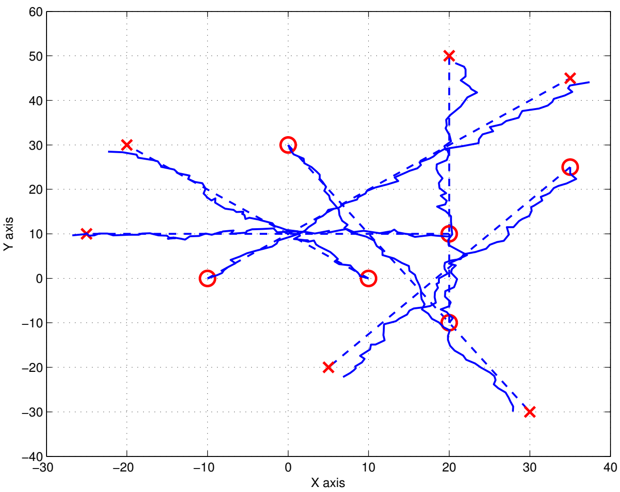

Figure 1: Nominal trajectories and one possible simulated path of each aircraft

Figure 1 shows only one simulated aircraft trajectory. By simulating a large number of times, we can obtain increasing accurate estimates of the air traffic density, which is a functional of the aircraft trajectories, based on the Law of Large Number. Precisely, H experiments is performed for each aircraft and the final expression of the air traffic density can be given as follows:

$$D(U_j, t) = \frac{1}{MH} \sum_{i=1}^{H} E\left[N^i(U_j, t)\right].$$

Comparing with the probabilistic conflict detection method proposed in our previous work [5], the advantage of the air traffic density estimation approach in this paper is that air traffic density not only indicates conflicts between two aircraft, but also gives a time varying high-level view of the estimated conflict among multiple aircraft sharing a common region of the airspace at any future time.

4 Congestion/Conflict Detection and Resolution

4.1 Congestion/Conflict Detection

This section presents the congestion/conflict detection algorithm using the estimated air traffic density obtained in the previous section. Based on the values of the air traffic density, we can not only detect the conflicts, but also measure their different levels of urgency. Thus, we are indeed detecting the degree of the congestion of different zones of the airspace in future time horizon.

Our approach can be described as follows. Given the estimated values of $D(U_j, t)$ in Section 3, consider a threshold value $\nu > 0$. Then, three different zones of the airspace, free zone, less congested zone, and congested zone can be defined according to the following rules:

$$U_{j,t} = \begin{cases} \text{free zone}, & \text{if } D(U_j, t) = 0, \\ \text{less congested zone}, & \text{if } 0 < D(U_j, t) < \nu, \\ \text{congested zone}, & \text{if } D(U_j, t) \geq \nu. \end{cases}$$

In this way, air traffic density enables us to specify varying level of danger at future times, so that the ATC and the pilot can take appropriate and immediate resolutions for more imminent threat.

4.2 Conflict Resolution

The last step of conflict resolution. To guarantee safety and smoothness of travel for the aircraft, we need to avoid the predicted conflicts from occurring by re-routing the involved aircraft trajectories away from the congested zones, while at the same time minimize the deviations from the original assigned trajectories in order to meet the scheduled arrival time and reduce fuel consumption. Accordingly, we formulate the conflict resolution problem as an optimization problem, where the cost function to be minimized is a weighted sum of the congestion along the trajectory and the deviations. We iteratively update a sequence of way points parameterizing the aircraft trajectories, until some stopping criteria are met. For other existing papers on ATM systems that also formulate the conflict resolution as an optimization problem, see e.g. [21, 22].

Let $X_{A_i}(t) \in \mathbb{R}^2$ be the position of aircraft A_i at time $t = t_1, t_2, ..., t_K$. Then, the trajectory planning problem for aircraft A_i is to minimize the following cost function:

$$\begin{aligned} V(X_{A_i}, t) &= \sum_{k=1}^{K-1} \|X_{A_i}(t_k) - X_{A_i}(t_{k+1})\|^2 \\ &+ \lambda \sum_{k=1}^{K-1} \oint_{X_{A_i}(t_k) \to X_{A_i}(t_{k+1})} F(X_{A_i}, t)dt. \end{aligned}$$

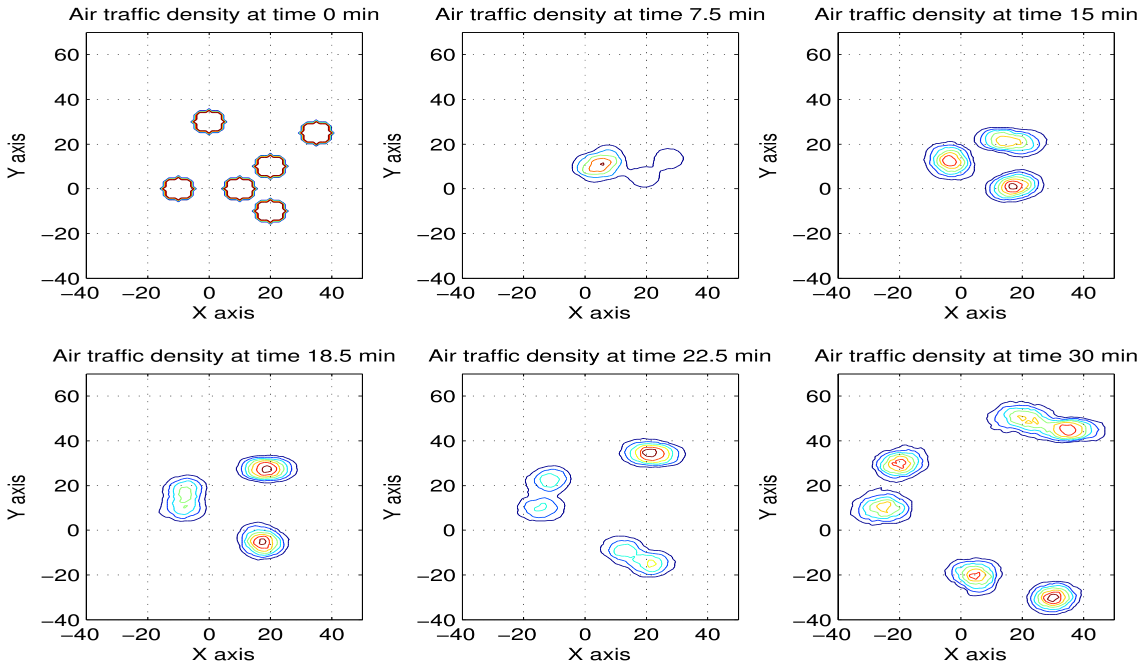

Figure 2: Air traffic density varying with time period T

In the above expression of the cost function, minimizing the first summation tends to result in a smooth trajectory with small deviation from a straight line motion. The second term characterizes the safety of flying along the different segments of the trajectory. The function $F(X_{A_i},t)$ is the conflict function and could be chosen to be the air traffic density function computed in the previous section. The parameter λ here is a weight parameter. A larger value of λ means drawing more focus on ensuring safety and less attention on smoothness of travel.

To solve the above optimization problem, we focus on three consecutive way points, $X_{A_i}(t_{k-1})$, $X_{A_i}(t_k)$, and $X_{A_i}(t_{k+1})$. Assume $X_{A_i}(t_{k-1})$ and $X_{A_i}(t_{k+1})$ are fixed and $X_{A_i}(t_k)$ could be freely allocated. This results in the following sub-problem:

$$\begin{aligned}\min_{X_{A_i}(t_k)} V(X_{A_i}(t_k),\lambda) &= \min_{X_{A_i}(t_k)} \|X_{A_i}(t_{k-1}) - X_{A_i}(t_k)\|^2 \\ &+ \|X_{A_i}(t_k) - X_{A_i}(t_{k+1})\|^2 \\ &+ \lambda \Bigg[\oint_{X_{A_i}(t_{k-1}) \to X_{A_i}(t_k)} F(X_{A_i},t)dt \\ &+ \oint_{X_{A_i}(t_k) \to X_{A_i}(t_{k+1})} F(X_{A_i},t)dt \Bigg] \quad (4)\end{aligned}$$

We can define similar sub-problems with respect to other way points. An iterative solution to the original optimization problem can be obtained by repetitively solving the sub-problem for way points $X_{A_i}(t_k)$ cyclically.

5 Numerical Example

Consider six aircrafts $X_{A_i}(t)$, $i = 1,2,\ldots 6$, traveling within the time period $T = [0,t_f]$, $t_f = 30$ min, in an open bounded domain of the airspace $U = (-30,40) \times (-40,60)$. Each aircraft has a constant nominal velocity from its starting position to its destination, but with different noise variance σ_a. Suppose the minimum horizontal separation between aircraft is $r_h = 5$ nmi, and the number of simulation trials for estimating the air traffic density is $H = 100$.

The algorithm in Section 3 can be applied to estimate the air traffic density in the airspace U within the time horizon T. Figure 2 shows the contour plots of the air traffic density map at different future times.

Next, assume another intruding aircraft is introduced that passes through the airspace domain U with a constant speed starting from $(-20,-20)$ and ending at the destination position $(40,60)$ within the same time period $T = [0,30]$. Figure 3 shows the air traffic density map overlapped with the intruding aircraft's nominal trajectory at various future times, where the ($\star$) represents the intruding aircraft's position $X_{A_7}(t)$. Note that, at time $t = 7.5$ min, the intruding aircraft is approaching a congested zone so an alert to the pilots to be caution is sufficient. However, at times 15, 18.5, and 22.5 min, the intruding aircraft would be inside congested zones, thus resolution maneuvers need to be designed.

To avoid these potential conflicts, we apply the optimal trajectory planning algorithm in Section 4. Here we search for the sequence of way points using equation (4) with $F(X_{A_i},t)$ being the air traffic density $D(U_j,t)$ and consider the two end points to be $X_{A_7}(t=0) = (-20,-20)$ and $X_{A_7}(t=30) = (40,60)$. Then, we solve the optimization problem with different values of the weight λ: λ=10, 25, and 5^5, to study its effects on the solution. Figure 4 shows the optimal trajectory for the different λ. Note that, the bigger λ is, the safer but less smooth the resolution trajectory is. Figure 5 plots together the air traffic density $X-Y$ plane, the original trajectory of $X_{A_7}(t)$, and its

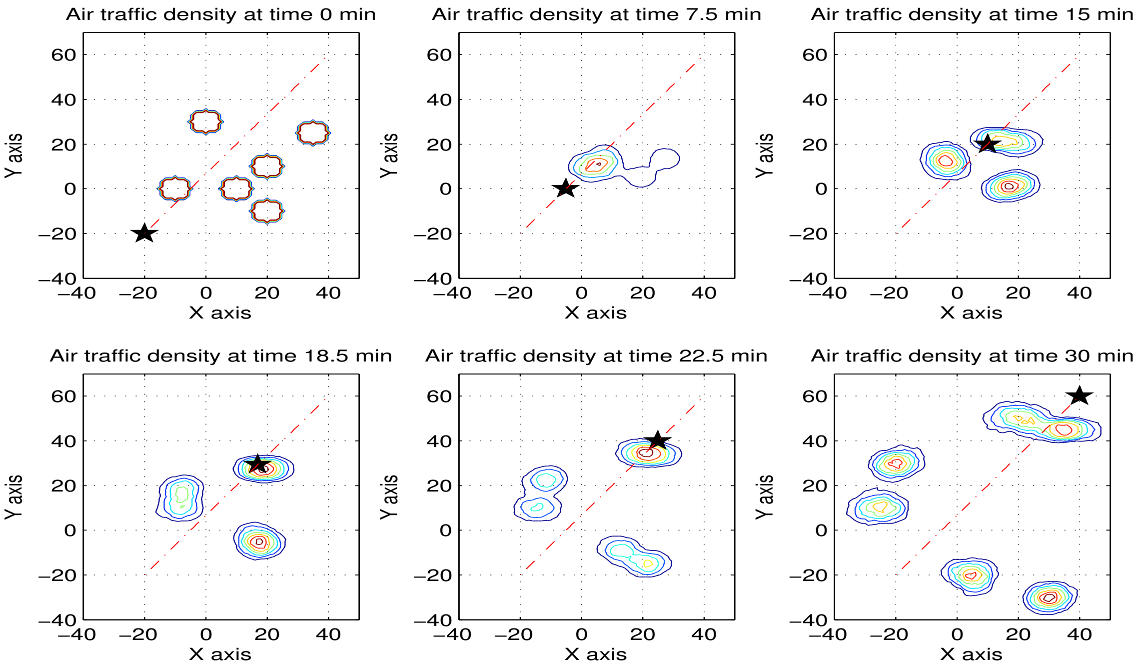

Figure 3: Air traffic density varying with time period T

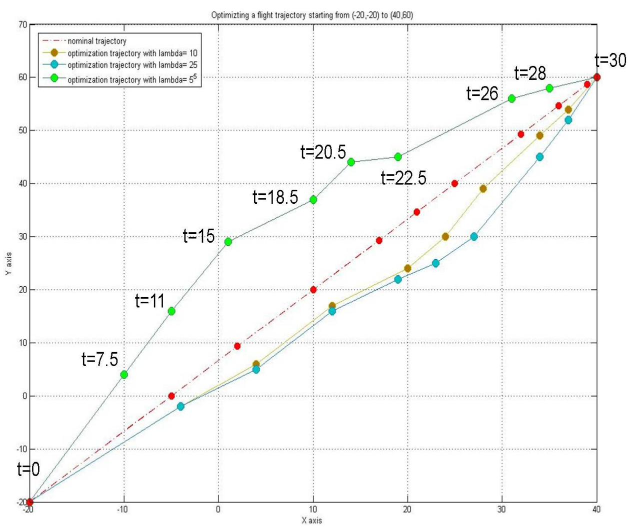

Figure 4: Optimization trajectory with different Lambdas

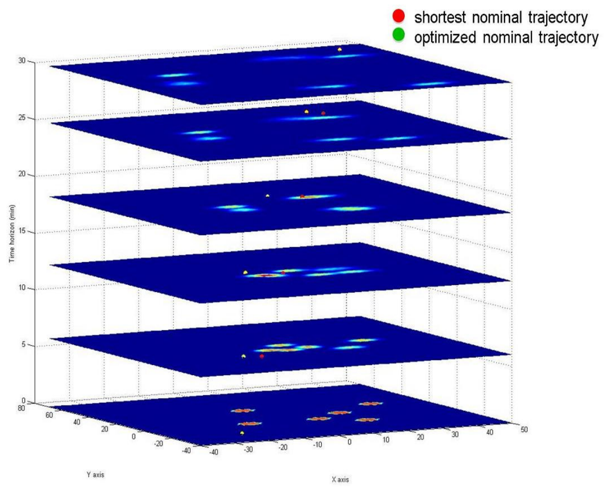

Figure 5: Optimization trajectory verses time

resolved trajectory.

6 Conclusion

Air traffic density estimation introduced in this paper is an efficient method for studying multi-aircraft conflicts. The estimated air traffic density is useful as its values indicate the degree of congestions and it provides a time-varying high-level view of traffic within a certain region of the airspace in future times. The air traffic density can also be used in the conflict resolution problem, where the aircraft trajectories are re-routed by solving an optimization problem. Although in this paper we focus on a simplified stochastic model of aircraft dynamics in two-dimensional airspace, our simulation method can be easily extended to 3D airspace with complicated aircraft dynamics (e.g. general wind field). Numerical experiments show that the proposed CDR algorithm is effective in resolving conflicts involving multiple aircraft.

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