D-Optimal Experimental Design for Calibration Deterministic Errors of DTG

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***Abstract*—This paper presents a novel experimental design for greatly improving the calibration accuracy of the acceleration-insensitive and the acceleration-sensitive biases of the dynamically tuned gyroscopes. A novel calibration procedure based on D-optimality criteria and real code genetic algorithm (RCGA) is established. In order to reduce experiment cost, the D-optimality criteria is constructed with constraints. The experiment results show that the new experimental procedure achieves better calibration results with only 12-position which is half the cost of the widely adopted 24-position experimental procedure. The experimental procedure proposed in this paper has been proved to be simple and efficient in calibrating Dynamically Tuned Gyro (DTG).**

I. Introduction

The Two-Degree-of-Freedom Dynamically Tuned Gyro (DTG) is the most widely adopted key component in platform inertial navigation system. Because of its inherent simplicity, DTG is being broadly applied to both military and civilian uses. Although recently ring laser gyroscopes are emerging as the new generation gyro, DTG is still the most widely used and is playing important roles in the field. How to effectively calibrate and compensate the deterministic error sources of DTG is crucial in improving the precision and performance of DTG-based navigation systems.

There are some existing works on the conventional calibration procedure design. A widely regarded 24-position test procedure design was designed for DTGs to achieve a high accuracy [1][2]. IEEE has presented a 8-position test procedures for testing DTG to extract estimates of the acceleration insensitive bias and the acceleration sensitive bias [3]. A six-position static and rate tests illustrated by Syed are among the most commonly used calibration methods for IMUs [4]. Zhang [5] proposed another 18-position calibration procedure to calibrate a static IMU without the need of an accurate turntable. Another method depended on the Earth's gravity as a stable physical calibration without external equipment was proposed to calibration IMUs [6]. However, all of the calibration processes used empirical parameter-setting and provided no evidence on optimization in terms of accuracy and cost. To the authors' best knowledge, the problem of designing an optimal calibration experiment for DTGs has not been studied before. It is clearly desirable to have a theory-based systematic approach to optimize the calibration procedure with maximized accuracy and minimized cost.

To achieve this goal, optimal calibration procedure based on D-optimal designs is proposed. Optimal designs for the estimation of some specific nonlinear function of the parameters have also been studied [7-10]. The fact that the design tools can be used for many different fields illustrates its usefulness. There are a number of criteria for the optimal designs and several algorithms for constructing optimal designs [14]. The most commonly and widely used criterion is D-Optimality. In the earliest years, searching methods for constructing D-optimal designs are Monte Carlo algorithms and heuristics algorithms. A comparison of these algorithms can be found in [11]. More recently, genetic algorithm (GA) for solving D-optimal problems is widely adopted [16], it has high performance of global optimization and search efficiency. In this paper, real code genetic algorithm is used for searching optimal positions of calibration procedure.

In our previous work [13], a D-Optimal experimental design was proposed to optimize the accuracy of estimates and minimize the cost. But the deterministic error model neglected the acceleration square sensitive drift and the searching method has low efficiency. In this paper, a complete deterministic error model and more efficient search algorithms are deployed for the experimental design, and the experiment results illustrate that the designed optimal 12-position experimental procedure reduces the fit uncertainty by 37% for the ox axis and 61% for the oy axis, respectively, compared with the traditional 24-position design, while the experimental cost is only half that of the 24-position experimental procedure.

Our contribution is three-fold. First, a deterministic error model for DTG where the experimental inputs are Euler angles of three axis platform or turntable is designed. This error model is a critical element in the D-optimal identification problem. Second, an optimal experiment by using D-Optimal experimental design criteria and RCGA is designed, in which all of the deterministic errors of DTG are optimally estimated under the designed D-Optimal criteria. The experimental results show that the approach is effective and promising.

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The last contribution is that our proposed methodology can be easily extended to test other kinds of gyroscopes.

II. MODEL EQUATIONS

For realization of calibration procedure, it is necessary to specify the relationship between the input elements and the gyro drift outputs, and identify the coefficients in the drift equations. The DTGs drift model in IEEE standard [3], which includes many general error terms, is usually regarded as a general and complex model. In this paper, we focus on how to calibrate the acceleration-insensitive bias and the acceleration-sensitive bias which are considered to be the major error terms that influence the performance of DTGs. [3][4]

Suppose the oz axis coincides with the DTG's spin axis, the ox axis and the oy axis are output axes of DTG. Axes OX, OY, OZ are the inner axis, the middle axis and the outer axis of the turntable, respectively. The angular rate measurements $\tilde{\omega}_X$ and $\tilde{\omega}_Y$ may be expressed mathematically in terms of the true angular rate and the drift error of the DTG in the test table frame ($OXYZ$) as follows:

$$\begin{aligned} D(X) &= D(X)_F + D(X)_X a_X + D(X)_Y a_Y + D(X)_Z a_Z \\ &\quad + D(X)_{XZ} a_X a_Z + D(X)_{YZ} a_Y a_Z + \eta_X \\ D(Y) &= D(Y)_F + D(Y)_X a_X + D(Y)_Y a_Y + D(Y)_Z a_Z \\ &\quad + D(Y)_{XZ} a_X a_Z + D(Y)_{YZ} a_Y a_Z + \eta_Y \end{aligned} \tag{1}$$

where $D(X)$ and $D(Y)$ are the DTG drift rates about the OX and OY axes; a_X, a_Y and a_Z are the accelerations along the OX, OY and OZ axes; $D(X)_F$ and $D(Y)_F$ are the acceleration-insensitive bias coefficients; $D(X)_X$ and $D(Y)_X$ are the drift rate coefficients about OX and OY axes, attributable to acceleration along the OX axis; $D(X)_Y$ and $D(Y)_Y$ are the drift rate coefficients about the OX and OY axes, attributable to acceleration along the OY axis; $D(X)_Z$ and $D(Y)_Z$ are the drift rate coefficients about the OX and OY axes, attributable to acceleration along the OZ axis; $D(X)_{XZ}$ and $D(Y)_{XZ}$ are the drift rate coefficients about the OX and OY axes , attributable to acceleration along the X and Z axis; $D(X)_{YZ}$ and $D(Y)_{YZ}$ are the drift rate coefficients about the OX and OY axes, attributable to acceleration along the Y and Z axis; n_X and n_Y are zero-mean random biases.

The angular rate measurement $\tilde{\omega}_x$ and $\tilde{\omega}_Y$ provided by DTGs may be expressed mathematically in terms of the true input angular rate and the drift terms in the DTG-fixed frame ($oxyz$) as follows:

$$\begin{aligned} \tilde{\omega}_x &= \omega_x + D(x) \\ \tilde{\omega}_y &= \omega_y + D(y) \end{aligned} \tag{2}$$

where $\tilde{\omega}_x$ and ω_y are the turn rates of the gyroscope about its input axes. Under the laboratory test, they are the earth rate projected in the sensor-fixed frame. $D(X)$ and $D(Y)$ are the drift rates along its input axes.

Based on the dynamics of the DTG, the input rate about the ox axis will be balanced by y torque, and the input rate about the oy axis will be balanced by x torque. Thus, the relationship between the currents applied to the torques and the angular rate measurements is shown below:

$$\begin{aligned} \tilde{\omega}_x &= K_{Ty} i_y \\ \tilde{\omega}_y &= K_{Tx} i_x \end{aligned} \tag{3}$$

where K_{Tx} and K_{Ty} are composite command rate scale factor about its input axes, i_x and i_y are torque currents about its input axes.

The DTG torques frame ($oxyz$) is usually a nonorthogonal frame, while the test table frame ($OXYZ$) is an orthogonal frame. ε is the angle between the OX axis and the ox axis, and ζ is the angle between the OY axis and the oy' axis. The angle $\frac{\pi}{2}+\zeta$ is between the ox axis and the oy axis. The angular rate measurements $\tilde{\omega}_X$ and $\tilde{\omega}_Y$ in the test table coordinates are the nonorthogonal transformation from torques to test table coordinates by:

$$\begin{bmatrix} \tilde{\omega}_X \\ \tilde{\omega}_Y \end{bmatrix} = \begin{bmatrix} \cos(\varepsilon) & -\sin(\varepsilon+\xi) \\ \sin(\varepsilon) & \cos(\varepsilon+\xi) \end{bmatrix} \begin{bmatrix} \tilde{\omega}_x \\ \tilde{\omega}_y \end{bmatrix} \tag{4}$$

Taking into account equations (1) (2) (3) and (4), we have

$$\begin{bmatrix} i_y \\ i_x \end{bmatrix} = \begin{bmatrix} \dfrac{\cos(\varepsilon+\xi)}{K_{Ty}\cos\xi} & \dfrac{\sin(\varepsilon+\xi)}{K_{Ty}\cos\xi} \\ -\dfrac{\sin\varepsilon}{K_{Tx}\cos\xi} & \dfrac{\cos\varepsilon}{K_{Tx}\cos\xi} \end{bmatrix} \begin{bmatrix} \tilde{\omega}_X \\ \tilde{\omega}_Y \end{bmatrix} \times \begin{bmatrix} D(X)_F + D(X)_X a_X + D(X)_Y a_Y + D(X)_Z a_Z + \omega_X + \eta_X \\ D(Y)_F + D(Y)_X a_X + D(Y)_Y a_Y + D(Y)_Z a_Z + \omega_Y + \eta_Y \end{bmatrix} \tag{5}$$

where ω_X and ω_Y are the earth rates projected into the test table frame.

Let

$$\begin{bmatrix} U_1 & U_2 \\ V_1 & V_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\cos(\varepsilon+\xi)}{K_{Ty}\cos\xi} & \dfrac{\sin(\varepsilon+\xi)}{K_{Ty}\cos\xi} \\ -\dfrac{\sin\varepsilon}{K_{Tx}\cos\xi} & \dfrac{\cos\varepsilon}{K_{Tx}\cos\xi} \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} U_0 \\ V_0 \end{bmatrix} = \begin{bmatrix} U_1 & U_2 \\ V_1 & V_2 \end{bmatrix} \begin{bmatrix} D(X)_F \\ D(Y)_F \end{bmatrix}$$

$$\begin{bmatrix} U_3 & U_4 \\ V_3 & V_4 \end{bmatrix} = \begin{bmatrix} U_1 & U_2 \\ V_1 & V_2 \end{bmatrix} \begin{bmatrix} D(X)_X & D(X)_Y \\ D(Y)_X & D(Y)_Y \end{bmatrix},$$

$$\begin{bmatrix} U_5 \\ V_5 \end{bmatrix} = \begin{bmatrix} U_1 & U_2 \\ V_1 & V_2 \end{bmatrix} \begin{bmatrix} D(X)_Z \\ D(Y)_Z \end{bmatrix}, \qquad (7)$$

$$\begin{bmatrix} U_6 & U_7 \\ V_6 & V_7 \end{bmatrix} = \begin{bmatrix} U_1 & U_2 \\ V_1 & V_2 \end{bmatrix} \begin{bmatrix} D(X)_{XZ} & D(X)_{YZ} \\ D(Y)_{XZ} & D(Y)_{YZ} \end{bmatrix},$$

$$\begin{bmatrix} \mu_X \\ \mu_Y \end{bmatrix} = \begin{bmatrix} U_1 & U_2 \\ V_1 & V_2 \end{bmatrix} \begin{bmatrix} \eta_X \\ \eta_Y \end{bmatrix},$$

It is now possible to rewrite equation (5) as

$$\begin{bmatrix} i_y \\ i_x \end{bmatrix} = \begin{bmatrix} U_0 & U_1 & U_2 & U_3 & U_4 & U_5 & U_6 & U_7 \\ V_0 & V_1 & V_2 & V_3 & V_4 & V_5 & V_6 & V_7 \end{bmatrix} \times \begin{bmatrix} 1 & \omega_X & \omega_Y & a_X & a_Y & a_Z & a_X a_Z & a_Y a_Z \end{bmatrix}^T + \begin{bmatrix} \mu_X \\ \mu_Y \end{bmatrix} \qquad (8)$$

Therefore, calibrating the drift of DTG depends on estimating the model's unknown parameters. The parameters in (8) to be estimated are $\vec{U} = [U_0 \ U_1 \ U_2 \ U_3 \ U_4 \ U_5 \ U_6 \ U_7]^{\mathrm{T}}$ about the OX axis and $\vec{V} = [V_0 \ V_1 \ V_2 \ V_3 \ V_4 \ V_5 \ V_6 \ V_7]^{\mathrm{T}}$ about the OY axis.

Model expression (8) involves acceleration square sensitive drift terms. In order to realize the independence of model terms, mathematical transformation must be done.

Let three-axis turn table initial coordinate system (r-frame) is up, north, and the west. Coordinate system after rotation is defined as the b-frame, thus

$$g^r = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T, \qquad \omega^r = \begin{bmatrix} \omega_U & \omega_N & 0 \end{bmatrix}^T \qquad (9)$$

where g^r are accelerations of gyro relative to r –frame (unit: g), ω^r are angular rates of gyro relative to r –frame (unit: deg/hour), ω_U , ω_N are components of the earth rotation angular rate about up and north, respectively.

According to Euler angles transformation, the three-axis turn table will reach any position, so the error terms of every position can be described by the Euler angles independently.

When three rotations performed in the order θ, γ, ϕ as follows:

- a rotation is made of the r –frame (x_r, y_r, z_r) about the x_r axis through an angle θ .

$$\begin{bmatrix} x'_r \\ y'_r \\ z'_r \end{bmatrix} = A \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} \qquad (10)$$

- A second rotation to be made by rotating the (x'_r, y'_r, z'_r) coordinates about the y'_r axis by the angle γ .

$$\begin{bmatrix} x''_r \\ y''_r \\ z''_r \end{bmatrix} = B \begin{bmatrix} x'_r \\ y'_r \\ z'_r \end{bmatrix} = \begin{bmatrix} \cos(\gamma) & 0 & -\sin(\gamma) \\ 0 & 1 & 0 \\ \sin(\gamma) & 0 & \cos(\gamma) \end{bmatrix} \begin{bmatrix} x'_r \\ y'_r \\ z'_r \end{bmatrix} \qquad (11)$$

- The third rotation about z''_r axis by the angle ϕ , rotate the (x''_r, y''_r, z''_r) coordinates to the b-frame (x_b, y_b, z_b). The transformation equation is:

$$\begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = D \begin{bmatrix} x''_r \\ y''_r \\ z''_r \end{bmatrix} = \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x''_r \\ y''_r \\ z''_r \end{bmatrix} \qquad (12)$$

Using equations (10), (11) and (12), the vector in b-frame is given by

$$\begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = DBA \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} \qquad (13)$$

From equation (11) and (13), the angular rates and the accelerations of test position are:

$$a_X = \cos(\phi)\cos(\gamma) \qquad (14)$$

$$a_Y = -\sin(\phi)\cos(\gamma) \qquad (15)$$

$$a_Z = \sin(\gamma) \qquad (16)$$

$$\omega_X = \cos(\phi)\cos(\gamma)\omega_U + (\cos(\phi)\sin(\gamma)\sin(\theta) + \sin(\phi)\cos(\theta))\omega_N \qquad (17)$$

$$\omega_Y = -\sin(\phi)\cos(\gamma)\omega_U + (-\sin(\phi)\sin(\gamma)\sin(\theta) + \cos(\phi)\cos(\theta))\omega_N \qquad (18)$$

$$a_X a_Z = \cos(\phi)\frac{\sin(2\gamma)}{2} \qquad (19)$$

$$a_Y a_Z = -\sin(\phi)\frac{\sin(2\gamma)}{2} \qquad (20)$$

Taking into account equations (8) and (14) ~ (20), the deterministic error model for DTG is (21),

$$\begin{bmatrix} i_y \\ i_x \end{bmatrix} = \begin{bmatrix} U_0 & U_1 & U_2 & U_3 & U_4 & U_5 & U_6 & U_7 \\ V_0 & V_1 & V_2 & V_3 & V_4 & V_5 & V_6 & V_7 \end{bmatrix} \times \begin{bmatrix} 1 \\ \cos(\phi)\cos(\gamma)\omega_U + (\cos(\phi)\sin(\gamma)\sin(\theta) + \sin(\phi)\cos(\theta))\omega_N \\ -\sin(\phi)\cos(\gamma)\omega_U + (-\sin(\phi)\sin(\gamma)\sin(\theta) + \cos(\phi)\cos(\theta))\omega_N \\ \cos(\phi)\cos(\gamma) \\ -\sin(\phi)\cos(\gamma) \\ \sin(\gamma) \\ \cos(\phi)\frac{\sin(2\gamma)}{2} \\ -\sin(\phi)\frac{\sin(2\gamma)}{2} \end{bmatrix} + \begin{bmatrix} \mu_X \\ \mu_Y \end{bmatrix} \qquad (21)$$

where $\omega_N = \Omega \cos L, \omega_U = \Omega \sin L$. Ω is the earth rate and L is the local latitude. In this paper, the earth rate is 15.041 deg/h and the local latitude is 39.9136 degree.

The model expresses the relationship between the rotation angles and the gyro drift outputs. The parameters in (21) to be estimated are $\vec{U} = [U_0 \ U_1 \ U_2 \ U_3 \ U_4 \ U_5 \ U_6 \ U_7]^{\mathrm{T}}$ about the OX axis and $\vec{V} = [V_0 \ V_1 \ V_2 \ V_3 \ V_4 \ V_5 \ V_6 \ V_7]^{\mathrm{T}}$ about the OY axis.

III. D-Optimal Experimental Procedure

According to the formulation (21), calibrating the DTG deterministic error model depends on estimating the model's unknown parameters. The formulation can be expressed in matrix form:

$$Y = F\beta + \xi \tag{22}$$

where β is the estimated parameter vector for each axis, Y is the torque currents values, F is the matrix of the contributions of gravity and the earth rate projected values in each position of the test table, and ξ is the experimental errors. This is the general formulation of a linear identified model. Because the experimental procedure during the data collection phase will affect the properties of the identified model, it is important to design an effective and cost-efficient experiment procedure, which is based on the optimal design criteria for collecting valuable measured data of torque currents

A. D-Optimal Experimental Design Criteria

The optimal experimental design based on optimal design theory has been the focus of the statistician's work on parameter estimation. To minimize the uncertainty of the parameter estimates, several optimality criteria have been proposed [14]. The most commonly used criterion is called the D-optimal experimental design criterion which minimizes the confidence regions of the estimated parameters.

For the formula (22), the estimated value of the unknown parameter $\hat{\beta}$ can be obtained by the least square method:

$$\hat{\beta} = (F^T F)^{-1} F^T Y \tag{23}$$

The covariance matrix of the least square estimate $\hat{\boldsymbol{x}}$ is

$$Var(\hat{\beta}) = \sigma^2 (F^T F)^{-1} \tag{24}$$

where σ^2 denotes the variance of the measurement errors. According to formula (24), the variance of the experimental errors is irrelevant to experimental designs, so the covariance of the estimated parameters depends on elements of the matrix $(\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F})^{-1}$. The matrix $(\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F})^{-1}$ contains the products of the sensitivity coefficients which are the derivatives of the experimental process variables with respect to the estimated parameters.

The matrix

$$M = \boldsymbol{F}^{\mathrm{T}}\boldsymbol{F} \tag{25}$$

is defined as the Fisher information matrix. Based on minimizing the asymptotic confidence regions of the maximum likelihood estimates, the D-optimal criterion is defined as

$$\Phi(M) = \left[\det(M)\right]^{-1} \tag{26}$$

A D-optimal design κ^* over the region Γ is to choose the setting of N measuring points to minimize $\Phi(M)$ or, equivalently, maximizing the determinant of the Fisher information matrix:

$$D(\kappa^*) = \min_{\kappa\in\Gamma} \Phi[M(\kappa)] = \max_{\kappa\in\Gamma} \det[M(\kappa)] \tag{27}$$

It is shown that a D-optimal design can be achieved when $p \le N \le p(p+1)/2$, where p is the number of parameters to be estimated [15].

From equation (21), one can see that there are 8 unknown parameters for each output axis, so the optimal number of measurement points must satisfy $N^* \in [8 \quad 36]$.

Let Ξ be an experimental procedure, which was used to calibrate the drift of the DTG described in equation (21), composed of n ($n \in N^*$) tests $(z_k)_{k\in[1 \quad n]}$. Each experiment z_k corresponds to a measurement point of Ξ, which is generated by the input vector $\vec{\mu}_k$. As a result, every experimental procedure can be mathematically represented as

$$\Xi = \left\{ z_k(\vec{\boldsymbol{\mu}}_k) \middle| k \in [1 \quad n], n \in N^* \right\}$$

$$\text{with associated outputs } Y_k = \left[i_x \quad i_y \right]^T \tag{28}$$

In practice, the important limitation is the experiment cost. The cost of calibration experiments rises in proportion to the number of measurement points. In order to minimize the cost, D-optimal experiment procedure with the least number of measurement points is preferred. We propose the D-optimal experimental design criteria

$$D(\kappa^*) = \min_{\kappa\in\Gamma} \Phi[M(\kappa)/n] = \max_{\kappa\in\Gamma} \det[M(\kappa)/n] \tag{29}$$

Taken into account the above limitations of experiments, the objective function D of the optimal experimental design for DTG is then given as follows

$$Find: \quad \Xi^* = \left[z_1 \quad \dots \quad z_k \right]^{\mathrm{T}} \in \mathrm{E} \quad with: \quad z_k = \left[\theta_k \quad \gamma_k \quad \phi_k \right]^{\mathrm{T}}$$

$$\Xi^* = \arg\max_{\Xi\in\mathrm{E}} \left\{ D_n(\Xi) = \det[M(\Xi)/n] = \det[\boldsymbol{F}_n^{\mathrm{T}}\boldsymbol{F}_n/n] \right\} \tag{30}$$

$$s.t.: \forall k \in [1 \quad n], \forall n \in [8 \quad 36]$$

where E is the global region of measurement points.

In this formulation, different from the typical D-optimal design, the factor of experiment cost is considered by adding the inverse of the number of measurement points in the objective function, which is guaranteed to improve the accuracy of parameters estimation while reducing cost. By solving this optimal experiment design problem, the global region search technique has been applied.

B. Real Code Genetic Algorithm

Finding the optimal setting of N measuring points in the design space is the purpose of the experimental design. In this section, the optimal points of D-optimal design are obtained by applying the real code genetic algorithm (RCGA).

RCGA has a better performance of global optimization and high search efficiency, and is now used as a tool for searching the large, poorly understood spaces that arise in many application areas of science and engineering. So RCGA is adopted to search for the optimal solution.

Among the procedure, some points should be noted:

From equations (30), the three Euler angles can describe one test location, so chromosomes are composed of $3 \times n$ floats from -180 to 180. Each float represents an Euler angle and every three floats represent one test location. Therefore, the chromosomes include the information of n tests.

The determinant of the information matrix $D_n(\Xi)$ is considered directly as a function of the genetic algorithm optimization goals to search the optimal experimental test points.

The objective function for the genetic algorithm requires that the matrix $\boldsymbol{F}_n$ must be full column rank over the optimization process. Attendant problem is that not every chromosome makes the matrix $\boldsymbol{F}_n$ full column rank. Therefore, constraints need to add in the searching algorithm: the fitness of chromosome would reset to zero if it cannot satisfy the full column rank condition of the matrix $\boldsymbol{F}_n$.

The parameters setting of the RCGA is described as follows: The population size is 100. The crossover and the mutation probabilities are 0.9 and 0.07-0.06(t/G), where t is the evolution generation at present and G is the evolution generation, respectively. The stopping criterion is when the maximum number of generations reaches 300.

C. D-Optimal Experimental Design

Through previous work, a D-Optimal experiment to calibration deterministic error sources of DTG can be designed.

According to the proposed D-optimal design criterion, the objective function

$$D_n(\Xi) = \det[M(\Xi)/n] = \det[\boldsymbol{F}_n^{\mathrm{T}}\boldsymbol{F}_n/n] \qquad (31)$$

indicates that the purpose of D-optimal design choose n test position to maximize the determinant of the information. The maximum value for each experimental procedure, which is composed of different quantities of test points in the input domain. It is shown that the optimal number of measurement points for DTG is 12. For the error model (21), the optimal experimental procedures with 12 test points are described in Table 1. It should be compared with the test procedures of traditional 24-positions experimental procedure for DTG[1][2].

Through these experimental procedures, objective function values (31) of the optimal 12-positions procedure is 2.2438, compared to the values 0.5406 of traditional 24-positions experimental procedure, which demonstrates that the quality of the estimations made in the test procedure of traditional 24-position experimental procedure can be improved. In fact, the objective function value increases by 4.2 times for the optimal 12-position test procedure than the traditional 24-position experimental procedure, respectively. From a physical point of view, we observe that these experimental procedures occupy a much wider space than traditional 24-position experimental procedure. Thus, it seems rational that the experimental procedures give better estimates.

D-optimal design are very robust with respect to mis-specifcation of the initial parameter and perturbation of the data. Many papers illustrates the robustness of D-optimal design to estimate parameters [17][18] , which give sufficient evidence to robustness of the proposed D-optimal experimental design method for calibration procedures.

Table 1 The optimal 12 positions experimental procedure result

test points	Euler angle of three axes turntable		
$n=12$	θ(degree)	γ(degree)	ϕ(degree)
1	-161.7508	42.0008	-60.9634
2	-169.0881	-139.0974	-85.0035
3	-179.6216	-38.9781	-159.7965
4	179.621	-38.7855	82.1056
5	-164.7615	-44.2223	-35.9103
6	0.7933	41.4692	37.0962
7	-0.2836	138.7051	-116.2196
8	-2.4169	-41.1344	-31.1635
9	175.1844	140.0902	104.1741
10	-174.9497	41.9743	-178.7836
11	-2.1103	41.6049	162.577
12	-6.0849	-42.1642	-149.6437

IV. Experimental Results

In order to validate the efficiency of D-optimal 12-positions experimental design in the previous sections, a calibration experiment for a tactical grade DTGs has been conducted in our laboratory setting to compare calibration results computed with the experimental procedure of traditional 24-positions.

Table 2 The four validating positions experimental procedure

test points	Euler angle of three axes turntable		
$n=4$	θ(degree)	γ(degree)	ϕ(degree)
1	0	135	0
2	0	315	0
3	0	45	90
4	0	315	90

According to minimizing sum of absolute residual error criterion, the estimation accuracy of two methods on the error model is compared. The residual error is defined as the difference between measurements. The validating experimental procedures with four test points affected by acceleration-square-item are described in Table 2.

The DTG was mounted on the three-axis turntable according to instructions provided by the gyro manufacturer. Care must be taken to minimize misalignments. The purpose of this test is to obtain estimate value of the parameters in the model (21) of the DTG by placing the turntable axes following the optimal 12-positions and traditional 24-positions experimental procedure, and collect the torque measurement required to balance the gyro in each of the orientations. This measurement is in the form of a voltage across a current-sampling resistor in the capture loop of each output axis, and the corresponding output is converted by the voltage-to-frequency converter. This output is sampled by the data acquisition system.

Table 3 Model (23) estimated parameters for different designs

procedures	U_0 Hz	U_1 $Hz*h/deg$	U_2 $Hz*h/deg$	U_3 Hz/g	U_4 Hz/g	U_5 Hz/g	U_6 Hz/g^2	U_7 Hz/g^2
optimal 12positions	6.4545	0.0940	0.4724	0.1462	-3.9661	-0.411	1.0574	-1.024
traditional 24positions	6.1243	-0.2952	0.5326	3.8302	-5.1124	0.1093	0.0128	-0.0677
procedures	V_0 Hz	V_1 $Hz*h/deg$	V_2 $Hz*h/deg$	V_3 Hz/g	V_4 Hz/g	V_5 Hz/g	V_6 Hz/g^2	V_7 Hz/g^2
optimal 12positions	4.2849	0.01022	-0.0022	4.5323	2.0232	3.1473	1.1939	-2.463
traditional 24positions	3.7504	0.0905	0.0105	2.6084	-6.3913	0.0908	-0.0862	0.1844

The three-axis turntable was carefully rotated according to optimal 12-positions and traditional 24-positions. At each position we recorded the torque outputs for 2 minutes after stabilizing. These measurements of the ox axis and the oy axis were used to estimate the parameters of the model (21) by the least squares method, and parameters values obtained are listed in Table 3.

It was observed that when the optimal 12-position procedure was used, the residual errors were mostly within 2 Hz for the ox axis, 4 Hz for the oy axis, and the sum of absolute residual error is smaller than traditional 24-positions. The sum of residual errors was about 62% for the ox axis and 39% for the oy axis respectively, of that of the traditional 24-position design. These confirm that, in comparison to the traditional 24-position design, the optimal designs yielded better calibration results with nearly half experimental cost.

V. Conclusion

A novel methodology for multi-position calibration procedures was designed to maximize the precision in the estimate of the deterministic error of DTG and to minimize the experiment cost. Based on Euler angles transformation, a linear deterministic error model for DTG has been built up to realize the independence of model terms. For this model, a novel calibration procedure based on D-Optimal Experiment design and RCGA was proposed. Experiment results show that the estimation accuracy of the D-Optimal 12-positions procedure on the error model is better than the traditional 24-positions experimental procedure and half of the cost can be saved. The calibration procedure presented here are not the only possible choices, but still simple calibration procedure with considering both accuracy and cost.

The D-optimal experimental method proposed in this paper has proved to be simple and efficient in calibrating DTG procedure, and can be easily extended to test other kinds of gyroscopes.

The calibration experiment illustrates that the novel design reduces the fit uncertainties compared to the traditional 24-position design and nearly save half of the cost. Due to high experiment cost, there have not been more experiments to verify the optimal accuracy and efficiency of the 12-position design. But the more experimental evidence can be found without hardness in future work. What's more, the proposed methodology will be easily extended to test other kinds of gyroscopes.

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