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MODELING AND ANALYSIS OF AN OPTICALLY-ACTUATED, BISTABLE MEMS DEVICE

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ABSTRACT

Bistable microsystems have drawn considerable interest from the MEMS/NEMS research community not only due to their broad applicability in commercial applications, such as switching, but also because of the rich dynamic behavior they commonly exhibit. While a number of prior investigations have studied the dynamics of bistable microsystems, comparatively few works have sought to characterize their transient behavior. The present effort seeks to address this through the modeling and analysis of an optically-actuated, bistable MEMS switch. The work begins with the development of a distributed-parameter representation for the system, which is subsequently reduced to a lumped-mass analog and analyzed through the use of numerical simulation. The influence of various system and excitation parameters, including the applied axial load and optical actuation profile, on the system's transient response is then investigated. Ultimately, the methodologies and results presented herein should provide for a refined predictive design capability for optically-actuated, bistable MEMS devices.

1 INTRODUCTION

As their name suggests, bistable systems feature two, coexistent stable states. Depending on the system's energy level, a given bistable device can oscillate about one of its two states and decay, switch one or more times between states then decay, or exhibit stochastic behavior, such as stochastic resonance [1, 2]. In the mechanical domain, research related to bistable systems stems largely from early investigations of structural buckling and post-buckled dynamics. Recent, yet representative, works include those of Emam [3], which considered the complex, quasiperiodic and chaotic response of post-buckled beams, and Yabuno and Tsumoto [4], which explored the transient behavior of a buckled beam in the presence of a high-frequency direct excitation. In the context of micro- and nanoscale systems, bistable devices have garnered significant interest not only due to their rich dynamics, but also due to their broad applicability in commercial applications, such as switching and microfluidics (see, for example, [5–7]). Representative of works in this area are the collective efforts of Saif and collaborators [8–10], which the present work seeks to build upon, the work of Qin [7], which considered the dynamics of an electrothermally-actuated, bistable MEMS device, and, more recently, the works of Krylov and collaborators, which considered the bistable behavior of electrostatically-actuated MEMS structures operating near pull-in and/or snap-in [11, 12].

The bistable MEMS device of interest is a long slender beam, clamped at both ends, which is connected to an actuator that provides an axial compressive load (see Fig. 1) [8]. In this system, as the compressive force is increased beyond a critical load the beam buckles (the response bifurcates), yielding two co-

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existent stable configurations and an intermediate unstable state, which is often analogous to the device's unbuckled configuration. In prior work, Sulfridge and collaborators modeled the static response of this representative microscale system, after it had buckled in the first mode. Building upon this, successive efforts by the same research group considered optically-induced switching, realized via the application of an external laser pulse, and the impact of system nonlinearities [9, 10]. These works explained the switching between bistable states through the use of numerical simulations, and their behavioral predictions were subsequently verified experimentally.

Although the dynamics of the bistable system of interest were previously investigated in [10], the analytical approaches utilized in that work have some limitations. First, the previously-developed model is based upon a lumped-mass representation, which itself was based on a previously-developed static model. Additionally, the work assumed that the beam buckles only in the first mode of a clamped-clamped beam, which might not always be the case in practical applications where intermediate boundary conditions may be present. In light of these limitations, the objective of this work is to develop a refined model for the system of interest, which can be subsequently used to characterize the bistable device's transient response under various forcing conditions and, ultimately, to efficiently predict switching times. To this end, the work begins with the development of a distributed-parameter model for the bistable system suitable for the analysis of both buckling and post-buckling dynamics. An optical actuation force is then appended to this model and the switching behavior of the device is studied for various optical actuation profiles. The work concludes with a brief summary and an overview of ongoing and future work.

2 SYSTEM MODELING

The particular bistable system of interest, previously considered in [10], is shown in Fig. 1. This system consists of a rectangular beam (the line segment between points A and B) with dimensions 1000 μm × 100 μm × 5 μm, which is anchored at both ends and buckled by a compressive force provided by the comb-driven actuator connected at B. The known actuator force contributes to the displacement of both the buckled beam and the actuator springs. The laser source, which provides the transverse actuation (switching) force is designated by L. For analytical purposes, this system can be simplified to the model depicted in Fig. 2.

In Fig. 2, $F(s,t)$ represents the transverse external force applied on the beam. This force is used to switch between stable states after the onset of buckling. M represents the mass of the actuator and P represents the force provided by the actuator. Note that this model neglects the effects of damping on the actuator. If ρ, A, E, I, and l represent the mass density, cross-sectional area, elastic modulus, area moment of inertia, and length of the

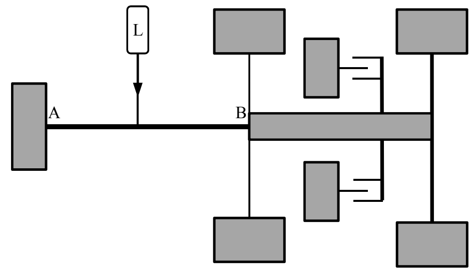

FIGURE 1. SCHEMATIC DIAGRAM OF THE BISTABLE MICROSYSTEM OF INTEREST.

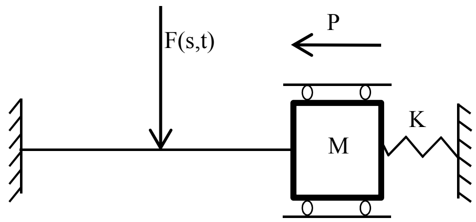

FIGURE 2. DIAGRAM OF THE SIMPLIFIED DEVICE REPRESENTATION USED FOR MODELING.

beam respectively, and s defines the local arc length variable, the kinetic and potential energies associated with the beam can be written as

$$T = \frac{1}{2}\int_0^l \rho A\left(\dot{u}^2 + \dot{w}^2\right) \mathrm{d}s, \tag{1}$$

$$V = \frac{1}{2}\int_0^l EI(w'')^2 \mathrm{d}s - P_{eff}\int_0^l u' \mathrm{d}s + \frac{EA}{2l}\left(\int_0^l u' \mathrm{d}s\right)^2, \tag{2}$$

where, u and w represent the displacements in the longitudinal and transverse directions respectively,

$$P_{eff} = P - M(\ddot{u})|_{s=l} - K(u)|_{s=l} \tag{3}$$

represents the effective external load acting on the beam (note that this takes into account the various corrections attributable to the mass and stiffness of the actuator), and $\dot{(\bullet)}$ and $(\bullet)'$ represent temporal and spatial derivatives taken with respect to the variables t and s.

The Lagrangian of the system is defined as

$$L = T - V + \frac{1}{2}\lambda\left[1 - (1+u')^2 - v'^2\right],$$

where λ is a Lagrange multiplier introduced to maintain the inextensibility constraint. Accounting for the damping and the transverse actuation load as non-conservative forces, and employing extended Hamilton's principle yields two equations governing the longitudinal and transverse vibrations of the system. Imposing the inextensibility constraint, solving for the Lagrange multiplier in the equation for longitudinal motion, and substituting the result, renders the equation of motion governing the transverse vibrations of the system:

$$\begin{aligned}\rho A\ddot{w}+C\dot{w}+EIw^{iv}+Pw''-\frac{EA}{2l}w''\int_0^l (w')^2\mathrm{d}s\\ -\frac{M}{2}w''\frac{\partial^2}{\partial t^2}\int_0^l (w')^2\mathrm{d}s-\frac{K}{2}w''\int_0^l (w')^2\mathrm{d}s=F(s,t).\end{aligned} \tag{4}$$

Equation (4) can be nondimensionalized by scaling the spatial variable with respect to the undeformed length of the beam, the beam displacements with respect to a characteristic length w_0 (for example the beam's thickness), and the time variable with respect to a characteristic period of the system's response, namely,

$$\hat{s}=\frac{s}{l},\qquad \hat{w}=\frac{w}{w_0},\qquad \hat{t}=\frac{t}{T}, \tag{5}$$

where

$$T=\sqrt{\frac{\rho A l^4}{EI}}.$$

This yields a final distributed parameter model for the system given by

$$\begin{aligned}\ddot{\hat{w}}+\hat{c}\dot{\hat{w}}+\hat{w}^{iv}+4n^2\pi^2\frac{P}{P_{cr}}\hat{w}''-\frac{Aw_0^2}{2I}\hat{w}''\int_0^1(\hat{w}')^2\mathrm{d}\hat{s}\\ -\frac{Mw_0^2}{2\rho Al^3}\hat{w}''\frac{\partial^2}{\partial \hat{t}^2}\int_0^1(\hat{w}')^2\mathrm{d}\hat{s}-\frac{Kw_0^2 l}{2EI}\hat{w}''\int_0^1(\hat{w}')^2\mathrm{d}\hat{s}\\ =\frac{l^4}{EIw_0}F(\hat{s},\hat{t}),\end{aligned} \tag{6}$$

where

$$\hat{c}=\frac{CT}{\rho A},\qquad P_{cr}=\frac{4n^2\pi^2 EI}{l^2}.$$

Note that the critical Eulerian buckling load associated with the system, P_{cr}, can be derived following the procedure outlined in [13].

The distributed-parameter model developed above can be reduced to a lumped-mass analog by decomposing the displacement variable $\hat{w}$ into its spatial and temporal components according to

$$\hat{w}=z(\hat{t})\Phi(\hat{s})$$

and projecting the result onto a single mode shape. This results in a final governing equation of motion for the system given by

$$m\ddot{z}+c\dot{z}+k_1 z+k_3 z^3+\alpha_n(z\dot{z}^2+z^2\ddot{z})=\eta(\hat{t}), \tag{7}$$

with nondimensional parameters defined as in Table 1.

TABLE 1. NON-DIMENSIONAL PARAMETERS FOR THE GOVERNING EQUATION OF MOTION.

$$m=\int_0^1\Phi^2\mathrm{d}\hat{s}$$

$$c=\hat{c}\int_0^1\Phi^2\mathrm{d}\hat{s}$$

$$k_1=\int_0^1\Phi\Phi^{iv}\mathrm{d}\hat{s}+\frac{4n^2\pi^2P}{P_{cr}}\int_0^1\Phi\Phi''\mathrm{d}\hat{s}$$

$$k_3=-\frac{Aw_0^2}{2I}\int_0^1\Phi\Phi''\int_0^1(\Phi')^2\mathrm{d}\hat{s}\mathrm{d}\hat{s}-\frac{Kw_0^2 l}{2EI}\int_0^1\Phi\Phi''\int_0^1(\Phi')^2\mathrm{d}\hat{s}\mathrm{d}\hat{s}$$

$$\alpha_n=-\frac{Mw_0^2}{\rho Al^3}\int_0^1\Phi\Phi''\int_0^1(\Phi')^2\mathrm{d}\hat{s}\mathrm{d}\hat{s}$$

$$\eta=\frac{l^4}{EIw_0}\int_0^1\Phi F(\hat{s},\hat{t})\mathrm{d}\hat{s}$$

The static buckled mode shapes associated with the system of interest can be explicitly obtained by dropping the time-varying terms in Eq. (6) and solving the resulting spatial differential equation:

$$\Phi^{iv}+\hat{P}\Phi''-6(1+\frac{Kl}{Ewh})\Phi''\int_0^1(\Phi')^2\mathrm{d}\hat{s}=0, \tag{8}$$

with

$$\hat{P} = \frac{Pl^2}{EI},$$

and associated boundary conditions given by,

$$\begin{aligned} \Phi &= 0 \quad \text{and} \quad \Phi' = 0 \quad \text{at} \quad x = 0, \\ \Phi &= 0 \quad \text{and} \quad \Phi' = 0 \quad \text{at} \quad x = 1. \end{aligned} \tag{9}$$

Given that $\int_0^1 (\Phi')^2 d\hat{s}$ is constant for a particular mode, the general solution of Eq. (8) can be expressed as,

$$\Phi = c_1 + c_2 x + c_3 \cos(\sqrt{\lambda} x) + c_4 \sin(\sqrt{\lambda} x) \tag{10}$$

where

$$\lambda = \hat{P} - (6 + \frac{6Kl}{Ewh})Q,$$

and $Q = \int_0^1 (\Phi')^2 d\hat{s}$. Substituting Eq. (10) into Eq. (9) and solving the resulting algebraic equations yields the following characteristic equation:

$$2 - 2\cos\sqrt{\lambda} - \sqrt{\lambda}\sin\sqrt{\lambda} = 0. \tag{11}$$

The solutions of Eq. (11) can be classified into two varieties: those corresponding to symmetric modes and those corresponding to asymmetric modes. These modes appear in alternating pairs with increasing λ. The first two solutions corresponding to asymmetric modes are 80.763 and 238.718 respectively. Using these values of λ, the asymmetric mode shape of the system can be evaluated to be

$$\begin{aligned} \Phi = & b_n \left[\frac{\frac{1}{\sqrt{\lambda}}\sin\sqrt{\lambda} - 1}{1 - \cos\sqrt{\lambda}} + x \right. \\ & \left. - \left(\frac{\frac{1}{\sqrt{\lambda}}\sin\sqrt{\lambda} - 1}{1 - \cos\sqrt{\lambda}} \right) \cos\sqrt{\lambda} x - \frac{1}{\sqrt{\lambda}} \sin\sqrt{\lambda} x \right]. \end{aligned} \tag{12}$$

The coefficients b_n can be explicitly evaluated by using the relationship between λ and Q. For symmetric modes, the values of λ are given by $4n^2\pi^2$, where n is an integer. The corresponding symmetric mode shapes are given by

$$\Phi = \frac{1}{2} b_n \left[1 - \cos(2n\pi x)\right], \tag{13}$$

where,

$$b_n = \sqrt{\frac{\hat{P} - 4n^2\pi^2}{3\left(1 + \frac{Kl}{Ewh}\right) n^2 \pi^2}}.$$

Using these equations, the non-dimensional parameters associated with the equation of motion can be explicitly evaluated.

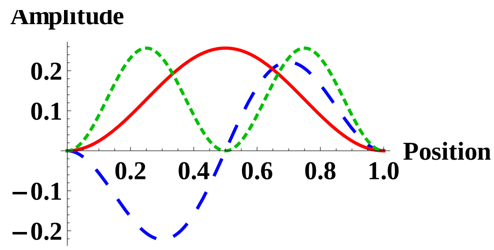

FIGURE 3. STATIC BUCKLED MODE SHAPES ASSOCIATED WITH THE SYSTEM OF INTEREST. THE RED (SOLID) AND GREEN (DOTTED) LINES REPRESENT THE FIRST AND SECOND SYMMETRIC MODES OF THE SYSTEM AND THE BLUE (DASHED) LINE REPRESENT THE FIRST ANTISYMMETRIC MODE OF THE SYSTEM.

The optical actuation force utilized for switching can be modeled using the theories of classical physics. Maxwell predicted that since light possesses momentum, if light is absorbed or reflected by a body, it exerts a pressure on that body. The resulting force is given by [9]

$$F = \frac{2W}{c}, \tag{14}$$

where W is the power of the (laser) source and c is the speed of light. In the context of MEMS, the scaling of an optical actuation force compares well with that of electrostatic forces. However, using a laser pulse as an actuation force does have some limitations, namely, diffraction, collimation, and local heating [9]. For example, no surface is completely reflective and hence, some amount of energy must be absorbed into a system being excited by an external laser pulse. This causes some degree of local heating. Previous works have considered the use of local heating as an actuation (switching) mechanism (see, for example, [14, 15]). In this work, following Sulfridge et al., [16], the pressure exerted by the incident radiation is considered to be quite significant compared to heating effects, and therefore radiation pressure is assumed to be the only external force acting on the device that is capable of inducing switching.

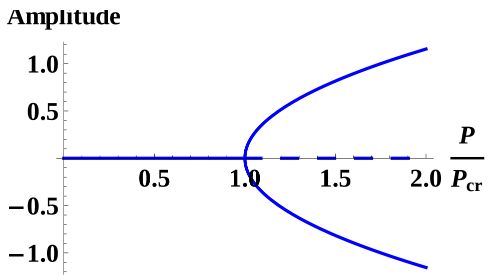

FIGURE 4. STATIC BEHAVIOR OF THE BEAM AT ITS FIRST SYMMETRIC BUCKLING MODE. THE BEAM SHOWS A SUPERCRITICAL PITCHFORK BEHAVIOR, WITH THE ZERO SOLUTION BECOMING UNSTABLE AND TWO ADDITIONAL STABLE SOLUTIONS ARISING AS THE AXIAL LOAD IS INCREASED BEYOND THE CRITICAL EULERIAN LOAD. SOLID LINES REPRESENT THE STABLE SOLUTIONS WHILE DASHED LINES REPRESENT THE UNSTABLE SOLUTION OF THE SYSTEM.

3 Analysis

The static post-buckled behavior of the system can be studied by neglecting the time derivatives and forcing in Eq. (7). Analysis of the resulting equation, after projection into the first symmetric buckling mode, reveals classical supercritical pitchfork behavior. Thus, static buckling behavior can be sufficiently captured by the model.

Though multi-mode approximations are commonly used to describe the dynamic behavior of buckled systems, in the case of a buckled beam with a transverse actuation force applied at the midpoint, the system's higher modes do not contribute appreciably to the accuracy of the solution [17]. Accordingly, a single symmetric mode approximation is used to investigate the post-buckling dynamics of the bistable system of interest. The dynamics of this system largely depend on how large the applied axial load is in comparison to the critical buckling load associated with the system. In order to study this effect, the phase plane associated with the system for two different initial conditions and two different axial load is considered. Figure 5(a) shows the phase plane for the system when the axial load is 1% higher than the critical buckling load and Fig. 5(b) shows the phase plane for the system when the axial load is 5% higher than the critical buckling load of the beam. It can be seen from these phase planes that there are two stable solutions (spiral points), corresponding to the extreme positions of the beam and one unstable solution (saddle point) corresponding to the unbuckled configuration. A small perturbation in the operating condition of the beam gives rise to oscillations in the system. Due to the presence of damping, in most situations the system settles down onto one of its stable states. Of particular note is that fact that for the same initial condition, depending on the value of the applied axial load (and hence, the maximum displacement of the beam), the response of the system may be fundamentally different.

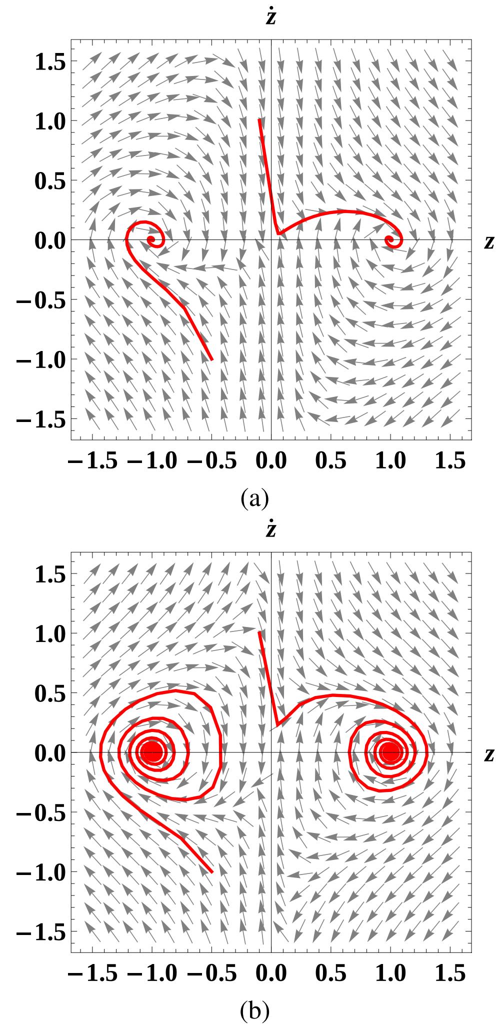

FIGURE 5. PHASE PLANE FOR THE BUCKLED BEAM IN ITS FIRST SYMMETRIC BUCKLING MODE, WHEN THE LOAD IS (a) 1% AND (b) 5% HIGHER THAN THE CRITICAL LOAD OF THE SYSTEM. THERE ARE TWO STABLE SPIRALS, AT ($\pm$1,0) AND A SADDLE POINT AT (0,0).

To study the effect of small perturbations on the system, the basins of attraction associated with the system have been plotted. Figure 6(a) shows the basins of attraction for the case where buckling load is 1% higher than the critical load and Fig. 6(b) shows the same for the case where buckling load is 5% higher than the critical load. It can be seen that the basins of attraction quantitatively change as the compressive load is increased, with each of the respective lobes increasing in size, but largely maintaining shape.

Using the above results as a benchmark, the transient (switching) behavior of the system can be investigated through the use of forward-time numerical simulation. For present pur-

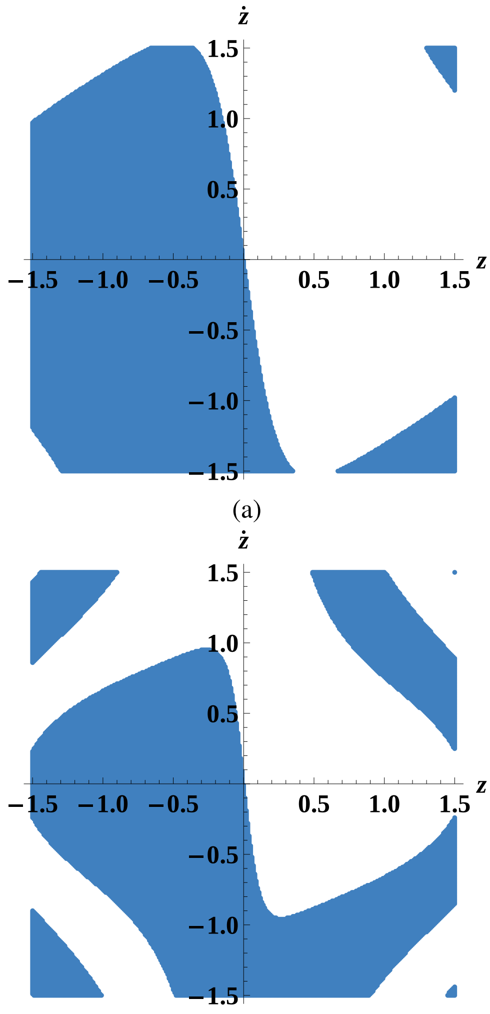

FIGURE 6. BASINS OF ATTRACTION FOR THE TWO STABLE CONFIGURATIONS OF THE SYSTEM BUCKLED IN ITS FIRST MODE WHEN THE LOAD IS (a) 1% AND (b) 5% GREATER THAN THE CRITICAL LOAD. NOTE THAT THE BLUE AREAS REPRESENT THE BASIN OF ATTRACTION FOR THE STABLE POINT (-1,0) AND THE WHITE AREA REPRESENTS THE BASIN OF ATTRACTION FOR THE STABLE POINT (1,0).

poses, the beam is initially considered to be in the stable position (-1,0) and to have an axial compressive load 1% higher than the critical load for buckling. A simple pulse is then applied. Figure 7 shows the response of the system for a simple laser pulse (1 W power) applied for 1 ms and 1.5 ms, respectively. As evident, the system switches to the other stable state for the pulse of 1.5 ms, but not for the pulse of 1 ms. Similarly, Fig. 8 shows the response of the system when a half-sine wave shaped laser pulse is applied for 4 ms and 6 ms respectively. Here, the system switches from one stable state to the other for a pulse 6 ms in

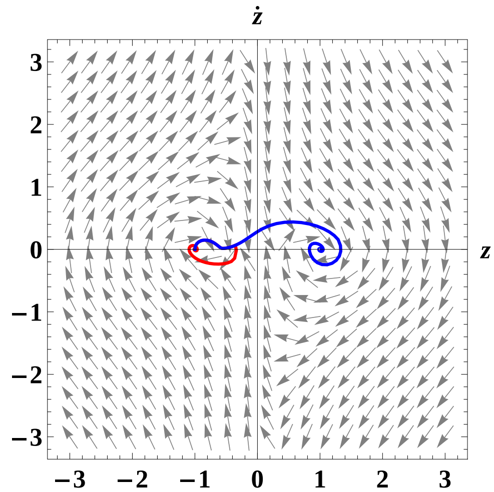

FIGURE 7. RESPONSE OF THE SYSTEM WHEN ACTUATED BY A SIMPLE PULSE INPUT (1 W) FOR 1 ms (RED) AND 1.5 ms (BLUE). THE SYSTEM IS INITIALLY AT THE STATE (-1,0). IT SWITCHES TO THE OTHER STABLE STATE FOR THE PULSE OF WIDTH 1.5 ms, BUT NOT FOR THE PULSE WIDTH OF 1 ms.

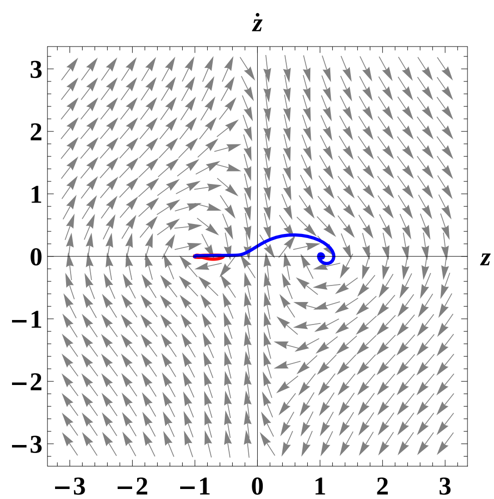

FIGURE 8. RESPONSE OF THE SYSTEM WHEN ACTUATED BY A HALF-SINE PULSE INPUT (1 W) FOR 4 ms (RED) AND 6 ms (BLUE). THE SYSTEM IS INITIALLY AT THE STATE (-1,0). IT SWITCHES TO THE OTHER STABLE STATE FOR THE PULSE OF WIDTH 6 ms, BUT NOT FOR THE PULSE OF WIDTH 4 ms.

duration, but not for the 4 ms pulse. This variance in the pulse duration required for switching is indicative of the fact that the activation energy required for switching is a function not only of pulse duration and amplitude, but of the pulse profile itself.

To compare the effectiveness of various pulse profiles in optical switching, two test cases are analyzed, namely, the simple pulse and a half-sine wave pulses noted above. Figure 9 highlights the relationship between (nondimensional) pulse widths and (nondimensional) switching times, as measured after the pulse has been removed, for each actuation profile for two different values of compressive axial load. Note that the pulse widths were kept constant here and the amplitudes of the pulses have been adjusted so that both pulses are of equal energy. The solid lines and the dashed lines represent cases when the axial load is higher than the critical load by 0.5% and 0.75%, respectively. The dashed lines both correspond to the same energy level, as do the solid lines. As evident, for a constant energy pulse, there exists regions where the switching times associated with simple pulse inputs are smaller than those recovered with half-sine inputs, and vice versa. As the axial load increases, these trajectories remain qualitatively alike, but shift towards the right. This results in increased minimum actuation times and increased switching times.

Figure 10 shows the relationship between the (nondimensional) pulse widths and the (nondimensional) switching times for each of the previously-utilized actuation profiles for two different optical power levels and an axial load that is 0.5% higher than the critical load. As before, the dashed lines correspond to the same energy level, as do the solid lines. It can be seen that the behavior is qualitatively invariant, but the pulse with a higher power has a reduced switching time for the same actuation time. In addition, it must also be noted that the settling times initially decrease and then increase as more energy is supplied to the system. This is attributable to the fact that the system can switch multiple times or oscillate excessively about a single stable position at higher energies. Accordingly, both the force and the duration of the force becomes critical in the design of optically-actuated bistable systems.

By extrapolating the methodologies detailed herein, the dynamics of the beam undergoing buckling in higher modes can be investigated. However, preliminary analysis reveals that for loads comparatively-higher than the critical buckling load, the system can exhibit rather complex dynamic behaviors, indicative of limit cycles. The onset of this complex behavior appears to have a non-trivial dependence on multiple system parameters, including damping, actuator mass, etc. and thus will be addressed in a separate study.

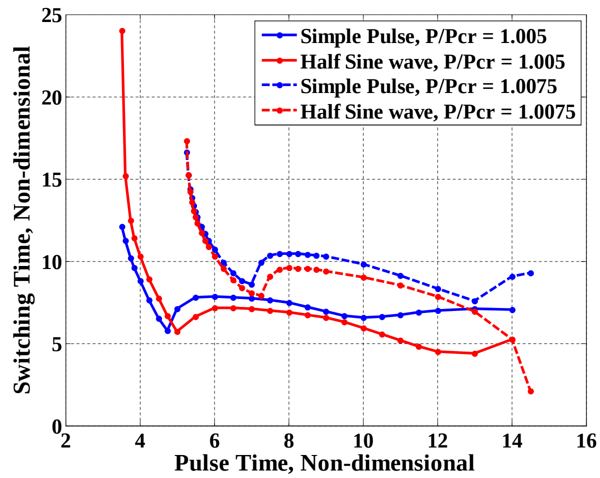

FIGURE 9. RELATIONSHIP BETWEEN SWITCHING TIMES AND PULSE WIDTHS FOR A SIMPLE PULSE INPUT AND A HALF-SINE WAVE INPUT FOR TWO DIFFERENT VALUES OF CRITICAL LOAD AT THE SAME EXCITATION LEVEL (1 W). THE PULSE WIDTHS ARE SUCH THAT BOTH THE INPUTS ARE OF EQUAL ENERGY. THE SWITCHING TIME IS DEFINED HERE TO BE THE TIME TAKEN BY THE SYSTEM TO SETTLE IN THE STABLE STATE (1,0), STARTING FROM THE (-1,0) CONFIGURATION, AFTER THE REMOVAL OF THE PULSE.

4 Conclusions and Future Directions

In conclusion, this work considered the modeling and analysis of the post-buckled dynamics of a representative, bistable MEMS device optically-actuated by an external laser pulse. Through the use of forward-time numerical simulation, the impact of various system and excitation parameters, including applied axial load and optical actuation profile, on the transient behavior (switching times) of the representative device have been investigated. The methodologies and results presented herein should provide for a refined predictive design capability for small-scale bistable devices. It is important to note that the efforts described here are still in their infancy. Ongoing work is aimed at incorporating refined modal approximations, experimentally verifying predicted behaviors, and extending the predictions to device-specific applications.

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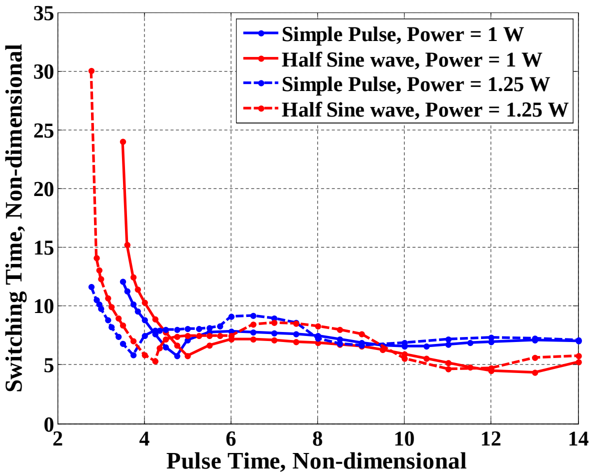

FIGURE 10. RELATIONSHIP BETWEEN SWITCHING TIMES AND PULSE WIDTHS FOR A SIMPLE PULSE INPUT AND A HALF-SINE WAVE INPUT FOR TWO DIFFERENT PULSE AMPLITUDES WHEN THE AXIAL LOAD IS 0.5% HIGHER THAN THE CRITICAL LOAD OF THE BEAM. THE PULSE WIDTHS ARE DESIGNED SUCH THAT BOTH OF THE INPUTS ARE OF EQUAL ENERGY. THE SWITCHING TIME IS DEFINED TO BE THE TIME TAKEN BY THE SYSTEM TO SETTLE IN THE STABLE STATE (1,0), STARTING FROM THE (-1,0) CONFIGURATION, AFTER THE REMOVAL OF THE PULSE.

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