# Multi-Agent Control for Centralized Air Conditioning Systems Serving Multi-Zone Buildings*

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***Abstract*— Coordinating different components in a complex air conditioning system is challenging for centralized controls due to the large number of optimization variables. In this scenario, de-centralized controls are more appropriate alternatives. This study proposes a multi-agent control methodology for the optimal control of centralized air conditioning systems that are typically adopted in multi-zone commercial buildings. A hierarchical multi-agent framework is developed in which the agents cooperate to find the optimal operating point. Two consensus-based distributed optimization algorithms are formulated for this specific type of problem, which form the underlying mechanism of intra-agent optimization and inter-agent cooperation. Finally, a 3-zone building case study is used to demonstrate the performance of the proposed approach.**

## I. INTRODUCTION

It is widely recognized in the building control community that one important barrier to the deployment of advanced controls in buildings is the high implementation cost. Buildings are unique in terms of both building construction and heating, ventilation and air-conditioning (HVAC) system configuration. As a consequence, building specific controller design is required which can be costly. Multi-agent control could provide a plug-and-play, low cost control alternative if each piece of the equipment was shipped with an agent representing the product design performance (meaning?).

For medium- to large-sized commercial buildings, the HVAC system configuration could be very complicated and it is not feasible to use centralized control. On the other hand, multi-agent control is a de-centralized control scheme in which a large complex problem is broken down into subproblems that can be solved by individual agents in parallel. For small-sized commercial buildings or even residential houses where HVAC component coordination is a relatively simple task, multi-agent control could still prove to be valuable for grid level coordination, although this is out of the scope of this paper.

There are several categories of methods in the literature related to building multi-agent controls. The first category mainly relies on heuristic or rule-based control embedded in each agent (e.g., [1], [2], [3]). For example, [1] provides an example in which room agents setup or setback room temperature setpoints based on the occupancy status from a personal agent. Another study [2] compared several typical lighting control strategies and also proposed a cooperative control between occupants and operator agents. A second type of building multi-agent control that is commonly studied adopts an optimization-based approach. However, most previous studies in this category have used centralized optimization for decision making (e.g., [4], [5], [6]) and were more focused on the modularity of multi-agent systems.

Distributed model predictive control (DMPC) in buildings has attracted growing attention in recent years (e.g., [7], [8], [9], [10], [11]). The goal is to shift the building heating/cooling load to reduce peak-time energy utilization and demand for cost reduction. For multi-zone buildings, an agent can be assigned to each zone for optimizing the local heating/cooling injection rate and different agents coordinate to achieve some type of consensus. However, most of the work under this category simplifies or even ignores the optimization on the HVAC system side. For example, [10], [11] and [9] all assumed the building energy consumption was proportional to the heating/cooling rate injected into the space with a constant ratio. One study [7] considered a cooling coil model and optimization of the combination of chilled water flow rate, air rate and supply air temperature; however, a constant efficiency was assumed for the chiller which was the main energy consumer in the HVAC system. Another study [8] considered near-optimal heuristics to control a direct-expansion (DX) air-conditioning system while each zone tried to schedule its local energy utilization in a distributed manner. However, it took significant effort to obtain the heuristic rules and the rule were only valid for the specific type of equipment under study.

This paper presents a general multi-agent framework and algorithm to optimally coordinate the components in a HVAC system. MPC tries to optimize the load profiles in a building while the method proposed in this paper is dedicated to optimizing the HVAC system with some given load at any time instance. (This is not clear: MPC means general MPC? What is the distinction?) It should be pointed out that [12] presented several ideas that coincide with those in this paper. For example, a component agent in [12] (called basic agent in this paper) has the capability of self-identification and an optimizer agent (called the same in this paper) was designed for each component agent for optimization as well as inter-component cooperation. However, the optimization-then-negotiation iteration follows a ping-pong scheme and the optimization relies on an exhaustive search method, both of which make the approach infeasible for large systems. This paper applies two consensus-based distributed optimization algorithms to minimize energy consumption of a centralized air-conditioning system while satisfying load requirements.

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The proposed multi-agent framework, presented in another paper (you mean this paper?), is able to automate the multi-agent system construction which makes the method plug-and-play and easily applicable to other types of HVAC systems.

I notice that when describing existing work, past tense and present tense are both used in this seciton. This should be consistent. I normally use present tense.

## II. System Overview

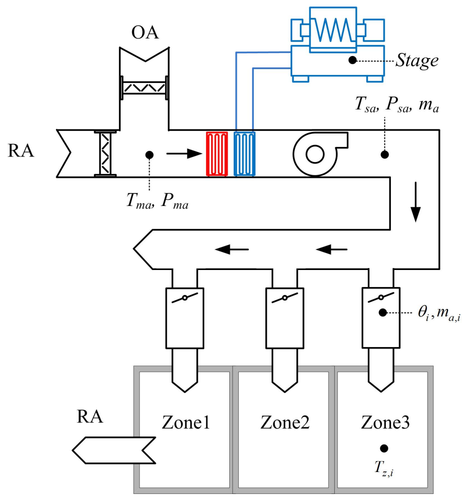


Fig. 1. Diagram of HVAC system for multi-zone buildings

Fig. 1 shows the diagram of a typical centralized air conditioning system serving a multi-zone building. Air is conditioned in the air handling unit (AHU) and then supplied to each conditioned zone through a dedicated variable-air-volume (VAV) box. The zone air temperature is regulated by varying the entering airflow rate through modulation of the VAV damper. The return air (RA) from the space is circulated back to the AHU and mixed with the outdoor air (OA) before going through the heating/cooling coil for air conditioning. Supply air temperature ($T_{sa}$) is controlled to a setpoint by varying the chilled water flow rate for chilled water systems or by changing the refrigerant evaporating temperature for direct-expansion (DX) systems. Fan speed modulates to maintain a constant supply duct pressure ($P_{sa}$) while outdoor air (OA) and return air (RA) dampers coordinate to keep a constant mixed-air pressure ($P_{ma}$).

## III. Component Models

### A. Direct-expansion unit

This study focuses on a direct-expansion cooling system in which the air exchanges heat directly with the refrigerant going through the cooling coil. There are six stages in the compressor excluding the off stage. By controlling the staging bandwidth, the system can achieve continuous capacity modulation; thus, *Stage* is a continuous variable with value between 0 and 6. Given the corresponding boundary conditions and compressor stage, the model will output cooling coil outlet air temperature, unit power consumption and sensible heat ratio (*SHR*, the ratio of sensible capacity to total capacity). A gray-box model was developed for this DX unit which was trained with field data and the details can be found in [13].

$$[T_{la}, Pow_{DX}, SHR] = DX\left(T_{ma}, w_{ma}, m_a, T_{amb}, Stage\right). \tag{1}$$

The first four variables and $T_{la}$ are not defined. I know some of them are defined below, but they should be defined at their first appearance.

### B. Fan

Energy is consumed by the fan to deliver conditioned air to the zone space. The instantaneous fan power is a function of pressure rise (external static pressure, *ESP*) across the fan and the airflow rate ($m_a$) delivered. In this study, a quadratic polynomial form shown in Eq. (2) is used and the coefficients were obtained through curve fitting using field data:

$$\begin{aligned} Pow_{fan} \quad &= \quad a_0 + a_1 m_a + a_2 m_a^2 + a_3 ESP \\ &\quad\quad + a_4 ESP^2 + a_5 ESP \cdot m_a. \end{aligned} \tag{2}$$

Fan energy contributes to a temperature rise in the air which is calculated as

$$T_{sa} = T_{la} + \frac{Pow_{fan}}{m_a \cdot c_p}, \tag{3}$$

where $c_p$ is the specific heat of air (1000 J/kg-K). ($T_{la}$ is not defined above) Let $P_{ma}$ be the mixed air pressure. Then a pressure rise is resulted due to the fan's work as

$$P_{sa} = P_{ma} + ESP. \tag{4}$$

### C. Damper

The VAV box damper has a feedback control based on the space temperature. By varying the damper opening, the airflow rate that enters the zone space can be modulated to regulate the space temperature. Let $P_{sa}$ be the air pressure in the supply duct and $P_z$ be the zone space pressure. Then the pressure drop across each air damper is

$$\Delta P = P_{sa} - P_z. \tag{5}$$

Airflow rate going through the air damper can be formulated as ([14])

$$m_a = sign(\Delta P) A_{damper} \left(\frac{2\rho \Delta P}{exp(a + b(1-\theta))}\right)^{1/2} \tag{6}$$

where $A_{damper}$ is the damper section cross area, $\rho$ is the air density, $\theta$ is the damper opening (%) and $a$, $b$ are parameters that are associated with damper characteristics. These two parameters were also obtained from field data.

### D. Zone

A gray-box model with a resistance-capacitance thermal network was trained from field data to represent the zone thermal behaviors (see [15] for details). A discrete-time state-space representation of the model is

$$\begin{aligned} \mathbf{x}_{k+1} \quad &= \quad \mathbf{A}\mathbf{x}_k + \mathbf{B}_w \mathbf{W}_k + \mathbf{B}_u \mathbf{Q}_{sen,k} \\ \mathbf{y}_k \quad &= \quad \mathbf{C}\mathbf{x}_k, \end{aligned} \tag{7}$$

where $\mathbf{x}$ is a state vector containing all the nodal temperatures in the thermal network; $\mathbf{W}$ contains all disturbance inputs including weather condition and internal heat gains from occupants and electrical appliances; $\mathbf{y} = [T_{z,1}, T_{z,2}, T_{z,3}]$ is the output vector containing the space air temperatures of the three zones; and $\mathbf{Q}_{sen} = [Q_{sen,1}, Q_{sen,2}, Q_{sen,3}]$ is a controllable input vector consisting of the sensible cooling/heating capacities (kW) of the zones. The sensible capacity of each zone is calculated by

$$Q_{sen,i} = m_{a,i} c_p (T_{sa} - T_{z,i}), \tag{8}$$

where $m_{a,i}$ and $T_{z,i}$ denote the current (step $k$) supply airflow rate and the air temperature of zone $i$. In order to achieve a set of next step (step $k+1$) zone air temperature setpoints $\mathbf{y}^{sp}_{k+1}$, the required sensible capacities can be calculated as

$$\mathbf{Q}_{sen,k} = (\mathbf{CB}_u)^{(-1)} \left( \mathbf{y}^{sp}_{k+1} - \mathbf{CAx}_k - \mathbf{CB}_w \mathbf{W}_k \right). \tag{9}$$

Note that, due to the model structure, the matrix $\mathbf{CB}_u$ is diagonal with nonzero diagonal elements, thus invertible.

### E. Models in input-output form

The aforementioned models are summarized in input-output form as below:

$$\begin{array}{rcl} Pow_{fan} & = & Fan_{Pow} \left( m_a, ESP \right) \\ Pow_{DX} & = & DX_{Pow} \left( \underline{T_{ma}}, \underline{w_{ma}}, m_a, \underline{T_{amb}}, Stage \right) \\ T_{la} & = & DX_{Tla} \left( \underline{T_{ma}}, \underline{w_{ma}}, m_a, \underline{T_{amb}}, Stage \right) \\ T_{sa} & = & Fan_T \left( T_{la}, m_a, ESP \right) \\ P_{sa} & = & \underline{P_{ma}} + ESP \\ m_{a,i} & = & Damper \left( P_{sa}, \underline{P_{z,i}}, \theta_i \right) \\ m_a & = & \sum_{i=1}^{3} m_{a,i} \\ Q_{sen,i} & = & m_{a,i} c_p \left( T_{sa} - T_{z,i} \right). \end{array} \tag{10}$$

The under-scored variables are boundary conditions provided at the beginning of each decision step. In practice, the mixed air pressure $P_{ma}$ is typically controlled to a constant level by modulating return air damper. The zone space pressure $P_{z,i}$ is also normally maintained at some positive value, assumed to be $P_{ma} = -0.3$ (inW.C.) and $P_{z,i} = 0.1$ (inW.C.) in this paper. The required sensible capacity $Q_{sen,i}$ is also taken as a boundary condition as it can be calculated as in (9) at the beginning of each decision step after the next step zone temperature setpoint has been prescribed. This can help simplifying the notation by dropping the time indices.

## IV. Problem Formulation

The optimization problem is to find the optimal operating point given the boundary conditions as well as the required cooling capacities such that the total power consumption is minimized. The problem needs to be solved at each decision step as the boundary conditions and capacity requirements change with time.

### A. Centralized formulation

Define a vector of the optimization variables as

$$\mathbf{Z} = [m_a, Stage, T_{la}, T_{sa}, P_{sa}, \theta_1, \theta_2, \theta_3, m_{a,1}, m_{a,2}, m_{a,3}]^T. \tag{11}$$

The (static) optimization problem to be solved at each time step is formulated as

$$\begin{array}{rrcll} \text{minimize} & & & Pow_{DX}(\mathbf{Z}) + Pow_{fan}(\mathbf{Z}) & (12) \\ \text{s.t.} & Q_{sen,i} & = & m_{a,i} c_p (T_{sa} - T_{z,i}), \quad i = 1,2,3, & (13) \\ & m_{a,i} & = & Damper(\theta_i, P_{sup}), \quad i = 1,2,3, & (14) \\ & m_a & = & \sum_{i=1}^{3} m_{a,i}, & (15) \\ & T_{la} & = & DX_{Tla}(m_a, P_{sa} Stage?), & (16) \\ & T_{sa} & = & Fan_T(T_{la}, m_a, P_{sa}), & (17) \\ & \theta_i & \in & [0, 100], \quad i = 1,2,3, & (18) \\ & Stage & \in & [0, 6], & (19) \\ & P_{sa} & \in & [0, 1.2] \quad (\text{inW.C.}). & (20) \end{array}$$

The cost function in Eq. (12) is essentially the total power consumption for the DX unit and the supply fan. The equality constraints in Eq. (13) to (17) correspond to the models described in the preceding section. The interval type constraints in Eq. (18) to (20) are due to the capacities in the physical components. The boundary conditions are omitted from the formulation above for ease of notation. Also the variable ESP is eliminated through the relationship $P_{sa} = P_{ma} + ESP$.

### B. Distributed formulation

In the centralized formulation, there are eleven optimization variables, nine (mostly nonlinear) equality constraints and five interval-type constraints. Solving this problem requires significant computations. However, this complex problem can be broken down into several sub-problems as follows.

*Sub-problem (1):*

$$\begin{array}{c} \min \left\{ Pow_{DX} \left( m_a^{(1)}, Stage^{(1)} \right) \right\} \\ \text{s.t.} \quad Stage^{(1)} \in [0, 6] \end{array} \tag{21}$$

*Sub-problem (2):*

$$\begin{array}{c} \min \left\{ Pow_{fan} \left( m_a^{(2)}, P_{sa}^{(1)} \right) \right\} \\ \text{s.t.} \quad P_{sa}^{(1)} \in [0, 1.2] \end{array} \tag{22}$$

*Sub-problems (3-5) for $i = 1,2,3$:*

$$\min \left\{ 1/\delta \left( Q_{sen,i} - m_{a,i}^{(1)} c_p (T_{sa}^{(i)} - T_{z,i}) \right) \right\} \tag{23}$$

*Sub-problems (6-8) for $i = 1,2,3$:*

$$\begin{array}{c} \min \left\{ 1/\delta \left( m_{a,i}^{(2)} - Damper(\theta_i^{(1)}, P_{sa}^{(i+1)}) \right) \right\} \\ \text{s.t.} \quad \begin{array}{rcl} \theta_i^{(1)} & \in & [0, 100] \\ P_{sa}^{(i+1)} & \in & [0, 1.2] \end{array} \end{array} \tag{24}$$

*Sub-problem (9):*

$$\min\left\{1/\delta\left(m_a^{(3)}-\sum_{i=1}^{3}m_{a,i}^{(3)}\right)\right\} \tag{25}$$

*Sub-problem (10):*

$$\begin{aligned}\min\left\{1/\delta\left(T_{la}^{(1)}-DX_T(m_a^{(4)},Stage^{(2)})\right)\right\}\\ \text{s.t.}\quad Stage^{(2)}\in[0,6]\end{aligned} \tag{26}$$

*Sub-problem (11):*

$$\begin{aligned}\min\left\{1/\delta\left(T_{sa}^{(4)}-Fan_T(T_{la}^{(2)},m_a^{(5)},P_{sa}^{(5)})\right)\right\}\\ \text{s.t.}\quad P_{sa}^{(5)}\in[0,1.2]\end{aligned} \tag{27}$$

*Consensus constraints:*

$$m_a = m_a^{(i)}, \quad \forall i\in\{1,2,3,4,5\}, \tag{28}$$

$$Stage = Stage^{(i)}, \quad \forall i\in\{1,2\}, \tag{29}$$

$$T_{la} = T_{la}^{(i)}, \quad \forall i\in\{1,2\}, \tag{30}$$

$$T_{sa} = T_{sa}^{(i)}, \quad \forall i\in\{1,2,3,4\}, \tag{31}$$

$$P_{sa} = P_{sa}^{(i)}, \quad \forall i\in\{1,2,3,4,5\}, \tag{32}$$

$$\theta_j = \theta_j^{(1)}, \quad \forall j\in\{1,2,3\}, \tag{33}$$

$$m_{a,j} = m_{a,j}^{(i)}, \quad \forall i\in\{1,2,3\}, \ \forall j\in\{1,2,3\} \tag{34}$$

where

$$\delta(x)=\begin{cases}0, & x\neq 0\\ \infty, & x=0\end{cases}$$

is the Dirac delta function.

It can be seen that solving the sub-problems is equivalent to solving the original problem: each of the *Sub-problems (3)-(11)* essentially adds an infinite penalty to the cost function whenever the corresponding constraint is violated. However, the sub-problems are formulated in a way that can be solved in a parallel and distributed manner. In particular, the consensus constraints ensure that different local copies of the same variable agree with one another. The interval type constraints are present in all the sub-problems that involve the corresponding variables to make sure the local optimization does not yield infeasible solutions.

With this distributed formulation, the original problem breaks down to eleven sub-problems most of which have dimension less than three. *Sub-problems (9)* and *(11)* have four design variables but *Sub-problem (9)* has an explicit solution that will be discussed in Section IV-C. In addition, each sub-problem with at most two constraints of interval type has a much lower solution complexity. This makes the proposed method scalable to increased problem size.

### C. Solution algorithms

Two distributed optimization algorithms are considered in this paper: subgradient method ([16], [17]) and alternating direction multiplier method (ADMM) ([18], [17]). It is important to emphasize that the problem considered in this study is nonconvex (most problems in HVAC system optimization are not convex) but both of the algorithms only guarantee convergence for convex problems with some additional requirements. These algorithms are used here as local optimizers to provide coordination mechnism for the multi-agent system; the issue of global convergence will not be addressed in this paper.

The local constraints in the sub-problems can be carried over to the solution algorithms in a straightforward way; hence they are omitted in the following formulations to simplify notation. Define a column vector $\mathbf{X}$ as

$$\begin{aligned}\mathbf{X}(1:5) &= \{m_a^{(i)}|i\in\{1,2,3,4,5\}\}\\ \mathbf{X}(6:7) &= \{Stage^{(i)}|i\in\{1,2\}\}\\ \mathbf{X}(8:9) &= \{T_{la}^{(i)}|i\in\{1,2\}\}\\ \mathbf{X}(10:13) &= \{T_{sa}^{(i)}|i\in\{1,2,3,4\}\}\\ \mathbf{X}(14:18) &= \{P_{sa}^{(i)}|i\in\{1,2,3,4,5\}\}\\ \mathbf{X}(19:21) &= \{\theta_j^{(1)}|j\in\{1,2,3\}\}\\ \mathbf{X}(22:30) &= \left\{m_{a,j}^{(i)}|(i,j)\in\{1,2,3\}\right.\\ &\qquad \left.\times\{1,2,3\}\right\},\end{aligned}$$

where $\mathbf{X}(i:j)$ represents the $i$-th to $j$-th elements of $\mathbf{X}$. So vector $\mathbf{X}$ stacks all the local copies of the design variables.

Let $\mathbf{X}_i$ denote the sub-vector of $\mathbf{X}$ that contains the variables associated with *Sub-problem (i)* and let $f_i$ denote the cost function of *Sub-problem (i)*. For example, $m_a^{(1)}$ and $Stage^{(1)}$ are local variables within *Sub-problem (1)*; hence $\mathbf{X}_1=[m_a^{(1)},Stage^{(1)}]=[\mathbf{X}(1),\mathbf{X}(6)]$ and $f_1(\mathbf{X}_1)=Pow_{DX}\left(m_a^{(1)},Stage^{(1)}\right)$. Then the problem formulation for *Sub-problem (i)* becomes

$$\begin{aligned}&\min\{f_i(\mathbf{X}_i)\}\\ &\text{s.t.}\quad \mathbf{X}_i\in\mathbf{C}_i\end{aligned}$$

where $\mathbf{C}_i$ is the feasible region of the local variables $\mathbf{X}_i$.

Let $\mathbf{E}\in\mathbb{R}^{30\times 30}$ be an identity matrix and let $F_{[i,j]}\in\mathbb{R}^{i\times 11}$ be such a matrix that the elements of the $j$th column are one and all other elements are zero. Define

$$\begin{aligned}\mathbf{F}= &\left[F_{[5,1]}^T,F_{[2,2]}^T,F_{[2,3]}^T,F_{[4,4]}^T,F_{[5,5]}^T,F_{[3,6]}^T,\right.\\ &\left.F_{[3,7]}^T,F_{[3,8]}^T,F_{[7,9]}^T,F_{[7,10]}^T,F_{[7,11]}^T\right]^T\in\mathbb{R}^{30\times 11}.\end{aligned}$$

Then the *Consensus constraints* becomes

$$\mathbf{X}=\mathbf{E}\mathbf{X}=\mathbf{F}\mathbf{Z}, \tag{35}$$

where $\mathbf{Z}$ is a vector of all the global variables as defined in Eq. (11). Further define $\mathbf{E}_i$ and $\mathbf{F}_i$ sub-matrices of $\mathbf{E}$ and $\mathbf{F}$, respectively, which contain only the rows corresponding to the constraints that belong to *Sub-problem (i)*. Then $\mathbf{E}_i\mathbf{X}=\mathbf{X}_i=\mathbf{F}_i\mathbf{Z}$ and the original optimization problem is equivalent to

$$\begin{aligned}&\min\sum_{i=1}^{11}f_i(\mathbf{X}_i)\\ \text{s.t.}\quad &\mathbf{X}=\mathbf{E}\mathbf{X}=\mathbf{F}\mathbf{Z}\\ &\mathbf{X}\in\mathbf{C}\\ \text{or}\quad &\mathbf{X}_i=\mathbf{F}_i\mathbf{Z}\\ &\mathbf{X}_i\in\mathbf{C}_i\quad\text{for } i=1,\ldots,11\end{aligned}$$

*1) Subgradient method:* The Lagrangian is

$$\mathbf{L}(\mathbf{X},\mathbf{Z},\mathbf{Y}) = \sum_{i=1}^{11} f_i(\mathbf{X}_i) + \mathbf{Y}^T(\mathbf{E}\mathbf{X} - \mathbf{F}\mathbf{Z}), \tag{36}$$

where $\mathbf{Y} \in \mathbb{R}^{48\times 1}$ is the Lagrange multiplier vector. Let $\mathbf{Y}_i$ be the sub-vector that corresponds to *Sub-problem (i)*. Then Eq. (36) becomes

$$\mathbf{L}(\mathbf{X},\mathbf{Z},\mathbf{Y}) = \sum_{i=1}^{11} \left(f_i(\mathbf{X}_i) + \mathbf{Y}_i^T\mathbf{X}_i\right) - \mathbf{Y}^T\mathbf{F}\mathbf{Z}. \tag{37}$$

Assume the infimums are all obtainable within the feasible region for all problems considered and define

$$g_i(\mathbf{Y}_i) = \min_{\mathbf{X}_i} \left(f_i(\mathbf{X}_i) + \mathbf{Y}_i^T\mathbf{X}_i\right). \tag{38}$$

Then the dual function is

$$\begin{aligned} g(\mathbf{Y}) &= \min_{\mathbf{X},\mathbf{Z}} \mathbf{L}(\mathbf{X},\mathbf{Z},\mathbf{Y}) \\ &= \textstyle\sum_{i=1}^{11} g_i(\mathbf{Y}_i) + \min_{\mathbf{Z}}\left(-\mathbf{Y}^T\mathbf{F}\mathbf{Z}\right). \end{aligned} \tag{39}$$

For the equation above to be valid, we must have $\mathbf{Y}^T\mathbf{F} = \mathbf{0}$, for otherwise the second term in Eq. (39) would be unbounded below. Thus, the dual problem is

$$\begin{aligned} \max_{\mathbf{Y}} g(\mathbf{Y}) &= \textstyle\sum_{i=1}^{11} g_i(\mathbf{Y}_i) \\ \text{s.t.} \quad \mathbf{Y}^T\mathbf{F} &= \mathbf{0}. \end{aligned} \tag{40}$$

The slave problem in Eq. (38) and the master/dual problem in Eq. (40) can be solved iteratively. Note that the slave problems are totally separable and they have a one-to-one correspondence to the sub-problems described in the previous section. So each sub-problem can be solved independently by an individual agent and the master problem can be tackled by some coordinator agent which collects and broadcasts information from and to the individual agents.

*Definition 1:* $s_g(\mathbf{Y}) \in \mathbb{R}^{n\times 1}$ is a subgradient of a convex function $g : \mathbb{R}^n \to (-\infty, \infty)$ at $\mathbf{Y} \in \mathrm{dom}(g)$ if

$$g(\overline{\mathbf{Y}}) + s_g(\mathbf{Y})^T(\mathbf{Y} - \overline{\mathbf{Y}}) \leq g(\mathbf{Y})$$

for all $\overline{\mathbf{Y}} \in \mathrm{dom}(g)$. Should $\overline{\mathbf{Y}}$ an $\mathbf{Y}$ switch role here?

*Theorem 2:* Let $\mathbf{X}_i^*$ be the optimal point in Eq. (38) corresponding to $\mathbf{Y}_i$. Then functions $g_i$'s are convex ([19])(If proof of this statement is in the reference, it should be stated in the proof, not the theorem body) and $\mathbf{X}_i^*$ is a subgradient of $g_i$ at $\mathbf{Y}_i$.

*Proof:* Let $\overline{\mathbf{Y}}_i$ be any point in $\mathrm{dom}(g_i)$. Then

$$\begin{aligned} g(\mathbf{Y}_i) - \mathbf{X}_i^{*T}(\mathbf{Y}_i - \overline{\mathbf{Y}}_i) &= f_i(\mathbf{X}_i^*) + \mathbf{Y}_i^T\mathbf{X}_i^* - \mathbf{X}_i^{*T}(\mathbf{Y}_i - \overline{\mathbf{Y}}_i) \\ &= f_i(\mathbf{X}_i^*) + \overline{\mathbf{Y}}_i^T\mathbf{X}_i^* \\ &\geq \min_{\mathbf{X}_i}\left\{f_i(\mathbf{X}_i) + \overline{\mathbf{Y}}_i^T\mathbf{X}_i\right\} = g(\overline{\mathbf{Y}}_i). \end{aligned}$$

■

For non-differentiable functions, the subgradient plays the same role as a gradient does for differentiable functions. Subgradient does not need to be unique but for differentiable functions, subgradient coincides with the gradient which becomes unique. Assuming $\mathbf{X}_{k+1}^*$ contain all the optimal solutions in Eq. (38) at $\mathbf{Y}_k$, then the ascent direction to the dual problem in Eq. (40) is simply the projection of $\mathbf{X}_{k+1}$ onto the hyperplane defined by $\mathbf{Y}^T\mathbf{F} = \mathbf{0}$, which is $\left(\mathbf{I} - \mathbf{F}(\mathbf{F}^T\mathbf{F})^{-1}\mathbf{F}^T\right)\mathbf{X}_{k+1}$. From Eq. (35), an estimate of $\mathbf{Z}$ would be

$$\mathbf{Z}_{k+1} = (\mathbf{F}^T\mathbf{F})^{-1}\mathbf{F}^T\mathbf{X}_{k+1}.$$

Due to the special structure of $\mathbf{F}$, $\mathbf{Z}_{k+1}$ is essentially the average of the local copies of different variables from $\mathbf{X}_{k+1}$. So the ascent direct becomes $\mathbf{X}_{k+1} - \mathbf{F}\mathbf{Z}_{k+1}$, which is the violation of consensus constraints in Eq. (35). The dual update is

$$\mathbf{Y}_{k+1} = \mathbf{Y}_k + \alpha(\mathbf{X}_{k+1} - \mathbf{F}\mathbf{Z}_{k+1}),$$

which tries to penalize more on the larger element mismatches and aims at reducing the mismatches during the next iteration. $\alpha$ is the step size that can be adjusted to change the level of penalization.

**Algorithm 1** : *Subgradient*

- **initialize** $\mathbf{Z}$, $\mathbf{X} = \mathbf{F}\mathbf{Z}$, $\mathbf{Y} = \mathbf{0}$, k=1
- **loop**
  - **set** $k = 1$
  - {*Solve each sub-problem*}
  - **for** $i = 1$ **to** 11 **do**
    - $\mathbf{X}_i^{[k+1]} = \mathrm{argmin}_{\mathbf{X}_i}\left\{f_i(\mathbf{X}_i) + \left(\mathbf{Y}_i^{[k]}\right)^T\mathbf{X}_i\right\}$
  - **end for**
  - {*Estimate of the global variable*}
  - $\mathbf{Z}^{[k+1]} = (\mathbf{F}^T\mathbf{F})^{-1}\mathbf{F}^T\mathbf{X}^{[k+1]}$
  - {*Dual update*}
  - $\mathbf{Y}^{[k+1]} = \mathbf{Y}^{[k]} + \alpha\left(\mathbf{X}^{[k+1]} - \mathbf{F}\mathbf{Z}^{[k+1]}\right)$
  - **if** (termination criterion met) **then**
    - Break
  - **else**
    - $k = k + 1$
  - **end if**
- **end loop**

*2) Alternating direct multiplier method:* An augmented Lagrangian shown in Eq. (41)

$$\begin{aligned} &\mathbf{L}(\mathbf{X},\mathbf{Z},\mathbf{Y}) \\ = &\textstyle\sum_{i=1}^{11} f_i(\mathbf{X}_i) + \mathbf{Y}^T(\mathbf{X} - \mathbf{F}\mathbf{Z}) + (\sigma/2)\|\mathbf{X} - \mathbf{F}\mathbf{Z}\|_2^2 \\ = &\textstyle\sum_{i=1}^{11} f_i(\mathbf{X}_i) + \frac{\sigma}{2}\|\mathbf{X} - \mathbf{F}\mathbf{Z} + \frac{\mathbf{Y}}{\sigma}\|_2^2 - \frac{1}{2\sigma}\|\mathbf{Y}\|_2^2 \end{aligned} \tag{41}$$

is considered in this alternating direction multiplier method (ADMM). It has an additional quadratic penalty to the consensus constraint violations compared with the Lagrangian in Eq. (36). Again, let $\mathbf{Y}_i$ denote the sub-vector of $\mathbf{Y}$ corresponding to the *Sub-problem (i)*. Then the augmented Lagrangian can be reformulated as

$$\begin{aligned} &\mathbf{L}(\mathbf{X},\mathbf{Z},\mathbf{Y}) \\ = &\textstyle\sum_{i=1}^{11}\left\{f_i(\mathbf{X}_i) + \frac{\sigma}{2}\|\mathbf{X}_i - \mathbf{F}_i\mathbf{Z} + \frac{\mathbf{Y}_i}{\sigma}\|_2^2\right\} - \frac{1}{2\sigma}\|\mathbf{Y}\|_2^2. \end{aligned} \tag{42}$$

It can be noticed that, unlike in Eq. (37), the slave problem in Eq. (42) is not decomposable between $\mathbf{X}_i$ and $\mathbf{Z}$ due to the existence of the quadratic term. An alternating direction procedure is taken which firstly solves the $\mathbf{X}_i$ problem while fixing $\mathbf{Z}$, and then solves for $\mathbf{Z}$ with fixed $\mathbf{X}$. It is trivial from Eq. (41) to see that the optimal value $\mathbf{Z}^*$ satisfies

$$\mathbf{X} - \mathbf{F}\mathbf{Z}^* + \frac{\mathbf{Y}}{\sigma} = \mathbf{0},$$

which gives an estimate $\mathbf{Z}^* = (\mathbf{F}^T\mathbf{F})^{-1}\mathbf{F}^T\left(\mathbf{X} + \frac{\mathbf{Y}}{\sigma}\right)$.

Similar to the subgradient method, the dual ascent direction is still on $\mathbf{X} - \mathbf{F}\mathbf{Z}$. However, ADMM uses the penalty multiplier $\sigma$ as the step size for the dual update.

**Algorithm 2** : *ADMM*

1: **initialize** $\mathbf{Z}$, $\mathbf{X} = \mathbf{F}\mathbf{Z}$, $\mathbf{Y} = \mathbf{0}$, k=1
2: **loop**
3: **set** $k = 1$
{*Solve each sub-problem*}
4: **for** $i = 1$ **to** 11 **do**
5: $\mathbf{X}_i^{[k+1]} = \mathrm{argmin}_{\mathbf{X}_i}\left\{f_i(\mathbf{X}_i) + \frac{\sigma}{2}\|\mathbf{X}_i - \mathbf{F}_i\mathbf{Z}^{[k]} + \frac{\mathbf{Y}_i^{[k]}}{\sigma}\|_2^2\right\}$
6: **end for**
{*Estimate of the global variable*}
7: $\mathbf{Z}^{[k+1]} = (\mathbf{F}^T\mathbf{F})^{-1}\mathbf{F}^T\left(\mathbf{X}^{[k+1]} + \frac{\mathbf{Y}^{[k]}}{\sigma}\right)$
{*Dual update*}
8: $\mathbf{Y}^{[k+1]} = \mathbf{Y}^{[k]} + \sigma\left(\mathbf{X}^{[k+1]} - \mathbf{F}\mathbf{Z}^{[k+1]}\right)$
9: **if** (termination criterion met) **then**
10: Break
11: **else**
12: $k = k+1$
13: **end if**
14: **end loop**

*Sub-problems (3-11)* in Step 5 of the ADMM algorithm can be reformulated as

$$\begin{array}{ll}\min_{\mathbf{X}_i} & \|\mathbf{X}_i - \mathbf{F}_i\mathbf{Z}^{[k]} + \frac{\mathbf{Y}_i^{[k]}}{\sigma}\|_2^2 \\ \text{s.t.} & f_i(\mathbf{X}_i) = 0,\end{array} \tag{43}$$

which is also equivalent to the Euclean projection of $\mathbf{F}_i\mathbf{Z}^{[k]} + \mathbf{Y}_i^{[k]}/\sigma$ onto the constraint hyper-surface:

$$\mathbf{X}_i^{[k+1]} = \Pi_{C_i}\left(\mathbf{F}_i\mathbf{Z}^{[k]} - \mathbf{Y}_i^{[k]}/\sigma\right), \tag{44}$$

where $C_i$ is the hyper-surface defined by $f_i(\mathbf{X}_i) = 0$ or equivalently, by the corresponding equality constraint among Eq.'s (13) to (17). In addition, if the corresponding equality constraint is linear in $\mathbf{X}_i$, say $\mathbf{D}_i^T\mathbf{X}_i - b = 0$, analytic solution can be obtained as

$$\mathbf{X}_i^{[k+1]} = \mathbf{F}_i\mathbf{Z}^{[k]} - \mathbf{Y}_i^{[k]}/\sigma - \frac{\mathbf{D}_i^T(\mathbf{F}_i\mathbf{Z}^{[k]} - \mathbf{Y}_i^{[k]}/\sigma) - b}{\|\mathbf{D}_i\|_2^2}\mathbf{D}_i.$$

The equality constraint for *Sub-problem (9)* is linear so the corresponding sub-problem can be easily solved. This is important because this specific constraint physically corresponds to an air splitter or merger and for centralized air conditioning system with large number of air splits, the computation requirement does not increase much due to this nice property.

*3) Comparison of subgradient method and ADMM:* The subgradient method is relatively easy to implement and computational burden is slightly lower. In addition, the slave problems are totally decomposible with the subgradient method while for ADMM they are not and an alternating direction procedure is needed. However, ADMM has better robustness. When the two algorithms were tested for the case study, the subgradient method failed frequently if the step size $\alpha$ was not small enough. That was because for poorly chosen $\alpha$, the cost function in Step 5 of **Algorithm 1** was unbounded below and optimization drove the variable to infinity. **Algorithm 2** does not have this issue since the cost function in Step 5 incorporates a quadratic penalty which makes sure the variable does not deviate too far from the nominal value. Although convergence of the algorithms is not considered in this paper, ADMM requires much weaker assumptions to guarantee convergence than the subgradient method. So ADMM is a preferred method and the results shown in the following section were obtained with ADMM.

## V. Case Study

The system that is considered as a case study is depicted in Fig. 1. There is one air handling unit (AHU) that provides conditioned air to three VAV boxes and each box controls an individual thermal zone. The temperature of each thermal zone is regulated by varying the entering airflow rate through modulation of the air damper in the corresponding VAV box. Only a cooling scenario is considered and a DX cooling system is used to cool the air that is delivered to the conditioned zones.

### A. Problem description

The problem has already been formulated in Section IV. In real implementation, the problem can be solved with the aforementioned algorithms on the fly under varying operating conditions. This paper only presents the optimization results for one specific operating condition to demonstrate the validity of the proposed multi-agent coordination method. The operating conditions that are considered are

$$\begin{array}{ll} w_{ma} = 0.009 & T_{ma} = 26°\text{C} \\ T_{z,1} = 25°\text{C} & T_{z,2} = 24.5°\text{C} \\ T_{z,3} = 24°\text{C} & T_{amb} = 31°\text{C} \\ Q_{sen,1} = 28\text{kW} & Q_{sen,2} = 26\text{kW} \\ Q_{sen,3} = 23\text{kW} & \end{array}$$

### B. Multi-agent framework

Fig. 2 shows the framework of the designed multi-agent coordination methodology. The bottom layer corresponds to the sensing network that collects the required operating conditions. The layer above includes all the basic agents which represent behaviors of all the components. The basic agents could be implemented by equipment manufacturers or could be identified on the fly from collected data. This paper assumes the basic agents are perfect representations of the corresponding component characteristics. In other words, the control model is an exact match to the plant behavior.

On top of the basic-agent layer, there is an optimizer-agent layer which is responsible for self-optimization of each sub-problem described in Section IV-B. Each optimizer agent calls the related basic agents iteratively to find the optimal point, independently and in parallel with the other optimization agents. Each slave problem only optimizes their local copies of variables ($\mathbf{X}_i$). For example, $m_a$ has a local variable ($m_a^{(1)}$) for the DX agent and another local variable ($m_a^{(2)}$) for the fan agent. They are two different variables with the same physical meaning, hence the term local copies of $m_a$. The DX agent only optimizes $m_a^{(1)}$ and the fan agent only optimizes $m_a^{(2)}$. A coordination layer is needed as shown in the top layer of Fig. 2. It collects the local copies of all the variables, updates the dual variables accordingly and feeds the updated dual variables back to the optimizer agents to let them re-optimize with respect to the updated information. This process iterates until certain termination criteria are met.

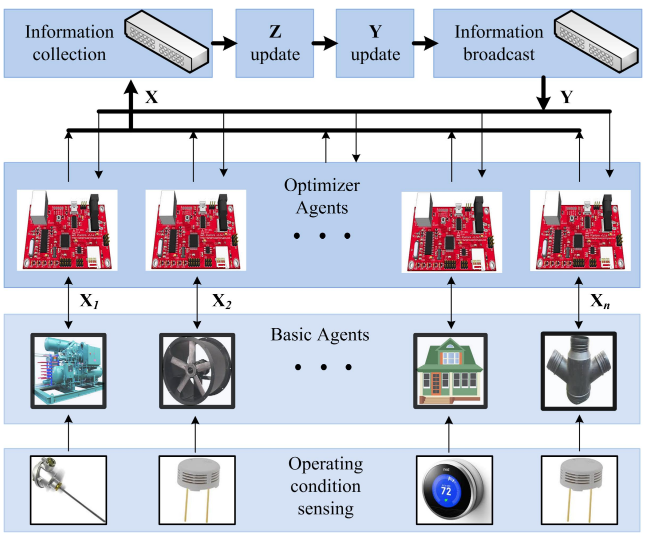


Fig. 2. Multi-agent coordination framework.

It needs to be noted that the multi-agent structure could be constructed automatically after some pre-configuration. Field engineers would just need to specify the connections among different agents and the algorithm extraction can be automated. This will reduce the site-specific engineering significantly and make the proposed methodology a plug-and-play solution.

### C. Optimization results

ADMM was used as the mechanism to run the multi-agent coordination. As a first step, the variables were normalized to have comparative scales to make sure that the penalties to the consensus violations were assigned fairly. The damper openings were scaled down by 10 and the pressures were scaled down by 25. Fig. 3 plots the variation of the normalized variables as the coordination procedure proceeds. Some variables have local copies among several optimizer agents and they are reflected by multiple curves within the same subplot.

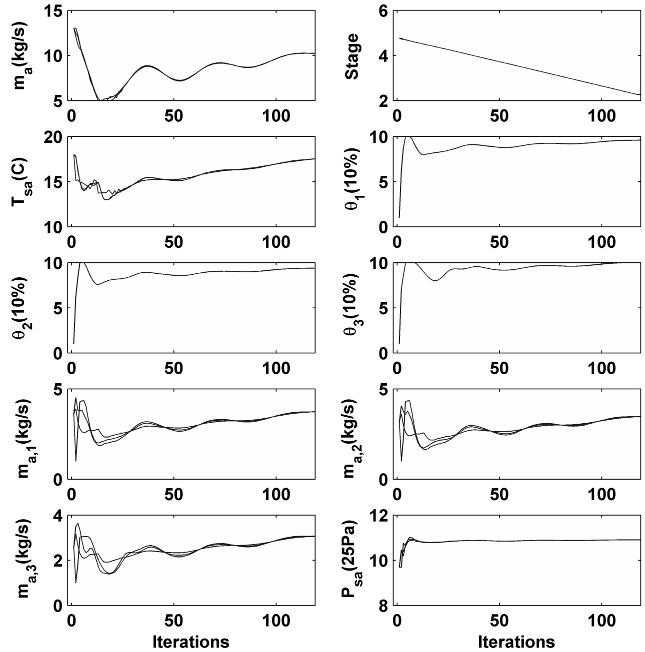


Fig. 3. Evolution of local variables

Plugging the estimated global variables $\mathbf{Z}$ into the centralized problem formulated in Section IV-A, the cost function as well as the constraint satisfaction were evaluated at different iterations. The top plot in Fig. 4 shows the constraint violations and each curve corresponds to one constraint listed from Equations (13) to (17). The constraint violations were normalized with some nominal values. The bottom figure plots the evolution of the total power consumption, which shows up as the cost function in Eq. (12).

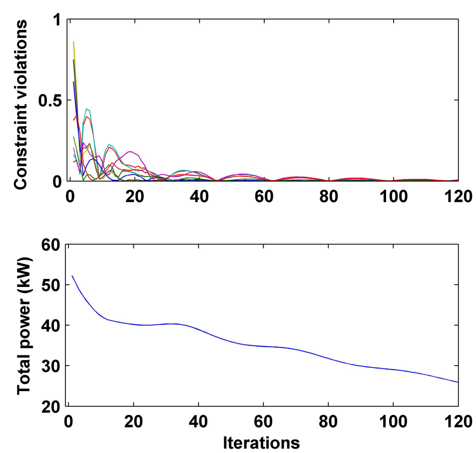


Fig. 4. Evolution of power and constraint violations

The centralized optimization has eleven variables and many constraints. Through some engineering, the problem was reduced to a 2 degree-of-freedom optimization that is relatively easy to solve ([13]) and the optimal point for this specific operating condition was: $Stage = 2.19$, $T_{sa} = 17.8°\text{C}$, $\theta_1 = 0.96$, $\theta_2 = 0.94$, $\theta_3 = 1$, $P_{sup} = 300\text{Pa}$ with a minimum total power consumption of 24.9kW. To assess the energy savings, a baseline control strategy for the conventional control was considered with $T_{sup} = 14°C$ and $P_{sa} = 280Pa$. The energy consumption with the baseline strategy is 43.5kW. So there is a 42.7% energy savings potential and the multi-agent control was able to find a solution with 25.9kW power consumption which covers 94.6% of the maximum energy savings.

Again, the problem considered is highly nonconvex so convergence is not guaranteed. Different initial guesses would give different types of solutions (e.g., local minimums, points without consensus or even divergent solutions). A practical approach is to implement a multi-start scheme and find the consensus solution with the minimum power. Testing results show that 60% of the randomly generated initial points converge to some solution that covers at least 92% of the maximum energy savings.

TABLE I

NOMENCLATURE

| Variables | Definition |
|---|---|
| $A_{damper}$ | Damper cross sectional area (m$^2$) |
| $\mathbf{A}, \mathbf{C}$ | State-space matrices (-) |
| $\mathbf{B}_w, \mathbf{B}_u$ | State-space **B** matrices corresponding to disturbance and control inputs, respectively (-) |
| $c_p$ | Specific heat of air (kJ/kg-K) |
| $m_a$ | Total air mass flow rate (kg/s) |
| $m_{a,i}$ | Air mass flow rate through VAV box $i$ (kg/s) |
| $ESP$ | External static pressure of fan (inW.C. or Pa) |
| $Pow$ | Power consumption (kW) |
| $P_{ma}$ | Cooling coil inlet air pressure (inW.C. or Pa) |
| $P_{sa}$ | Fan outlet air pressure (inW.C. or Pa) |
| $P_{z,i}$ | Air pressure in zone $i$ (inW.C. or Pa) |
| $Q_{sen,i}$ | Sensible cooling to zone $i$ (kW) |
| $SHR$ | Sensible heat ratio of cooling coil (ratio of sensible to total capacity) (-) |
| $Stage$ | Compressor stage (-) |
| $T_{amb}$ | Ambient temperature (°C) |
| $T_{la}$ | Cooling coil outlet air temperature (°C) |
| $T_{ma}$ | Cooling coil inlet air temperature (°C) |
| $T_{sa}$ | Supply (fan outlet) air temperature (°C) |
| $T_{z,i}$ | Air temperature of zone $i$ (°C) |
| $\mathbf{X}$ | Vector of all local copies of the design variables |
| $\mathbf{Y}$ | Dual variable vector |
| $\mathbf{Z}$ | Vector of the design variables |
| $\mathbf{X}_i, \mathbf{Y}_i$ | Sub-vector corresponding to *Sub-problem (i)* |
| $w_{ma}$ | Cooling coil inlet air humidity ratio (kg water/kg air) |
| $\mathbf{x}_k, \mathbf{y}_k$ | State and output vectors at time step $k$ (-) |
| $\mathbf{y}_{k+1}^{sp}$ | Desired (setpoint) output at time step $k+1$ (-) |
| $\alpha$ | Step size for dual update |
| $\theta_i$ | Damper opening in VAV box $i$ (%) |
| $\rho$ | Air density (kg/m$^3$) |
| $\sigma$ | Factor for the augmented multiplier |
| **Subscripts** | **Definition** |
| $fan$ | Fan |
| $DX$ | DX unit |
| **Supscripts** | **Definition** |
| $(i)$ | The $i$th copy of the corresponding local variable |
| $[i]$ | The $i$th iteration of optimization |

This table should appear earlier, say around the problem formulation section

## VI. CONCLUSIONS

This paper presented a multi-agent control approach for centralized air-conditioning systems. A three-zone building served by one air handling unit was considered as a case study and an optimization problem was formulated to find the optimal control under any operating condition. The problem was reformulated as a consensus distributed optimization problem to decompose the centralized problem into several sub-problems, each of which can be solved by an individual agent. Subgradient ADMM methods were implemented under a slave-master scheme which forms the mechanism of inter-agent coordination. The ADMM method showed better robustness and was able to find a close-to-optimal point under the test operating condition.

Along with the distributed algorithm, a general multi-agent framework was briefly described. With the help of this framework, the construction of a multi-agent system can be automated and requires moderate upfront engineering effort, if the component agents come pre-equiped with the needed mechanism. This is critical in real building deployments since buildings differ from one to another which makes plug-and-play solutions highly desirable.

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