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NONLINEAR DYNAMICS AND ITS APPLICATIONS IN MICRO- AND NANORESONATORS

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ABSTRACT

This review provides a summary of the work completed to date on the nonlinear dynamics of resonant micro- and nanoelectromechanical systems (MEMS/NEMS). This research area, which has been active for approximately a decade, involves the study of nonlinear behaviors arising in small scale, vibratory, mechanical devices that are typically integrated with electronics for use in signal processing, actuation, and sensing applications. The inherent nature of these devices, which includes low damping, desired resonant operation, and the presence of nonlinear potential fields, sets an ideal stage for the appearance of nonlinear behavior, and this allows engineers to beneficially leverage nonlinear dynamics in the course of device design. This work provides an overview of the fundamental research on nonlinear behaviors arising in micro/nanoresonators, including direct and parametric resonances, parametric amplification, impacts, self-excited oscillations, and collective behaviors, such as localization and synchronization, which arise in coupled resonator arrays. In addition, the work describes the active exploitation of nonlinear dynamics in the development of resonant mass sensors, inertial sensors, and electromechanical signal processing systems. The paper closes with some brief remarks about important ongoing developments in the field.

INTRODUCTION

This paper describes past developments, ongoing work, and a vision of future topics for a research area with a relatively short history – the application and exploitation of nonlinear dynamic behavior in micro- and nanoelectromechanical systems (MEMS/NEMS). In the 1950s, Richard Feynman presented a prescient paper entitled,"There's Plenty of Room at the Bottom" (reprinted in [1]), and a follow-up paper "Infinitesimal Machinery" (reprinted in [2]), in which he outlined a vision for building very small devices that had the potential to revolutionize a wide range of technologies. These ideas lay essentially dormant for many years, awaiting the means of manufacturing mechanical devices on very small scales. The first notable progress in the area of microresonators was the work of Nathanson and coworkers on resonant gate transistors [3, 4]. The driving force that made the fabrication of such devices possible came from the electronics industry, specifically the production of miniaturized integrated circuits (ICs). In 1982, Kurt Peterson described the next step in the development towards MEMS with his provocative paper, "Silicon as a Mechanical Material" [5], in which it was demonstrated how one could use IC manufacturing techniques to build small mechanical devices with movable components. The field has accelerated rapidly since that time, and the community is well on its way to at least partially fulfilling Feynman's vision of miniature machines. Applications abound and are having an impact in a wide variety of fields including communications, sensing, basic physics, and biotechnology.

Another research area that has seen a similar growth in interest and applications over the past few decades is nonlinear dynamics. While the history of this field dates to Newton's solution of the two-body problem and Poincare's classical work on celestial mechanics (described from a modern point of view by Holmes [6]), the field accelerated significantly in the 1980s, building on a number of developments from the 1960s, including Lorenz's discovery of chaos in a simple model for convective fluid flow [7] and Smale's work on the topology underlying

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chaos (recounted in [8]). The technology that drove this field was the wide availability of computers, which brought to life the mathematical ideas from dynamical systems. This has produced a thriving community which spans mathematics, physics, engineering, operations research, and biology.

This paper focuses on a class of problems that lie at the intersection of the two topical areas detailed above. Thus, of the various types of MEMS and NEMS, those of interest here utilize a resonant mode of operation and exhibit nonlinear behavior, either intentionally or otherwise (Note that detailed reviews of linear micro/nanoresonators can be found in [9–11]). Such devices are used in a wide variety of applications, including sensing, signal processing, switching, and timing. Many of these applications are driven by the recognition that some purely-electrical components can be advantageously replaced by electromechanical analogs. The benefits of such an approach include smaller size, lower damping, and improved performance metrics. These devices also offer the desirable feature of being easily integrated with solid-state circuits, thus enabling the development of IC chips with integrated mechanical and electrical functionality. It is important to note that the transition to electromechanical components requires that analysts and designers deal with dynamic behavior resulting from classical forces acting on the mechanical components, as well as multi-physics interactions, many of which do not have counterparts in purely-electrical or macroscale mechanical systems. Accordingly, the vast experience developed for the analysis and design of electrical circuits and (independently) for mechanical structures are not, in themselves, sufficient for understanding the behavior of resonant micro/nanosystems. This has, in turn, opened a new realm of system design. Some of the unique aspects of MEMS/NEMS present difficulties for device realization, but they also can be exploited to achieve some interesting device designs. From a nonlinear dynamics point of view, MEMS/NEMS offer greater flexibility in terms of designing for specific types of nonlinear behavior, and thus provide fertile ground for designs that actively exploit the rich behavior of nonlinear systems.

Given that the field of interest is relatively new, this work provides a quite thorough overview of previous and ongoing research, and includes a quite comprehensive review of the literature.[1] However, one area that falls under the umbrella of nonlinear dynamics in MEMS/NEMS, which is covered only briefly here is atomic force microscopy (AFM) and other forms of probe-based microscopy which utilize micro/nanoresonators. These technologies, which typically use individual or uncoupled arrays of microbeams to characterize the surface properties of materials, often using a vibratory mode of operation, are commercially available and widely used in industry, yet also remain the subject of interesting research investigations. Accordingly, this line of work is deserving of an independent review, such as that provided by Garcia and Perez or Raman, et al., which detail the modeling and analysis of nonlinear effects in atomic force microscopes (AFMs) [12, 13].

It should be noted that a recent review by Lifshitz and Cross [14] covers some of the same ground as this paper. Specifically, the work includes a detailed description of the modeling and analysis of the near-resonant behavior of isolated and coupled micro/nanoresonators with quadratic and cubic nonlinearities. In contrast, the aim of the present work is to provide an engineering perspective of nonlinear micro/nanoresonators and a more comprehensive summary of the current literature, albeit with markedly less analytical detail.

The paper is organized into the following sections: the basics of modeling, the phenomenon of pull-in, systems with direct excitation, systems with parametric excitation, systems with combined direct and parametric excitation, self-excited systems, vibro-impact systems, single-element systems with a few degrees of freedom, and coupled resonator arrays. The focus in all but the latter two sections is on systems that are accurately described by single-degree-of-freedom (SDOF) models. The work closes with some thoughts about the potential for further progress in this field and some areas of research that will become increasingly important as the area advances.

MODELING

Resonant MEMS/NEMS are miniature machines, designed for specific functionality and fabricated using a wide variety of additive and subtractive methods adopted from classical IC fabrication. MEMS/NEMS are quite simple in terms of mechanical design and typically consist of a few common elements such as beams, lumped masses, etc. Many of the forces that act on these elements are the same as those encountered at the macroscale, such as those arising from elastic, magnetic, electromagnetic, and aerodynamic sources. However, due to the length scales involved, additional forces also come into play, or have a more significant role, including van der Waals, adhesion, Casimir, and electrostatic forces. In addition, actuation of these devices can be achieved through the forces noted above, as well as through base excitation and induced stresses, both of which are commonly generated by piezoelectric elements. It is the combination of these effects, and the fact that the mechanical device is often integrated directly with electronics, that provides both challenges and opportunities for the design of dynamic MEMS/NEMS. This section focuses on electrostatic effects, since these are widely used for the actuation, sensing, and tuning of micro/nanoresonators.

The resonators of interest here range in overall size from $10^1 - 10^4$ μm (often with feature sizes on the order of a few nanometers) and have been designed with natural frequencies in the range of $10^4 - 10^9$ Hz and dissipation corresponding to Q values in the range $10^0 - 10^5$. They are prime candidates for experiencing nonlinear behavior, since they are often very lightly damped, and are driven at or near resonance and thus can experience relatively large amplitudes. These devices are large enough that one can use continuum approaches as the starting point for

[1] As is always the case, there are undoubtedly papers of which the authors are not aware – the authors apologize for such oversights.

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the modeling of inertial and material effects. Typical devices are designed to behave in a predictable manner, and thus are configured to look like lumped-mass elements, beams, plates, or other well-understood structures. In this regard, one does not need any special tools for modeling their inertia and elastic restoring forces, whether linear or nonlinear, as the required methodologies are quite standard and finite element techniques are widely accessible for MEMS/NEMS design. Most common amongst the various nonlinear elastic effects are those attributable to finite displacements, such as mid-plane stretching in fixed-fixed beams. In contrast, as is the case with macroscale vibratory systems, the issue of energy dissipation often requires special treatment (see, for example, [15–27]).

Accurate models are essential for the prediction of device behavior, and are becoming increasingly more important for resonant MEMS/NEMS as designers push devices into new operating regimes. For most microscale devices Newtonian mechanics is sufficient for model development, while for certain nanoscale devices that are coupled to components such as single-electron transistors, some quantum-level modeling is required. In addition, many devices utilize feedback, which provides additional possibilities for interactions. In all cases, the devices involve physics from multiple fields, and this, along with the fact that they are fabricated from a variety of materials using a wide range of techniques, makes the field inherently multi-disciplinary. This section outlines some of the basic features of models developed for resonant micro/nanosystems, paying particular attention to their nonlinear mechanical and electrostatic aspects. A number of computational modeling tools have been developed for these purposes, including SUGAR [28] and CoventorWare^{TM}. These focus primarily on mechanical-electrostatic interactions, and allow for nonlinear designs to be considered. Reviews of system modeling in the presence mechanical and electrostatic interactions, incorporating quite extensive reference lists, can be found in [29, 30]. De and Aluru [31] describe a systematic Lagrangian approach to modeling electromechanical devices with dynamic interactions. These types of computational methods provide a detailed description of system behavior, as required for refined designs. However, many devices, especially those built for "proof-of-concept" purposes do not require such thorough analysis, and often lumped-mass models suffice for initial designs. Reviews of modeling from the structural mechanics community include those of Lin and Wang [32] on structural dynamics and Wittwer et al. [33], which addresses the important problem of modeling nonlinear static structural elements in the face of uncertainties. More specific to the development of reduced-order models that capture the qualitative behavior of micro/nanoscale devices with nonlinear characteristics are the works of Gabbay, Mehner, and Senturia [34, 35], which utilize computer-generated, reduced-order macromodels; Younis et al. [36] and Xie et al. [37], which utilize expansion methods; Hung and Senturia [38] and Liang et al. [39], which use proper-orthogonal decomposition techniques; and Nayfeh et al. [40], which compares domain and point-wise expansion techniques, concluding that domain methods provide better results, provided that the basis functions are judiciously selected. The books of Senturia [41], Pelesko and Bernstein [42], Lobontiu [43], and Cleland [44] also provide systematic views of MEMS/NEMS modeling from first principles.

The present discussion specifically focuses on simple lumped-mass models for uncoupled resonators, from which one can build models of systems with additional degrees-of-freedom, such as coupled resonator arrays. The reader should keep in mind, however, that these models are typically the result of a more detailed modeling approach, which begins with partial differential equations or finite element models and subsequently employs model reduction techniques. When using lumped-mass models for experimental work, as opposed to designing for mass production, the dynamics involve only a few degrees of freedom and it is often sufficient to build a device and carry out some form of system identification in order to determine pertinent parameters, such as frequencies, quality factors, and nonlinear characteristics, that describe the behavior of the device.

The small scales involved here give rise to effects that are insignificant at the macroscale, and this is the most interesting aspect of modeling these systems. A particularly important class of forces are those arising from electrostatic effects. If two elements of a system experience a voltage difference between them, attractive Coulomb forces arise. The simplest model for this effect is to consider a parallel-plate capacitor with gap g, which generates an attractive force between the plates of the form

$$F_e = \frac{\varepsilon A V^2}{2g^2}, \tag{1}$$

where ε is the permittivity of the space, A is the area of the electrodes, and V is the voltage difference between the plates (this simple expression ignores fringe field effects). It is important to note that F_e will, for any non-zero voltage, possess a DC term. Furthermore, if the voltage is composed of DC and AC terms,

$$V(t) = V_{DC} + V_{AC}\cos(\omega t), \tag{2}$$

then F_e is composed of a DC component as well as harmonic components with frequencies ω and 2ω. In fact, the DC component of the electrostatic force depends on V_{AC} as well as V_{DC}. These facts have some interesting consequences on the behavior of systems as one sweeps through the excitation parameters V_{AC} and ω. This form also allows for tuning of the excitation, through independent manipulation of V_{DC} and V_{AC}.

Perhaps the simplest nonlinear model for a micro/nanoelectromechanical system can be developed by considering a two-plate capacitor with one plate mechanically fixed and the other, of mass m, flexibly suspended such that it can move relative to the fixed plate. Ignoring gravity, let y denote the inertially-measured displacement of the movable element such that $y = 0$ is the equilibrium of the system for zero

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voltage, that is, for $V = 0$ ($F_e = 0$). Also, let g_o denote the gap corresponding to $y = 0$. In this case the equation of motion for the suspended mass m can be expressed as

$$m\ddot{y} = F_m(y) + F_e(y,t) + F_d(y,\dot{y}) + F_a(t), \tag{3}$$

where $F_m(y) = -F_m(-y)$ is the (assumed symmetric) conservative mechanical restoring force provided by the suspension, F_d accounts for dissipative effects [with $F_d(y,0) = 0$], and F_a accounts for time-dependent, externally-applied excitations. In the case of no external excitation ($F_a = 0$) and no AC voltage ($V_{AC} = 0$), the equilibrium position is given by solving

$$F_m(\bar{y}) + \frac{\varepsilon A V_{DC}^2}{2(g_o - \bar{y})^2} = 0, \tag{4}$$

that is, by balancing the mechanical and electrostatic forces that result from the DC voltage. At this stage one can already uncover some interesting nonlinear behavior related to this equilibrium condition, namely, the pull-in instability, which will be considered in a subsequent section. In terms of dynamics, one can develop a generic model that captures much of the important nonlinear behavior by defining the local coordinate $x = y - \bar{y}$, and expanding the equation of motion out to cubic order in x and its derivatives, resulting in,

$$\begin{aligned} m\ddot{x} &+ (c_1 + c_3 x^2)\dot{x} + \left\{ k_1 + 3k_3\bar{y}^2 - \frac{2\gamma[V_{DC} + V_{AC}\cos(\omega t)]^2}{(g_o - \bar{y})^3} \right\} x \\ &+ \left\{ 3k_3\bar{y} - \frac{3\gamma[V_{DC} + V_{AC}\cos(\omega t)]^2}{(g_o - \bar{y})^4} \right\} x^2 \\ &+ \left\{ k_3 - \frac{4\gamma[V_{DC} + V_{AC}\cos(\omega t)]^2}{(g_o - \bar{y})^5} \right\} x^3 \\ &= F_a(t) + \frac{\gamma\left[2V_{DC}V_{AC}\cos(\omega t) + V_{AC}^2\cos^2(\omega t)\right]}{(g_o - \bar{y})^2}, \end{aligned} \tag{5}$$

where γ is a coefficient which accounts for the permittivity of the surrounding medium and the area of the electrodes, k_1 and k_3 are linear and cubic nonlinear mechanical stiffness coefficients, and c_1 and c_3 are linear and nonlinear damping terms used to capture important dissipative effects [14]. Note that the form of the voltage excitation renders a number of interesting effects. First, it generates both direct and parametric excitations, and, in particular, leads to a parametric excitation that acts on the entire restoring force, including its linear and nonlinear components. In fact, when one drives the system near resonance, that is, when ω is close to the natural frequency, there also exists parametric driving terms at 2ω, which are capable of inducing parametric instabilities. Therefore, one cannot apply a resonant excitation to such a system without at least the possibility of these additional resonant interactions. In addition, the AC voltage leads to shifts in the time-invariant linear and nonlinear coefficients, since $\cos^2(\omega t) = \frac{1}{2}[1 + \cos(2\omega t)]$ results in a DC term. This implies that the linear natural frequency and the strength of the nonlinearities (which cause shifts in the frequencies of oscillation) both depend on the amplitude of the excitation. This does not occur in most macroscale mechanical systems, wherein one can view the excitation to be applied to a system with a fixed natural frequency and nonlinearity.

In some cases it is desirable to have a simpler form for the excitation. Accordingly, one can utilize a voltage of the form $V(t) = V_o\sqrt{1 + \alpha\cos(\omega t)}$ with $|\alpha| \leq 1$, which renders a single frequency AC component of amplitude αV_o^2 and frequency ω, in addition to the DC component V_o^2 (see, for example, [45]). In this case the AC voltage (captured by the parameter α) has only an ω harmonic, but the other effects described above are present. The key to this excitation is that one can independently drive the system into primary or parametric resonance, without interference. If one drives the system with ω near the natural frequency, direct excitation is achieved (in this case the parametric terms are non-resonant). Similarly, by driving ω near twice the natural frequency parametric resonance is obtained (here the direct excitation term is non-resonant).

A particularly important embodiment of electrostatic effects occurs in comb drives. These components use arrays of fingers, as shown in Fig. 1, to allow for an effectively large area over which voltage differences can act. In the more common case of interdigitated comb fingers, shown in Fig. 1a, two sets of combs move relative to one another in such a manner that the change in area generates a change in capacitance, which results in an electrostatic force of the form

$$F_{ic} = \frac{bV^2}{g}, \tag{6}$$

where b is a coefficient that depends on the permittivity of the space, the geometric particulars of the combs, and the number of comb fingers, g is the gap between adjacent fingers, and V is the voltage applied across the fingers. This arrangement is used to induce direct excitation forces which act along the longitudinal axes of the comb fingers. This arrangement can also be used to capacitively sense the motion of a given device. Another, less traditional, arrangement exists in non-interdigitated comb drives (shown in Fig. 1b). Here the combs move perpendicular to the finger axes, and the forces are generated by fringe field effects. The most interesting feature of this arrangement is that when the fingers are aligned directly across from one another, or evenly staggered, zero net force is generated. If this configuration corresponds to the mechanical static equilibrium of the device, forces arise only when the symmetry of the finger alignment is broken. A general form for the force in this situation, in terms of the dis-

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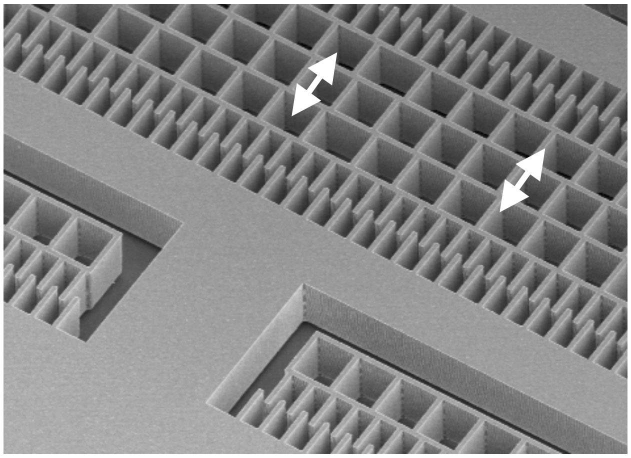
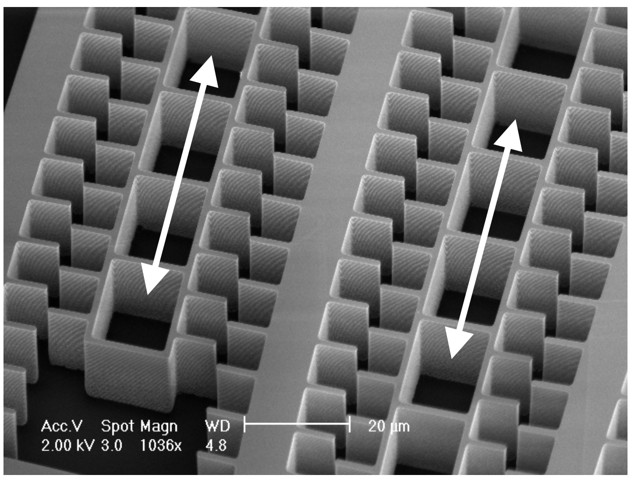


(a)



(b)

Figure 1. (a) AN INTERDIGITATED ELECTROSTATIC COMB DRIVE. (b) A NON-INTERDIGITATED ELECTROSTATIC COMB DRIVE. NOTE THAT THE INCLUDED ARROWS INDICATE THE DIRECTION OF DOMINANT MOTION FOR THE MOVEABLE PART OF THE DEVICE. THE OTHER BANKS OF COMB FINGERS ARE FIXED (PICTURE COURTESY OF B. DEMARTINI).

placement x from the symmetric alignment position, is

$$F_{nic} = f(x)V^2, \tag{7}$$

where V is the voltage across the combs and $f(x)$ describes the displacement-dependent nature of the force, which depends on a number of geometric factors [46]. Note that in the symmetric cases $f(0) = 0$, and that if there are a large number of fingers (such that end effects are essentially negligible), $f(x)$ is essentially periodic, that is, $f(x+s) = f(x)$, where s is the spacing between fingers. Locally near the equilibrium position, the electrostatic force can be linearly or nonlinearly hardening or softening [46]. Note that this arrangement is a convenient way to generate parametric excitation in MEMS, and is utilized extensively in the works described in the later section on Parametrically-Excited Systems. It should also be noted that though one generally restricts the movement of non-interdigitated devices such that $x << s$, interesting dynamics, including chaos, can arise for larger motions [47]. In summary, comb drives lead to equations of motion of the general form given above in Eq. (5), albeit with different forms for the coefficients. Perhaps the most important aspect of these components is that one can design them to produce desired linear and nonlinear forces, and, in fact, tune these forces by adjusting the amplitude of DC and AC excitation voltages [46, 48], or even the comb-finger geometry [49].

The particular details of Eq. (5) are not as important as its general form, which is encountered in a number of situations involving electrostatic, and in some cases piezoelectric or electromagnetic, actuation. For example, it is found in the modeling of electrostatically-actuated torsional devices, which are mounted by mechanical supports with torsional flexibility and actuated by placing electrodes away from the axis of rotation, thereby generating driving torques. Likewise, these effects arise in multi-degree-of-freedom systems, in which the voltage differences between adjacent resonators produce coupling effects. With this in mind, the essential features of the systems of interest are those of lightly-damped nonlinear resonators (or a set of coupled resonators) which are subjected to periodic excitations, either direct, parametric, or combined in nature. Arguably the most unique feature of these systems is the manner in which the excitation and parameter coefficients are intertwined, and the fact that one can use a variety of actuators to tune devices and achieve complex, yet tailored, dynamic behaviors.

Before closing this section on modeling, it is worth mentioning some of the other types of forces that arise at small scales, and are thus not important in macroscale resonators. The first of these are the van der Waals forces, which arise from atomic-level interactions. These forces are very weak except at close distances (approximately 1-10 nm), and become increasingly attractive at shorter interaction lengths. At very close distances, however, these forces become repulsive, since electron orbitals cannot overlap. This effect is very strong at extremely close interaction distances, and in fact, becomes essentially unbounded near contact (to avoid penetration). These combined attraction and repulsion effects, which are particularly important in probe-based microscopy applications, are conveniently modeled by Lennard-Jones potentials, which capture the qualitative nature of the interactions. Another small-scale force, which is entirely non-classical and is very strong at scales of $\sim$10 nm, is the Casimir-Polder force [50, 51]. This is a purely quantum effect and arises from zero-point fluctuations of the quantum field of the empty space between two uncharged conductors separated by a distance g. This is an attractive force that scales like g^{-4} for simple geometries. Basic notions from harmonically-forced nonlinear oscillations were instrumental in providing some of the first experimental evidence of these forces [50].

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PULL-IN

The competition between elastic restoring forces and attractive electrostatic or van der Waals forces acting on a mechanical structure leads to a situation in which an instability can occur, essentially a form of buckling. In fact, the most commonly used model to demonstrate nonlinear behavior in MEMS is a mass suspended by a linear spring with a DC voltage applied between the mass and a fixed electrode. For small voltages there exists two stable static equilibrium for this system, one near the purely-mechanical equilibrium and another associated with contact, which have an additional unstable equilibrium in-between. As the DC voltage is increased (or, for example, the zero-voltage gap between electrodes is decreased) the original, non-trivial stable equilibrium and the unstable equilibrium move towards one another. At a critical voltage these two equilibria coalesce in a saddle-node bifurcation, leaving only the stable contact equilibrium, and so-called "jump to contact" occurs. This overall behavior is referred to as "pull-in". Likewise, the critical saddle-node point is known as the "pull-in instability". As a rule of thumb, if the mechanical spring is linear, this instability will occur if the system experiences an amplitude of about 1/3 of the initial zero voltage gap [52]. A number of studies have considered the effects of more complicated geometries and flexibility in the mechanical structure. See, for example, the works of Krylov et al. [53–56].

Of interest here is the dynamic version of pull-in. Specifically, it is of interest to know how a system behaves near the pull-in instability described above. The simplest model is for a SDOF system of the type described above, whose global dynamics can be described by a relatively simple phase plane. In the bistable situation it has a classic form with two stable equilibria and a saddle point, and the basins of attraction for the stable equilibria are separated by the stable manifold of the saddle point (one of the stable equilibria is at a boundary in the phase plane, imposed by the constraint of the fixed electrode, but this does not alter the qualitative picture). As the parameter is varied a saddle-node bifurcations occurs in which the saddle point coalesces with the stable equilibrium at a critical value of the DC voltage. This model for the dynamics is well understood, and can also be derived for continuous systems with distributed electrostatic forces, so long as the mechanical system response is dominated by a single mode [57]. A systematic investigation of this situation, involving experimentation and comparison with a mathematical model for a microbeam, is described by Krylov and Maimon [58]. They consider a three-mode beam model with an attached lumped mass, and also include the effects of nonlinear squeeze film damping, which plays an important role in the dynamics near the instability point. They consider the system response to step inputs starting from the non-contact equilibrium, and examine condition that lead to jump to contact. Experimental measurements of this transient behavior are remarkably well described by the model. Krylov [59] has also demonstrated that Lyapunov exponents provide a reliable indicator of the beam response for these systems, indicating whether or not jump to contact will occur. Such results are of practical importance in applications such as switches, where the instability is repeatedly encountered. Krylov et al. [60] have also carried out a systematic analytical and experimental investigation of systems which experience both buckling and electrostatic instability, and have examined the interplay between these effects.

More subtle is the situation when periodic excitation is added to the system. In this case the situation is ripe for complicated dynamics, including fractal basin boundaries and chaos, as is known to occur when periodic excitation is imposed on systems with multiple equilibria [61]. Ashab and co-workers were the first to consider this problem from a modern dynamical systems point of view [62–64] . The authors used a global analysis approach, including Melnikov's method, to address the problem of chaos and fractal basin boundaries between the non-contact and contact steady-states in a system where the attractive force is van der Waals. Similarly, Luo and Wang [65] considered global dynamics in electrostatically-actuated systems with periodically time-varying capacitance. Lenci and Rega [66] have shown that one can manipulate the appearance of fractal basin boundaries, and thus the subsequent appearance of chaos, by judiciously adding higher harmonic components to the excitation. A number of analytical studies have been carried out that make use of the amplitudes of steady-state responses in relation to the pull-in response amplitude in order to develop approximate dynamic pull-in threshold criteria [67–69]. Some studies have shown that one can actually drive these systems with combined AC and DC voltage amplitudes $V_{DC}+V_{AC}$ that exceed the pull-in value of V_{DC}, which implies a type of dynamic stabilization against pull-in. For example, Ai and Pelesko [70] have carried out a detailed mathematical analysis of the existence of periodic responses for a simple SDOF model with parallel-plate capacitive actuation, including the limiting cases of damping dominated (for which inertia is ignored) and zero damping. They mathematically prove that in the zero damping case there exist frequency values for which the AC amplitude can exceed the DC pull-in value without experiencing the instability. Fargas-Marques et al. [52] have experimentally demonstrated this effect and offer another analytical predictive approach using energy methods. Likewise, Krylov and collaborators have demonstrated stabilization against pull-in using parametric excitation [71].

SYSTEMS WITH DIRECT EXCITATION: PRIMARY RESONANCE

Of the numerous investigations of nonlinearity in micro/nanoelectromechanical resonators, the earliest, and certainly most prevalent, are those focusing on systems with Duffing, or Duffing-like, characteristics (note that the term is used in its loosest sense here in an attempt to accommodate systems with asymmetric potential functions), which arise from Taylor Series expansions of elastic restoring forces and/or electrostatic, electromagnetic, or piezoelectric effects. The present section summarizes the various research investigations of these systems when

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they are subjected to direct (as opposed to parametric) excitation, and the primary resonance is excited. These works begin with largely-observational studies published in the late-1980s and early-1990s, and then continue with the more mature analytical and experimental investigations of the late-1990s and 2000s. The section concludes with a review of directly-excited nanoscale resonators, a brief discussion of application-specific literature, and an overview of recent efforts emphasizing dynamic transitions between bistable states.

To the best of the authors' knowledge, the earliest report of nonlinearity in a resonant microsystem appeared in the 1987 work of Andres, Foulds, and Tudor [72]. In this effort, the authors utilized a silicon microbeam, driven by an adjacent piezoelectric element, to recover a frequency response with distinct hardening characteristics. Though not explicitly detailed within the work, the observed nonlinearity appears to arise from large-amplitude effects, induced by driving the resonator with a hard excitation in a low-pressure environment ($Q = 21,000$). Subsequent to the publication of Andres et al.'s work, a number of research papers reported observations of nonlinear frequency responses, attributable to Duffing-like effects, in alternative contexts. Ikeda, Tilmans, and their respective collaborators, for example, reported hardening responses in resonant strain gauges undergoing large elastic deformations [73–75]. Likewise, Nguyen, Legtenberg, Bourouina, Piekarski, and their respective collaborators, noted hardening responses in electrostatically-actuated, and subsequently electromagnetically- and piezoelectrically-actuated, devices [76–80]. These latter works are of particular note, as they drew attention to the fact that nonlinearities in resonant microsystems can arise from multiple sources. Specifically, the authors noted the potential for interplay between nonlinearities arising from mechanical mechanisms, which are often hardening in the first mode, and transduction effects, which are often softening. This observation provided an important research framework for the efforts that would follow.

Building upon the works detailed above, Ayela and Fournier [81] reported in 1998 what is believed to be the first microresonator frequency response with softening characteristics. Though there appears to be some discrepancy regarding the source of the softening response, the authors stated that it arose from mechanical sources. In 2000, Camon and Larnaudie [82] observed softening response characteristics in an electrostatically-actuated micromirror excited by a comparatively-large actuation voltage. Though not directly addressed in the paper, the softening nature of the recovered response was almost certainly due to the dominance of softening electrostatic nonlinearities, especially in light of the large angles of deflection that were reported.

In 2002, Abdel-Rahman, Nayfeh, and Younis presented the first of several early, purely-theoretical works detailing the nonlinear behavior of electrostatically-actuated microbeams [83–87]. These works, which approached microsystems analysis from a classical dynamical systems perspective, emphasized the development, and subsequent reduction, of robust, distributed-parameter models, incorporating both mechanical and electrostatic nonlinear effects. The authors subsequently utilized these models in conjunction with multiple-scales and numerical analyses in order to identify the various nonlinear behaviors that could be recovered with a capacitively-driven microbeam. These nonlinear behaviors included not only the classical, hysteretic responses associated with direct excitations applied in the presence of Duffing-like nonlinearities, but also super- and sub-harmonic resonances, internal resonances, and limit cycles.

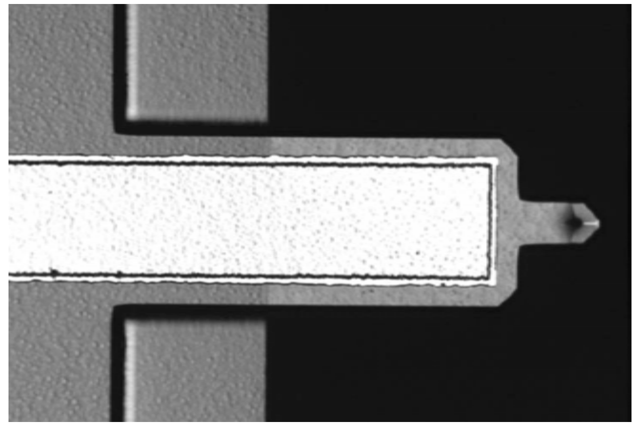

Figure 2. A REPRESENTATIVE PIEZOELECTRICALLY-ACTUATED MICROBEAM RESONATOR – A VEECO DMASP PROBE.

From 2002 onward, analytical and experimental investigations of directly-excited microresonators flourished. Amongst the various analytical and experimental efforts focusing on electrostatically-actuated systems, for example, were: (i) the 2004 effort of Kaajakari et al. [88], which examined the nonlinear response of silicon, bulk acoustic wave (BAW) resonators, demonstrating that these systems, in comparison to their flexural counterparts, had appreciably-higher energy storage capabilities; (ii) the 2005 work of Jeong and Ha [89], which developed a predictive model for the linear displacement limits of comb-driven resonant actuators; (iii) the 2007 and 2008 efforts of Agarwal et al. [90, 91], which detailed the modeling, analysis, and optimization of double-ended-tuning-fork resonators simultaneously actuated by two, variable-gap electrostatic forces; and finally, (iv) the 2008 investigation of Shao et al. [92], which thoroughly detailed the nonlinear response of free-free microbeam resonators.

Apart from the aforementioned investigations of electrostatically-actuated microresonators, a number of works addressed Duffing-like behaviors that arise in directly-excited piezoelectrically-actuated devices. Foremost amongst these are the works of Li, Dick, Mahmoodi, and their respective collaborators [93–96]. The first of these [94], by Li et al., detailed the modeling and analysis of the clamped-clamped, piezoelectrically-actuated microstructure previously described in [80]. In this work the authors utilized composite beam theory to develop a distributed parameter representation of their nonlinear system. This model was subsequently reduced to a

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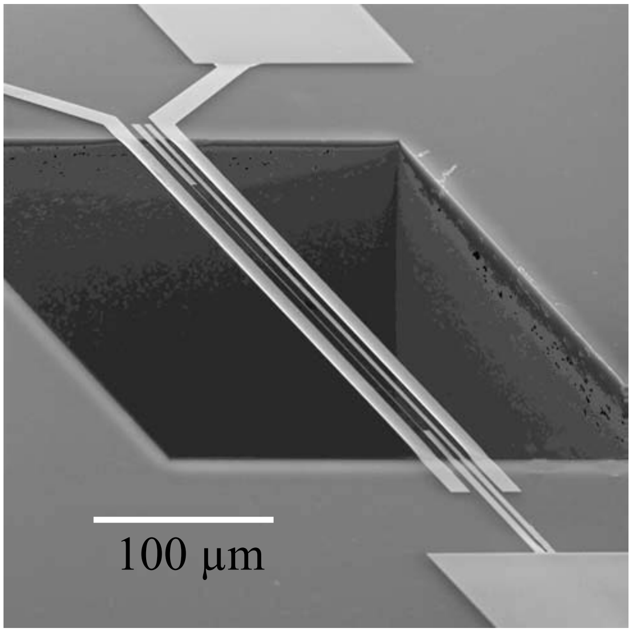

Figure 3. AN ELECTROSTATICALLY-ACTUATED NANORESONATOR (PICTURE COURTESY OF E. BUKS).

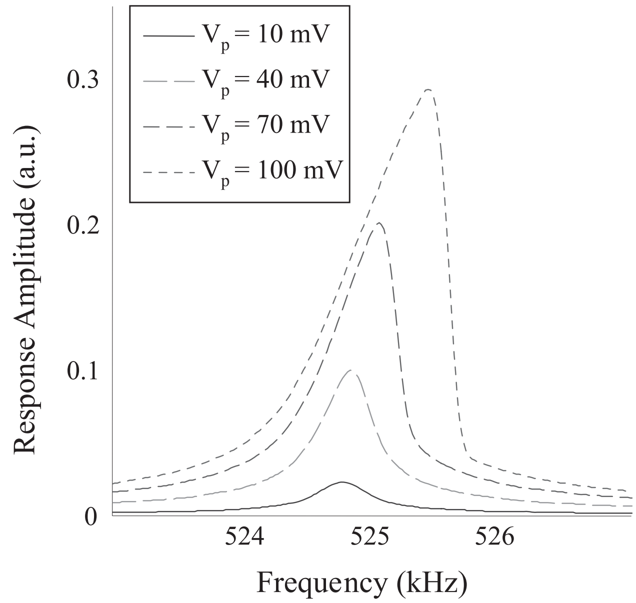

Figure 4. NONLINEAR FREQUENCY RESPONSE OBTAINED FROM THE DEVICE DEPICTED IN FIG. 3. NOTE THAT BECAUSE THESE RESPONSES WERE OBTAINED SOLELY THROUGH UP-SWEEPS, HYSTERESIS IS NOT EVIDENT (PICTURE COURTESY OF E. BUKS).

lumped-mass analog, and used to characterize the free and forced responses of the device in its post-buckled configuration. The work of Dick et al. [93] extended this effort, demonstrating that parametric identification techniques could be used to extrapolate pertinent model parameters, which, in turn, facilitated predictive design. The later works of Mahmoodi et al. [95, 96] detailed the nonlinear characteristics of a silicon microcantilever actuated by a ZnO patch. In these works the authors utilized the constitutive relationships of the piezoelectric material and Euler-Bernoulli beam theory to develop a distributed-parameter representation of their system. This model was subsequently reduced, using Galerkin methods, and experimentally validated through the use of a Veeco DMASP probe, such as that shown in Fig. 2. Somewhat surprisingly, the acquired analytical and experimental frequency responses depicted a distinct softening nonlinearity, which softened further with increasing voltage. This was attributed to the dominance of material nonlinearities.

As nanofabrication techniques matured, there was a re-birth of largely-observational reports of Duffing-like nonlinearities, this time in directly-excited nanosystems. In 1999, for example Evoy et al. reported the existence of hardening frequency response characteristics in nanofabricated paddle resonators [97]. This would be followed by subsequent reports of hardening responses in suspended, electrostatically-actuated carbon nanotube resonators [98], magnetomotively-excited SiC resonators [99], platinum and silicon nanowires [100, 101], and electrostatically-actuated nanobeams [102, 103], an example of which is shown in Fig. 3. A key distinction between these reports and earlier reports of Duffing-like behaviors at the microscale was that the nonlinear responses arising in nanoresonators appeared to be endemic to the devices. This was rectified by the work Postma et al. [104] in 2005, which demonstrated that many nanoscale resonators exhibit a severely-diminished dynamic range, and, thus, when driven above the thermomechanical noise floor, can transition into a nonlinear response regime rather quickly.

While the majority of studies detailed above approached the investigation of nonlinear behaviors from a largely application-independent perspective, a number of works over the past decade have addressed these nonlinear behaviors within the context of very specific applications. Amongst these various works are efforts focusing on electromechanical signal processing, signal amplification, resonant mass sensing, and magnetic field detection, as briefly highlighted below.

One of the earliest efforts emphasizing electromechanical signal processing in a directly-excited system with Duffing-like nonlinearities is that of Erbe and collaborators from 2000 [105]. In this work, the authors reported the ability to mechanically mix two direct excitation signals with a magnetomotively-driven nanoresonator. Specifically, the authors demonstrated that by driving the nanomechanical resonator with two hard excitation signals, one slightly detuned in frequency from the other, they could recover a mechanical output with distinct, tunable sidebands. In 2005 and 2006, Alastalo and Kaajakari investigated the impact of electrostatic and mechanical nonlinearities on capacitively-coupled filters [106, 107]. The first of these efforts utilized a lumped-mass model of the electrostatically-actuated system to characterize the filter's third-order intermodulation and intercept points. The latter effort built upon this work, incorporating out-of-band distortion mechanisms and developing a

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closed-form expression for the filter's signal-to-interference ratio. Building upon the success of prior nonlinear electromechanical signal processing efforts, Koskenvuori and Tittonen recently reported the development of a micromechanical down-converter [108]. This work demonstrated, for the first time, that amplitude-modulated signals in the GHz frequency range could be converted into MHz-range signals through the use of a double-ended-tuning-fork resonator. As noted in [109], these positive results could potentially stimulate the development of MEMS-based radios.

Apart from the investigations of directly-excited micro/nanoresonators detailed above, a series of works by Almog and collaborators have investigated the potential of Duffing-like nonlinearities in signal amplification and noise squeezing applications [110, 111]. The first of these works, which appeared in 2006, utilized signal mixing techniques, similar to those described above in relation to the work of Erbe et al. [105], to realize a signal amplifier capable of yielding high intermodulation gains. In particular, the authors demonstrated that by exciting an electrostatically-actuated microresonator with two direct excitation signals – a weak carrier signal and an intense pump, driven in the proximity of the system's saddle-node bifurcation – amplifier gains approaching 15 dB could be recovered. The latter work, appearing in 2007, adopted similar methods, in conjunction with a homodyne detection scheme, to realize phase-sensitive signal amplification and noise squeezing in a nanomechanical resonator. Similar principles to those utilized in this latter work were also proposed for use in resonant mass sensing applications by Buks and Yurke in 2006 [112].

Though application-specific investigations of nonlinear, directly-excited micro/nanoresonators with Duffing-like nonlinearities have primarily emphasized signal amplification and signal processing applications, a handful of works, in addition to the work of Buks and Yurke detailed above, have highlighted alternative uses. The 2005 work of Greywall [113], for example, proposed the development of a sensitive magnetometer based on a Duffing-like resonator embedded in an oscillator circuit. This device utilized magnetic-field-induced changes in damping to alter the resonant amplitude of a current-carrying, clamped-clamped beam. This approach resulted in a sensor design theoretically capable of detecting ppm changes in the Earth's magnetic field. It should be noted that a recent effort by Choi et al. proposed a similar technical approach [114]. The recent work of Liu et al. examined the impact of Duffing-like nonlinearities in an alternative context, namely MEMS microphones. In this effort, the authors detailed the lumped-mass modeling, analysis, and experimental validation of a dual-backplate system. Particular emphasis was placed on system identification, within the paper, as a well-defined system model was deemed essential to future closed-loop, force-feedback studies. Another novel use of primary resonance in nonlinear systems is the work of Chan et al. [50], which used nonlinear resonance measurements of a microscale torsional resonator to verify the existence of the Casimir force, a purely-quantum effect.

Though the notion of nonlinear tuning in micro/nanoscale resonators can be traced to the works of Adams et al. [46], amongst others, a series of works have recently re-visited the issue within the context of directly-excited resonators. The works of Agarwal et al. [115, 116], for example, detailed how, with proper tuning and the effective cancelation of nonlinearity, at least to first order, improved power handling characteristics could be recovered. Specifically, the authors demonstrated how second-order and third-order electrostatic effects could be leveraged against third-order mechanical hardening nonlinearities to 'straighten' the nonlinear backbone. Similar results are detailed in the recent work of Shao et al. [117]. Kozinsky and collaborators subsequently extended the approach to the nanoscale in their investigation of fixed-fixed nanoresonator actuated through magnetomotive and electrostatic effects [118]. In this work the authors demonstrated the feasibility of bi-directional linear frequency tuning and nonlinear reduction. The latter method proved highly-advantageous to dynamic range enhancement, with the authors reporting more than 6 dB of dynamic range improvement in their initial work.

Despite the present section's emphasis on conventional, hysteretic behaviors, it is important to note that a number of research efforts have identified or predicted the occurrence of more-complex dynamical behaviors, including chaos, in directly-excited micro/nanoresonators with Duffing-like nonlinearities. In 2002, for example, Scheible and collaborators noted the existence of chaos in a 'clapper' nanoresonator – a free-free beam resonator suspended by intermediate point – excited through magnetomotive and electrostatic effects [119]. Specifically, the authors demonstrated, through experiment, that in applications like signal mixing, where multiple frequencies are present, there is some degree of likelihood for the emergence of chaos, principally arising through the Ruelle-Takens route. This observation would be further addressed by Gottlieb et al. [120] in 2007. In contrast to the works of Scheible and Gottlieb, Liu and collaborators in 2004 predicted, using simulation, the emergence of chaos in closed-loop, electrostatically-actuated microcantilevers through period-doubling routes [121]. Similar results were reported in the analytical investigations of De and Aluru [122,123]. In 2008, Park et al. [124] noted the potential for chaos control in micro/nanoscale systems. Specifically, the authors demonstrated, through simulation, that by adopting an appropriate feedback rule chaotic solution trajectories could be effectively converted into periodic responses. This was shown to not only increase the operating range of the electrostatically-actuated microresonator of interest, but also increase its effective power output.

While the bulk of efforts described above emphasize steady-state behaviors, transient responses are of distinct interest, as well. The utility of transitional behaviors in resonant micro/nanosystems was first addressed by Aldridge and Cleland in 2005 [125]. In this work, the authors demonstrated that noise-induced transitions between bistable states in a nanomechanical, Duffing resonator could be used to extrapolate the system's

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resonant frequency and cubic nonlinearity with a degree of accuracy unobtainable in a linear system. This, as the authors noted, facilitates highly-sensitive parametric sensing. Building upon Aldridge and Cleland's work, Stambaugh and Chan characterized noise-activated switching in a torsional micromechanical resonator with softening characteristics [126, 127]. Though the premise of these latter works were quite similar to their predecessor, it should be noted that an alternative activation energy scaling was reported. The latter works' conclusions were recently confirmed in the work of Kozinsky and collaborators [128], which provided experimentally-measured basins of attraction in the bistable region.

While the investigations of transient behaviors in bistable micro/nanomechanical resonators are still in their infancy, a number of practical applications have been reported. In 2004, for example, Badzey et al. detailed the development of a controllable switch based on a Duffing-like response [129]. In this work the authors applied a square-wave modulation signal to a clamped-clamped nanobeam driven near resonance to induce dynamic transitions between the resonator's bistable states. The binary nature of the resulting time response was viewed as an ideal platform for primitive memory elements. Subsequent investigations furthered this work by addressing the temperature-dependence of the dynamic switching event, as well as phase-modulation-induced switching [130, 131]. In 2006, Chan and Stambaugh, applied their previous investigations of noise-induced switching to micromechanical mixing, demonstrating that can be used to facilitate frequency down-conversion [132]. Almog and collaborators extended this approach for signal amplification purposes [133].

PARAMETRICALLY-EXCITED SYSTEMS

Systems which experience time-varying parameters are said to be parametrically excited. This form of forcing arises in many macroscale systems, the prototype being the simple pendulum with vertical base excitation. Of interest here are MEMS/NEMS with periodic parametric excitation. This arises very naturally in the devices of interest, as one can see from an examination of the prototypical model developed in the preceding section on Modeling, in which time-varying voltages provide both direct and parametric excitation. The most basic model for this class of systems is the linear Mathieu equation,

$$\ddot{x}+\frac{2}{Q}\dot{x}+[\omega_0+\beta\cos(\omega t)]\,x=0, \qquad (8)$$

which is a simple oscillator with a time-varying stiffness term. The Mathieu equation has been widely studied, and governs the forced motion of a swing, the stability of ships, Faraday surface wave patterns on water, and the behavior of parametric amplifiers based on electronic or superconducting devices. A linear stability analysis predicts that parametric resonances occur at drive frequencies that satisfy $\omega = 2\omega_0/n$ where ω_0 is the system's natural frequency and n is an integer greater than or equal to unity [134]. When instability occurs, the steady-state response is governed by nonlinear effects, such as those described in the section on Modeling. It is worth noting that damping does not have a strong effect on the resulting steady-state amplitude, in contrast with the resonant response of directly-forced linear systems.

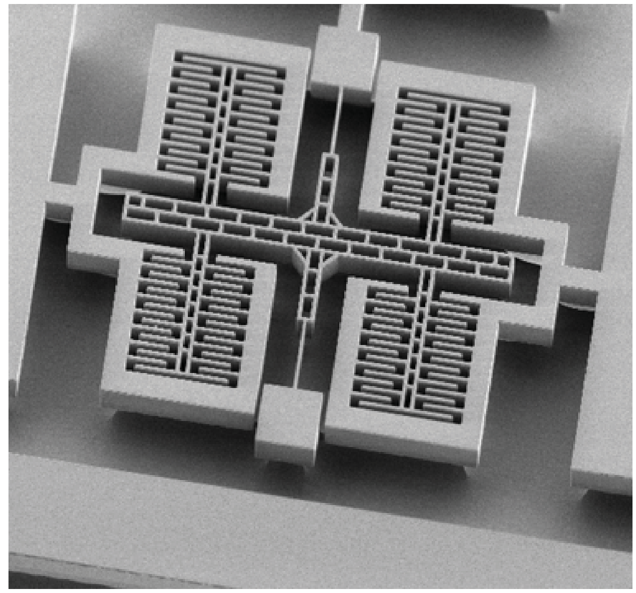

Figure 5. A PARAMETRICALLY-EXCITED TORSIONAL MICRORESONATOR DRIVEN THROUGH THE USE OF NON-INTERDIGITATED COMB DRIVES.

In macroscopic systems only the first instability region ($n = 1$) is typically observed, due to the levels of damping and the exponential narrowing of the regions with increasing n. However, in microsystems, the internal damping in resonators is small (when compared with macroscale systems), and the aerodynamics damping is rather predictable (and can be made very small by operating in vacuum). This leads to a condition wherein the parametric resonance effects are significantly more visible, and can be utilized in many applications. In 1997, Turner et al. [45] verified the existence of five such instability regions in a micro-electromechanical torsional oscillator. This oscillator, which is shown in Fig. 5, was an electrostatically-driven torsional device developed for data storage applications. An out-of-plane fringing field actuator gave this device its unique properties, wherein the electrostatic torque is non-zero only for non-zero angles of rotation. Following this initial demonstration at the microscale, this phenomena was seen in other torsional microresonators [135, 136], resonant microscanners [137–140], piezoelectrically-actuated micro/nanoresonators [141, 142], laser-heated nanoresonators [143], and nanowires [144–146].

Nonlinearity (from mechanical and/or electrostatic sources) proves to significantly impact key aspects of the behavior of parametrically-resonant micro/nanosystems. In 2002, Zhang et al., showed the effects of tuning on parametrically-excited mi-

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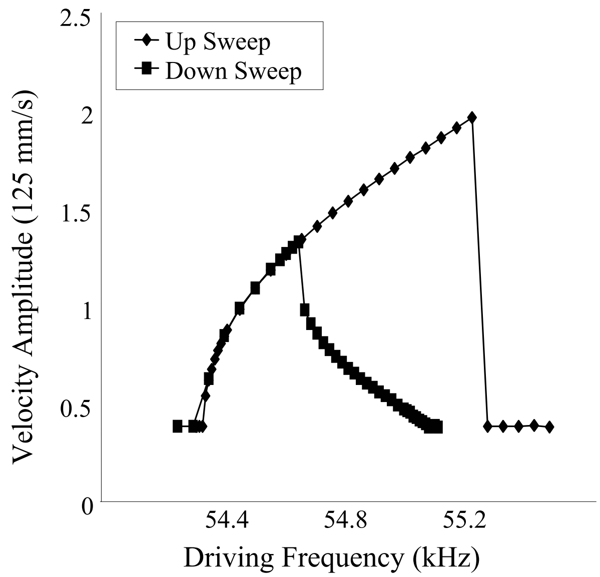

Figure 6. A REPRESENTATIVE FREQUENCY RESPONSE OBTAINED FROM AN ELECTROSTATICALLY-ACTUATED, PARAMETRICALLY-EXCITED RESONATOR. NOTE THAT THE MIXED RESPONSE CHARACTERISTICS REPORTED HERE ARE YET TO BE OBSERVED IN A MACROSCALE DEVICE (ADAPTED FROM [154]).

croresonators [147–149]. By designing a device which had fringing field actuators for both tuning and excitation, they were able to independently tune the linear and nonlinear stiffness coefficients of their device, thereby changing the shape of the instability region. Through this tuning, the device performance could be quantitatively and qualitatively adjusted merely by manipulating the amplitudes of the AC and DC excitation signals. This proved to be essential in expanding the application space of parametrically-excited micro/nanoresonators. Napoli and collaborators extended this result to include resonant microbeams, and verified the existence of nonlinear parametric instabilities in capacitively-actuated cantilevers [150, 151]. As detailed in later sections, the combined excitation of electrostatically-actuated microbeams has been studied more recently by Zhang and Meng [152, 153].

The accurate modeling of parametrically-excited micro/nanoresonators is essential to the effective design of devices with practical application. Rhoads, DeMartini, and collaborators [48, 69, 155, 156] improved on prior modeling efforts, and more comprehensively analyzed parametrically-excited microresonators, in order to determine the achievable parameter space, effectively explaining in greater detail previously obtained results [157]. Building upon this result, Rhoads et al. created a filter utilizing two coupled MEMS devices [149, 158]. One was tuned to be hardening, one was tuned to be softening, and when combined they created a bandpass filter. Although the frequencies were relatively low, as the devices were large, the stage was set for the later creation of logic elements using nonlinear tunable MEMS oscillators [48]. In addition, Rhoads et al. analyzed a model in which the parametric excitation acted on both linear and nonlinear stiffness terms in order to explain certain unique response characteristics, including the mixed hardening/softening behavior shown in Fig. 6 [154].

In parametric resonance the boundary between stability and instability is extremely sharp, and leads to a very dramatic jump in amplitude on the subcritical side of the parametric resonance zone (the frequency step in the original Turner result is 0.001 Hz [45]). In addition, in an unstable region, the amplitude achieved is determined not by the damping, as in a directly-forced linear oscillator, but by the nonlinearity in the system. Therefore, parametrically-forced systems are capable of significant amplitudes while operating in the unstable region. By tracking this stability boundary in parameter space, a very precise sensing mechanism can be exploited. This has been used with varying degrees of success in mass sensing, non-contact atomic force microscopy, and microgyroscopes.

The parametrically-forced mass sensor was the first microsensor to demonstrate the benefits of such a mechanism. Early attempts at parametric resonance-based mass sensing were completed by Zhang et al. [148, 159]. The sensitivity of this phenomena compares favorably to linear micro-oscillators, showing itself to be $\sim$1-2 orders of magnitude more sensitive in air operation. In this work, Zhang and collaborators detected humidity changes by measuring the mass of condensed water on a planar device. This device utilized a fringing-field, in-plane actuator of the type described in Adams et al. [46, 160]. This was followed up with a nanoscale investigation by Yu and collaborators [144], who considered mass-loaded nanowires. Furthering the work of Zhang et al. [161], Requa and Turner [162, 163] built a self-sensing mass sensor based on parametric resonance (see Fig. 7). This sensor was much smaller and more sensitive, and utilized Lorentz force sensing and actuation. Noise imposes limits on the resolution of these sensors, as described by Cleland [164], and as experimentally investigated by Requa and Turner [165]. Transitions to chaos are also possible in parametrically-excited nonlinear systems, and this was investigated in a MEMS resonator by DeMartini et al., both experimentally and by using Melnikov analysis on a system model [47]. This effort built upon the earlier work of Wang and collaborators [166].

Parametric resonance has also been applied to the design of microscale gyroscopes. The resonant Coriolis force sensor has been long-studied for rate measurement and inertial tracking (see, for example, [168]). Contrary to accelerometers, though, the Coriolis rate sensor requires at least two degrees of freedom. In its most straightforward application, the Coriolis force generated by external rotation couples two perpendicular modes, one for drive and the other for sensing. Ideally, in order to achieve full amplification the two modes are tuned to the same resonance frequency. However, in practice this is very difficult to achieve, and significant amplification gains are lost. Oropeza-Ramos et al. [169, 170] utilized the broadband response of parametric resonance to achieve robust amplification in a Coriolis force sensor

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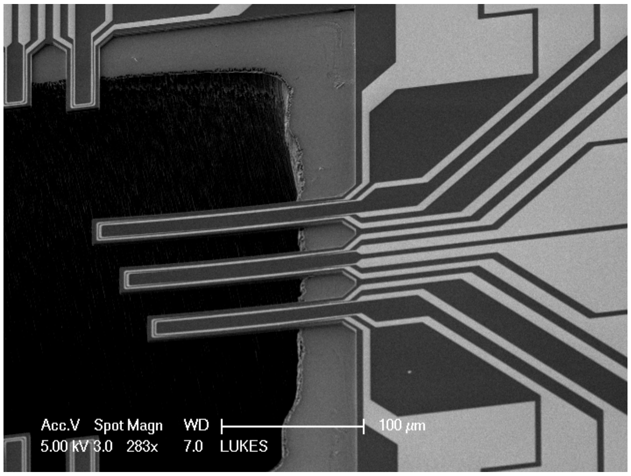

Figure 7. AN ELECTROMAGNETICALLY-ACTUATED MICROBEAM RESONATOR DRIVEN BY LORENTZ FORCES (FROM [167], PICTURE COURTESY OF K. (LUKES) MORAN).

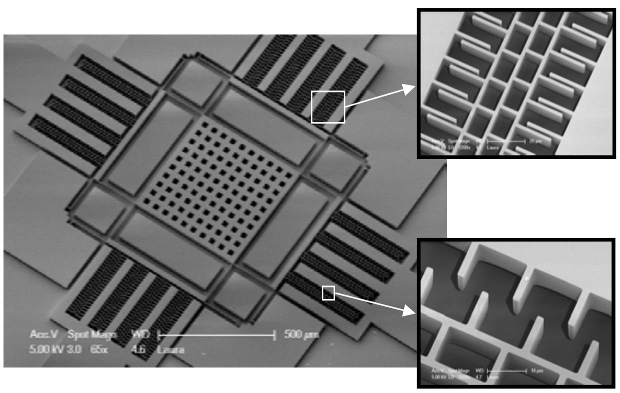

Figure 8. A PARAMETRICALLY-EXCITED GYROSCOPE (ADAPTED FROM [170], PICTURE COURTESY OF L. OROPEZA-RAMOS.).

(see Fig. 8). The sensor operation is similar to the linear types of rate sensor in that it is a single mass, electrostatically-driven in one axis, and electrostatically-sensed along a perpendicular axis. Here, however, the device is driven into its first parametric instability. This band of excitation is typically broader than a high-Q resonance mode, and is limited in amplitude only by nonlinearity. This allows for full sense mode amplification, even in the presence of fabrication-induced irregularities. The device can be driven at the frequency producing the maximum sense direction amplification, *without the need for tuning*. Although this early device was not optimized, the proof of concept is compelling. Subsequent analytical studies by Miller et al. using perturbation techniques have shown that these nonlinear devices can be tuned to achieve nearly-linear sense response in the rotation rate [171, 172].

The discussion above focuses on steady-state behavior resulting from harmonic, deterministic excitation. However, investigations of transient types of excitation, including frequency sweeps and noise, and combinations of these, are also of interest. The work of Requa and Turner [165] considers the effects of frequency sweep rates on the response of a parametrically-excited microbeam. Here there is a tradeoff, since faster sweeps lead to better sensor performance (in terms of response time), but pay a price in terms of precision, since the ability to precisely locate a subharmonic instability depends on this rate. Noise can also play an important role in these system. Chan and Stambaugh [173, 174] have carried out an analytical and experimental study of noise-induced switching between different steady-states in parametrically-excited microsystems. The effects of noise and sweep rates and their interplay, will be an increasingly important consideration as devices are reduced in scale.

SYSTEMS WITH COMBINED EXCITATIONS

Though the majority of micro- and nanoresonators independently utilize direct or parametric excitation, a number of resonant micro/nanosystems exploit the excitations simultaneously, using the combined result to render beneficial response characteristics. While there exists a fairly large number of combined excitation studies in the MEMS/NEMS literature, the vast majority of works published to date can be classified into two distinct groups: (i) efforts focusing on parametric amplification, and (ii) efforts focusing on nonlinear phenomena induced through combined excitations, which arise from Taylor Series expansions of electrostatic or electromagnetic forces. The current section details literature from each of these groups, placing particular emphasis on works that exploit dynamical phenomena that arise *solely* in the presence of a combined excitation. Works that exhibit combined excitation, yet appear to utilize direct or parametric phenomena independently, have been included in alternate sections.

The most prevalent investigations of combined excitation at the micro/nanoscale are those emphasizing low-noise parametric amplification – the process of amplifying a harmonic, external drive signal through the use of a parametric pump. While macroscale investigations of this linear phenomenon first appeared in the literature nearly fifty years ago [175–177], microscale implementations have garnered attention only since the early-1990s. In their now-seminal effort of 1991 [178], Rugar and Grütter demonstrated that the resonant response of a microcantilever, base-excited by a piezoelectric bimorph actuator, could be amplified by pumping the system with a parallel-plate electrostatic drive. In this work, the author's specifically utilized the degenerate form of parametric amplification – a phase-dependent variant, wherein the parametric pump is locked at twice the frequency of the resonant, direct excitation signal – to drive the system's resonant amplitude to approximately twenty times its unpumped state. This was achieved by exciting the system with a constant-amplitude harmonic signal and introducing a parametric excitation that was very near, but above, the Arnold tongue (i.e., the wedge of instability) associated with the onset

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of parametric resonance. By exploiting the phase-dependent nature of the degenerate amplifier's gain, the authors demonstrated that resonant amplitude reduction, subsequently termed parametric attenuation, could be readily obtained, as well.

In subsequent years, a number of works would build upon Rugar and Grütter's efforts by examining the feasibility of both degenerate and non-degenerate (where the parametric pump is locked at a frequency distinct from twice that of the external signal) parametric amplification in torsional microresonators [135, 157], optically-excited micromechanical oscillators [143], micro ring gyroscopes [179,180], MEMS diaphragms [181], micromechanical mixers [182,182], and resonant micro/nanobeams [183, 184, 184–186]. Of particular note amongst these latter efforts is the work of Olkhovets and collaborators, which demonstrated experimental gains in excess of 40 dB in a non-degenerate parametric amplifier based on two electrostatically-coupled torsional microresonators [136], and the work of Roukes and collaborators, which demonstrated experimental gains in excess of 65 dB in a degenerate amplifier based on a stiffness-modulated, fixed-fixed nanobeam [184].

Though the works noted above considered parametric amplification from a largely linear perspective, recent efforts have considered the effects of nonlinearity on parametric amplifiers [14, 187]. These largely analytical efforts were motivated by the discrepancies between analytically-predicted and experimentally-recovered gains reported in many of the works cited above. In an attempt to address these gain deficits, each of the efforts appended geometric nonlinearities to conventional, linear parametric amplifier models and quantified the resulting impact on amplifier gain. Not surprisingly, the works concluded that nonlinearity not only severely limits the gain of a parametric amplifier, but also renders frequency-response behaviors that are appreciable more complicated than those predicted by linear theory, due largely to the possible co-existence of resonances.

While the majority of works on combined excitations in MEMS and NEMS emphasize linear, parametric amplification, a handful of works have considered various nonlinear behaviors that arise in the presence of combined excitations resulting from a Taylor Series expansion of an electromagnetic or variable-gap, electrostatic force. The work of Zhang and Meng [152, 153], for example, considers the nonlinear response of an electrostatically-actuated microcantilever, driven by a combined excitation, operating in the presence of quadratic and cubic electrostatic nonlinearities and squeeze-film damping. This effort adopts harmonic balance methods, in addition to numerical techniques, in an attempt to identify regions of periodic, quasiperiodic, and chaotic response. Other notable efforts include those of Abdel-Rahman et al. (see, for example, [86]), which for the sake of categorization have been detailed above.

SELF-EXCITED SYSTEMS

As with macroscale systems, a number of MEMS/NEMS can experience self-excited oscillations. These are typically oscillatory system responses generated by an instability of an otherwise stable equilibrium, specifically, from a Hopf bifurcation. Such systems require an energy source coupled to an oscillator, with classical examples including aeroelastic flutter and wheel shimmy. In MEMS/NEMS applications self-excited behavior has been observed in optically-heated mechanical resonators, systems that emit particles, and systems that utilize feedback to generate limit-cycle behaviors.

Thermally-excited resonators have been considered in a number of studies by Zehnder and co-workers [188–191], in which a continuous wave (CW) laser provides a thermal input, which is coupled to the vibratory response of the heated element by thermoelastic effects. A particularly attractive feature of these systems is that one can add a small pump input to them, in the form of base excitation provided by a piezoelectric element, at either the main resonance frequency ω_n, or, by thermally modulating the stiffness, at $2\omega_n$. Doing so causes entrainment, that is, self-sustained oscillations, over a much wider bandwidth (by factors up to 400) than can be achieved using direct excitation [191]. These devices have potential for high-frequency signal processing and communications applications. The instability arises from the interaction of the disk with a continuous power laser, in which the absorbed and reflected components of the laser are modulated by the deflection of the disk via interferometric effects induced by the laser beam reflecting between the vibrating structure and a substrate. It is observed that there is a critical laser power above which the self-oscillations ensue, and small harmonic pump signals are added at frequencies near ω_n, using base excitation, which provides direct resonant forcing, or near $2\omega_n$, by thermally modulated the stiffness using a second laser. In both cases the response locks onto, or entrains, the input signal, resulting in a periodic output at the pump frequency over a relatively wide range of frequencies. Detailed numerical simulations of the model introduced in [191] demonstrate that it is sufficient to capture the experimentally-observed entrainment behavior [192]. Aubin et al. [188] consider in detail the mathematical model introduced in [191], compare it with experimental results, and also provide a number of references on prior work on optically self-excited microbeams. Their model, which focuses only on the self-excited behavior (and not the entrainment problem) includes a SDOF nonlinear oscillator and a first-order ODE for the thermal dynamics, which are coupled by the fact that the stiffness of the resonator depends on the temperature via thermoelastic effects, and the amount of power imparted to the structure from the laser depends on the displacement of the mechanical resonator through interferometric effects, as described above. This third-order, thermomechanical system has a number of parameters that are estimated using first principles and used to predict the onset of self-excited oscillations via a Hopf bifurcation. Good qualitative agreement is found between the theory and experiments, and for some cases the laser power threshold for self-excited responses can be accurately predicted. Pandey et al. [189] carry out a detailed perturbation analysis of a model that includes the direct base excitation and parametric excitation

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via thermal stiffness variation. The results show the underlying resonance structures that lead to the experimentally observed behavior for both the direct and parametric excitation. Sahai and Zehnder [190] carry out an analysis of entrainment and synchronization for a pair of coupled dome resonators, showing conditions for excitation and mistuning under which synchronous entrainment will occur.

Self-excited resonance behavior can be generated by providing destabilizing feedback that generates limit-cycle behavior. This has been carried out for a variety of MEMS, mostly in the area of sensors. An example of such a device is that described in Sung et al. [193, 194] who designed, fabricated, and tested a navigation-grade MEMS accelerometer using such an approach. In this system a self-excited oscillator is used to precisely track shifts in the accelerometer resonance frequency that arise from accelerations. A describing function approach is used to design a nonlinear feedback loop that results in good performance in terms of sensitivity and robustness. Similarly, a resonant chemical sensor based on a feedback-induced self-sustained resonator is described by Bedair and Fedder [195]. A patch on a cantilever beam is functionalized for the absorption of certain chemical gases so that its resonance frequency changes due to the added mass of the cantilever in the presence of the target chemical. Feedback circuitry is employed to generate a self-sustained oscillation, and changes in the frequency of this oscillation are successfully used for chemical detection.

Self-excited oscillations in a microresonator by alpha particle emission has been proposed and analyzed by Feng et al. [196]. The idea is to use particle emission to provide a type of follower force, akin to a fluid jet emitted from a pipe, although it is admitted in the paper that it would be difficult to experimentally achieve the forces required to generate self-oscillations.

In closing this section it should be noted that the term "self-excited" is not consistently used to designate limit-cycle behavior in the MEMS/NEMS literature, and sometimes refers simply to periodically-driven resonators.

VIBRO-IMPACT SYSTEMS

Vibro-impact problems arise in mechanical systems in which components make intermittent contact with one another. This type of interaction leads to a very severe nonlinear (hardening) behavior that is commonly characterized using models in which different smooth equations are valid in separate regions of the system's state space, and specific rules apply when the system response crosses the regional boundaries (for example, the Newtonian impact law). This field has been the subject of intense study in recent years, under the label "non-smooth systems", which are covered extensively in the recent monograph by di Bernardo et al. [197]. The area has a long history of investigation in mechanical engineering and mechanics, with applications in machinery dynamics and structural vibrations. The unique features of these systems are that they can undergo changes in system response that have no analogies in smooth system models. These include discontinuity-induced bifurcations that arise when responses interact in a particular manner with the boundaries that separate distinct regions of the system phase space [197]. Microscale systems have provided additional applications of this class of systems, most prominently in switching, positioning, and tapping-mode atomic force microscopy.[2] Of interest here are applications of micro- and nanoscale systems subjected to periodic excitation that can experience impacts. It is well known that such systems can experience a wide range of behavior, including all of the standard phenomena of nonlinear systems, plus the rich possibilities of non-smooth systems. Attention is focused here on positioning systems, switches, and bouncing mirrors.

Certain micro- and nanoscale positioning devices make use of impacts between two components, generally a driver and a slider, using impacts of the driver on the slider in order to overcome friction and nudge the slider (or the entire actuator) along. The use of small impact velocities allows for precise step-wise positioning of the slider along a single direction, and can do so over relatively large accumulated displacements. Such a device was fabricated on the microscale and tested by Mita [198], and it achieved incremental position resolution of $\sim$10 nm. In such systems the driver is a resonator with nonlinearities due to impacts (and possibly other effects), and the overall system is conveniently modeled by two lumped masses. Such a model was considered by Zhao and collaborators [199–201], who carried out a detailed analysis of the vibro-impact dynamics. They determined the influence of various system and input parameters using bifurcation analysis of a non-smooth system model. It was found that a wide range of dynamic behavior is possible, much of which, for example, chaos, is not suitable for precision positioning. It should be noted that this micro-positioning device is just one of many that have been developed, but it is one that has been examined with an eye towards understanding its nonlinear behavior in a systematic manner. The reader is referred to Zhao et al. [199] for a more thorough background on micropositioning systems that utilize impacts. Studies such as those of Zhao and co-workers should help guide future designs of vibroimpact positioning devices by allowing for systematic investigations of how system and excitation parameters influence performance. It is important to note that the dynamics must be studied using nonlinear methods, since impact dynamics are inherently nonlinear, and that brute-force simulation studies by themselves may not be sufficient, since these models can experience subtle transitions that are unique to non-smooth systems.

Switches form an important element in many IC devices, so it is natural to consider the use of micromechanical switches to replace their electrical counterparts. Mechanical vibroimpacting microswitches are a promising technology for which an understanding of nonlinear dynamics may play an important role. These elements are essentially oscillators that experience impacts in order to make electrical contact. They possess all of the rich dynamic features of macroscale vibro-impact systems, as

[2] As noted in the introduction the authors do not undertake a review of AFM dynamics here.

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well as all of the attendant difficulties. The main benefit of these systems, in terms of nonlinear behavior, is that one can control the bandwidth of the response by simply varying the input voltage amplitude. This can be imagined by considering an oscillator whose amplitude is limited on either side by rigid stops placed symmetrically about the equilibrium. When subjected to harmonic excitation, the frequency response can be roughly viewed as a usual linear resonance, except with an upper limit imposed by the mechanical stops that limit the amplitude. As one raises the forcing amplitude, the frequency range over which impacts must occur becomes wider, roughly in a symmetric manner centered at the resonance frequency. The width of this band can be estimated by a simple calculation using linear frequency response analysis, as follows. For a normalized frequency response of the form

$$|x| = \frac{\omega_n^2}{\sqrt{(\omega^2 - \omega_n^2)^2 + \left(\frac{\omega\omega_n}{Q}\right)^2}}, \tag{9}$$

where Q is the quality factor, setting $f|x| = a$, where f is the force amplitude and a is the limiting amplitude, solving for the resulting frequencies at which this amplitude condition is met, and taking the difference provides the bandwidth. By assuming $Q >> 1$ and $a >> 1$ the bandwidth is found to be approximated by the simple expression $f\omega_n/a$. Thus, it is seen that one can control the bandwidth by simply varying the force amplitude f, so long as impacts are maintained. Of course, impacts are a severely hardening nonlinearity, which leads to impacting steady-state motions that occur above the resonance, even coexisting with the smaller amplitude non-resonant response in this frequency range. It is also well known that the response of these systems can experience various bifurcations leading to subharmonic responses and chaos [202, 203]. However, near the main resonance the system is well behaved, undergoing motions that impact both stops in a symmetric manner. Two basic types of micro-vibro-impact switches have been considered to date, one is the type described above, in which the oscillator experiences impacts on both sides of equilibrium, and the other has its amplitude restricted on only one side. Ngyuen et al. [204] describe a centrally supported disk resonator that oscillates in plane, resonantly forced by squeezing the disk with electrodes placed on opposing sides of the device, say at 0 and 180 degrees. From Poisson effects the resonator "breathes" in such a manner that it simultaneously impacts electrical contacts placed at 90 and 270 degrees. Since the motion is dominated by the first in-plane mode (it has two such modes, and one is strongly preferred by the excitation), it behaves essentially like the SDOF impact oscillator described above. It is experimentally observed that the bandwidth indeed increases as the force levels are increased and the system experiences impacting responses further above the resonance than predicted by simply limiting the linear response. It should be noted that impacts generate high frequency vibrations that may interfere with the switch operation, so there is a tradeoff between increasing the bandwidth and minimizing these contaminating vibrations. In general, the transfer of energy to higher modes at impact can be included as part of an effective coefficient of restitution in the single mode model, since these vibrations typically are of small amplitude and dissipate relatively quickly. Zhang et al. [205] also considered two micro-switch systems, one in the form of a lumped mass moving in plane and striking a bumper, and the other a cantilever microbeam striking a substrate. In these systems a piecewise nonlinear model was employed, which showed some softening at amplitudes below the impact threshold. This generated frequency response curves that first bent slightly to lower frequencies as amplitudes were increased, and then experienced the drastic hardening due to impacts. Overall, the study showed that a simple SDOF model for these systems was quite adequate to capture the behavior near the main resonance.

A different class of applications is that of using impacts to tailor the harmonics of a system response to meet specific needs. An application of this is for video display technology, in which an oscillating mirror is used to scan a video signal. In order to generate a clean signal this scan should be of constant velocity in both directions of its motion during oscillation, resulting in a triangle wave. Bucher et al. [206] describe a clever multi-degree-of-freedom system based on linear behavior to achieve such a response in a microsystem. In contrast, Krylov and Barnes [207] have described an electrostatically-actuated tilting mirror that experiences a bouncing mode of operation that generates a triangular-wave response. Their model includes nonlinear effects from both electrostatic actuation and the rebound of the mirror against the bumpers, and is used to outline a systematic design and analysis of the desired nonlinear response.

SINGLE-ELEMENT SYSTEMS WITH MULTIPLE DEGREES OF FREEDOM

While most scientific investigations of uncoupled, micro/nanoresonators treat the devices as SDOF systems, a number of recent works have demonstrated that the behavior of some micro/nanosystems, such as resonant micromirrors and suspended carbon nanotube (CNT) resonators, cannot be adequately captured with a SDOF modeling approach [208–211]. Rather, these studies suggest that multi-degree-of-freedom (MDOF) models are required to accurately capture the salient features of these representative systems' response. The present section details the investigations of the MDOF systems alluded to above, as well as the recent efforts of Vyas et al., which suggest that internal resonances arising in uncoupled, MDOF devices may prove beneficial in signal mixing and resonant mass sensing [212, 213]. Note that other investigations of systems of this type, including those related to pull-in and parametrically-excited gyroscopes, for example, have been included in previous sections.

Amongst the various investigations of single-element nonlinear MDOF micro/nanoresonators, are a series of works fo-

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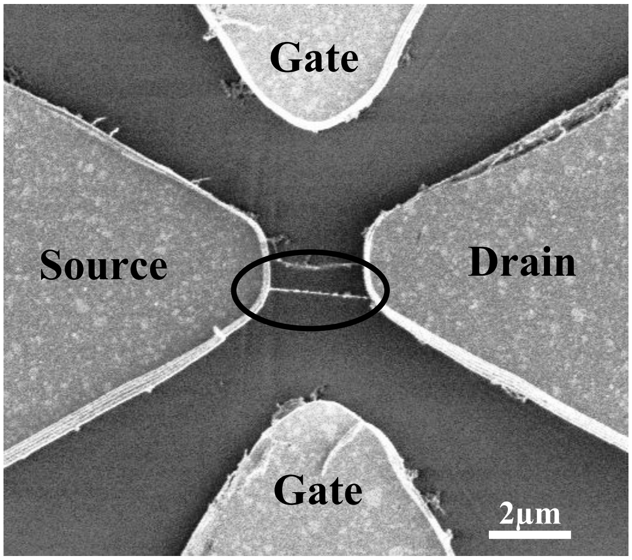

Figure 9. SCANNING ELECTRON MICROGRAPH OF A SUSPENDED, SINGLE-WALL CARBON NANOTUBE RESONATOR (PICTURE COURTESY OF S. MOHAMMADI).

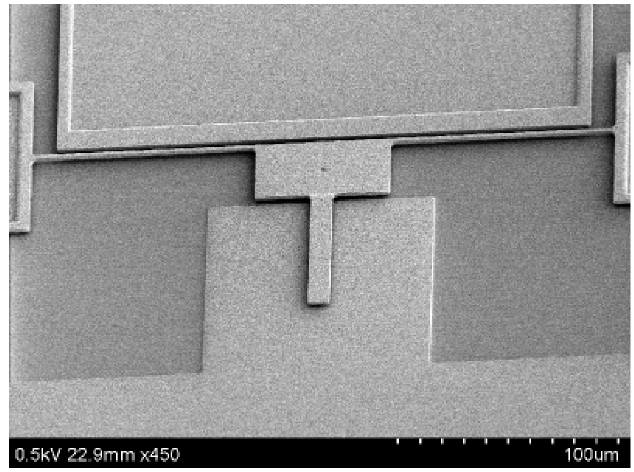

Figure 10. SCANNING ELECTRON MICROGRAPH OF A PEDAL RESONATOR UTILIZING 2:1 INTERNAL RESONANCE (PICTURE COURTESY OF A. VYAS, D. PEROULIS, AND A. K. BAJAJ).

cusing on the behavior of torsional micromirrors. The first of these, by Zhao et al. [211], details the coupled bending/torsional dynamics of an electrostatically-actuated device. In this work, the authors employ lumped-mass modeling, simulation methods, and finite element techniques, to demonstrate that nonlinearity and modal coupling result in a number of distinct dynamic behaviors, which cannot be captured with a linear and/or SDOF model. Recently, Daqaq, with collaborators, expanded upon this work [209,210], by developing a distributed-parameter representation of the 2-DOF micromirror, reducing this model to a refined, lumped-mass analog, and characterizing the result through the use of multiple-scales analyses. This analytical approach revealed that, over certain voltage excitation ranges, the electrostatically-actuated micromirror was capable of exhibiting hysteresis, quasiperiodic response, and 2:1 internal resonance, all of which can be detrimental to device performance.

Another investigation of single-element MDOF micro/nanoresonators that has drawn recent interest is that of Conley et al., which focuses on the dynamics of suspended nanotube and nanowire resonators, an example of which is shown in Fig. 9 [208]. This work demonstrates that nanotubes and nanowires, much like their macroscale counterparts (i.e. strings or cables), feature no preferred plane of oscillation, and as such, in the presence of nonlinearity, are capable of exhibiting rather rich, planar and non-planar dynamic responses. Perhaps the most intriguing of these responses is a whirling motion, which arises, even at fairly low excitation voltages, through a bifurcation on the resonator's backbone curve. Since the transition to this non-planar response is accompanied by a significant reduction in current modulation, the authors note that it, and other planar to non-planar response transitions, may have significant utility in switching and sensing applications.

Unlike the works detailed above, which emphasize largely-inadvertent MDOF behaviors, the works of Vyas et al. focus on the modeling, analysis, and design of microresonators with intentional MDOF responses [212, 213]. Specifically, these works detail the development of T-shaped microresonators, whose first two modes are tailored, such that they interact through a 1:2 internal resonance. A sample of such a device is given in Fig. 10. This arrangement proves beneficial, because by directly exciting the higher-frequency flexural mode of their resonator, the authors can autoparametrically-excite its lower-frequency counterpart. Since the autoparametric-excitation of this lower-frequency mode is a threshold phenomenon, similar to that seen in the section on Parametrically-Excited Systems, the authors can exploit the transitions across this threshold as a tunable switching event. This, as demonstrated in [161], facilitates highly-selective mass sensing. Furthermore, since the nonlinear excitation of the lower-frequency mode yields an output signal at half the frequency of the excitation, the authors, in essence, have realized a frequency divider or signal mixer.

ARRAYS OF COUPLED MICRO/NANORESONATORS

Though SDOF micro/nanoresonator implementations have historically garnered the most research attention, arrays of coupled resonators have elicited some level of interest since the early-1990s. While the bulk of investigations in this area have focused on linear device implementations, a number of recent works have demonstrated the potential of coupled systems utilizing nonlinear, resonant subsystems and/or nonlinear coupling. The present section first reviews those research efforts related to coupled *linear* systems in an attempt to provide some useful background and context. The focus then shifts to recent investigations of coupled, *nonlinear* systems, utilizing dynamic phe-

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nomena ranging from synchronization to vibration localization, which have built upon the aforementioned linear efforts.

In 1992, Lin et al. presented what is believed to be the first use of coupled microresonators in their seminal work on bandpass signal filters [214]. The premise of this work was to utilize chains of planar, comb-driven resonators, coupled through common elastic flexures, to mimic the behavior of LC ladder filters and early mechanical analogs [215]. Specifically, the authors exploited weakly-coupled micromechanical resonators to recover frequency response functions with narrow, well-defined, multi-resonance passbands. This approach proved to be an immediate success, as it allowed access to relatively-high center frequencies, without compromising effective quality (Q)-factor and stopband rejection metrics. In subsequent years, Nguyen and others would build upon the success of this initial work by developing a variety of new devices incorporating alternative resonator geometries [216–224], alternative coupling mechanisms [225–229], higher-dimensional arrays [230–233], and frequency-mistuned subsystems [230]. While these works unequivocally advanced MEMS-based filter technology, as well as a number of parallel applications, including increased bandwidth inertial sensors [234, 235], few, if any, utilized novel dynamical phenomena.

Of particular note amongst the references included above is the work of Judge et al. [230]. This work highlighted the impact of structural impurity (mistuning) and its relationship to classical vibration localization – a mechanical analog of Anderson localization [236] that results in the spatial confinement of energy in coupled systems with structural impurity (mistuning) – and the electromechanical filter design problem. By adopting a relatively broad research perspective, the work effectively bridged the research gap that existed between filter designers from the electrical engineering and mechanical engineering communities.

In recent years vibration localization has also been utilized to render improved sensitivity and inherent signal processing in elastically-coupled linear arrays. For example, in 2005, Spletzer et al. demonstrated that localized eigenmodes arising in arrays of elastically-coupled microbeams could be exploited for highly-sensitive mass detection [237–239]. Specifically, the work showed that structural impurities (frequency mistunings) induced through resonator-analyte interactions could be identified, with high selectivity, by examining distortions in the system's vibratory modes. In 2006, the authors of the present work adopted a slightly different approach by utilizing localized eigenmodes to reduce the input/output order of an elastically-coupled microcantilever array [240–243]. This was achieved by attaching a number of frequency-mistuned microbeams to a common shuttle mass, tailoring the resulting system's response to ensure kinetic energy was largely confined to a single microbeam near each of the sensor's resonances, and tracking the composite system's behavior using solely the response of the shuttle resonator. The net result was the first MEMS-based single-input, single-output (SISO) sensor capable of uniquely identifying multiple analytes within a gaseous mixture.

Another linear research effort of note is that of Bucher et al. from 2004 [206], which is quite distinct from its counterparts due to clever use of Fourier decomposition and structural optimization. This work specifically focused on the development of a resonant micromirror capable of triangular-wave scanning. Given the non-standard nature of this output, the authors spectrally decomposed the desired triangular waveform into its Fourier components. With these pertinent frequencies identified, the authors proceeded to reconstruct a lumped mass representation of the multi-degree-of-freedom system using inverse methods. Continuous system models, tolerant to manufacturing imperfections, were then extrapolated from the lumped-mass representation using structural optimization, and the resulting system was fabricated and tested. Not surprisingly, given the robustness of the analytical approach, excellent correlation between the desired triangular output and the near-resonant, experimental waveform was obtained.

Aside from the aforementioned investigations of sensors, filters, and micromirrors, a number of efforts have considered the response of coupled microsystems in a more generic setting. The recent works of Porfiri and Zhu et al., for example, adopt a traditional dynamical system's perspective and characterize the linear behavior of identical microbeam resonators coupled through electrostatic interactions [244, 245]. Porfiri's work specifically focuses on the derivation of closed-form representations of the coupled system's resonant frequencies and mode shapes. Zhu et al.'s work considers the impact of boundary conditions on the system's collective response. Along similar lines, the recent works of Gaidarzhy et al. have examined chains of nanomechanical resonators coupled to a common elastic backbone [246, 247]. These efforts, much like those noted above, adopt analytical and numerical methods to recover modal behaviors, but do so without placing particular emphasis on a target application.

Though linear resonator arrays offer distinct utility, arrays of nonlinear micro/nanoresonators have drawn increasing research interest as MEMS/NEMS technologies have matured. While numerous research efforts fall within the broad scope of this topic, most, if not all, of the current literature on nonlinear micro/nanoresonator arrays can be categorized into three distinct groups: (i) works focusing on nonlinear extensions of the linear array technologies detailed above; (ii) works focusing on nonlinear arrays which exhibit intrinsic localized modes (ILMs), or so-called discrete breathers; and (iii) works focusing on arrays of nonlinear resonators, which synchronize during the course of operation. Each of these categories is considered below.

To date, the traditional nonlinear dynamics community's contribution to nonlinear micro/nanoresonator array research has primarily been through the modeling and analysis of nonlinear variants of the linear array technologies detailed above. For example, the works of Hammad et al. [248–251] detail the development of refined nonlinear models for electrostatically-actuated, elastically-coupled filters similar in design to those originally proposed by Bannon et al. in 1996 [223]. These largely-theoretical works utilize multi-physics, continuous-system mod-

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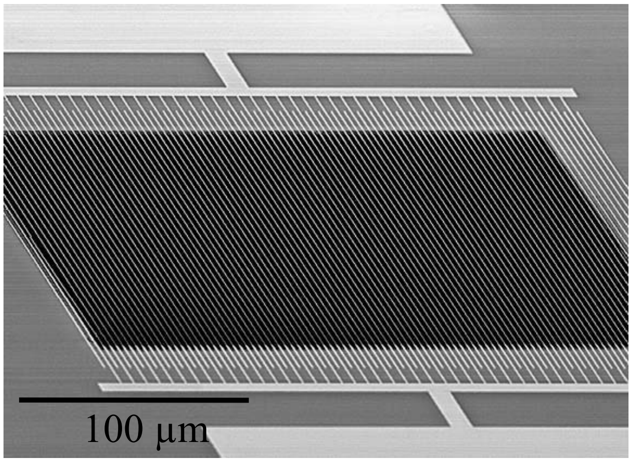

Figure 11. A 67-ELEMENT ARRAY OF ELECTROSTATICALLY-COUPLED, FIXED-FIXED MICROBEAMS (ADAPTED FROM [252], PICTURE COURTESY OF E. BUKS).

eling, model reduction, and perturbation analysis in an attempt to more accurately characterize pertinent filter metrics, pull-in behavior, and the effects that manufacturing imperfections have on a representative system's dynamic response.

Though elastically-coupled devices have been the traditional focus of nonlinear micro/nanoresonantor array research, a number of recent efforts have considered the behavior of comparatively-large arrays of microbeams coupled through electrostatic interactions. While these works could be seen as natural extensions of the investigations of Pourkamali, Zhu, Profiri, and their collaborators [226, 227, 244, 245], many, in actuality, predate these linear efforts. Representative amongst the nonlinear investigations of electrostatically-coupled micro/nanoresonator arrays are work of Napoli et al. [253], which examines the response of parametrically-excited microcantilevers coupled through both elastic and electrostatic interactions, and the collective efforts of Buks, Roukes, Lifshitz, and Cross [14,252,254, 255]. The earliest paper in the latter series of works is an experimental endeavor by Buks and Roukes examining the collective behavior of a 67-element array of electrostatically-actuated, doubly-clamped gold microbeams, driven near the principal parametric resonance (see Fig. 11) [252]. In this work, the authors utilize the microbeam array as a diffraction grating, and integrate it with an optical fiber light source and a photo diode detector to recover real-time representations of the system's modal behavior under varying drive conditions. While some of the relatively-complex behaviors reported in this work are left unexplained, a series of subsequent works by Lifshitz and Cross have addressed the observed collective response using both analytical and numerical techniques [14, 254, 255]. In [255], for example, Lifshitz and Cross propose a nonlinear model for the microbeam array incorporating Duffing-like elastic nonlinearities, in addition to linear electrostatic and nonlinear dissipative coupling. The authors then utilize this model, in conjunction with secular perturbation methods, to characterize the behavior of representative, two- and three-degree-of-freedom systems. Not surprisingly, relatively-rich frequency response behaviors are shown to arise in even these comparatively-small arrays. The high dimension of Buks and Roukes' 67-element array is prohibitive to analytical investigation, but a purely-numerical investigation in [255] nicely captures the mean response of this system. These authors subsequently continued their work in Bromberg et al in 2006 [254], by approaching the 67-element array problem from its continuous limit and transforming the spatially-discrete system considered in [255] into a spatially-continuous analog. This approach facilitated bifurcation analysis in the proximity of the parametric resonance, and, in turn, opened the problem to predictive system design.

Relevant to the efforts noted above are the recent works of Zhu et al. and Gutschmidt and Gottlieb [256–259]. The first of these efforts, by Zhu et al., extends the work of Lifshitz and Cross to incorporate the nonlinear, parametric interactions that arise from higher-order approximations of the electrostatic force. More specifically, the authors employ harmonic balance methods, in conjunction with numerical approaches, to characterize the behavior of a three-resonator system wherein the outer resonators are fixed (in essence, the system represents a variant of that considered in [69]). These results are subsequently used to show that the inclusion of nonlinear electrostatic effects renders higher-order sub-harmonic parametric resonances, which cannot be captured with models incorporating only linear, electrostatic coupling. Along similar lines, the works of Gutschmidt and Gottlieb extend the efforts of Lifshitz and Cross by adopting a spatiotemporal modeling approach which captures a broader range of excitation inputs [259]. These refined models are subsequently used in conjunction with multiple time scale perturbation methods and numerical continuation techniques to classify the various internal and combinational resonances that arise in two- and three-element arrays of electrostatically-coupled microbeams [257, 258]. These results capture a wider range of dynamic behavior, although corroboration with experimental results remains incomplete.

A second category of literature emphasizing nonlinear behaviors which arise in coupled micro/nanoresonator arrays is that focused on intrinsic localized modes (ILMs) or so-called discrete breathers (DBs) – analogs of classical localized modes which arise in the presence of strong nonlinearity, rather than structural impurity (mistuning) [260–269]. While the investigation of ILMs dates to the earlier analyses of discrete lattice vibration, ILMs were first reported to occur within microresonator arrays by Sato et al. in 2003 [267]. In this defining effort, the authors concluded that energy could be spatially confined in periodic arrays of identical microresonators, provided a strong mechanical nonlinearity and an appropriate drive mechanism were present. To verify this, the authors fabricated a spatially-periodic, 248-element array of alternating length, silicon nitride cantilevers, which were strongly coupled through a common elastic overhang. This coupled system was then driven at the base using

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a PZT element, and the resulting system response was recorded using a one-dimensional CCD camera. By driving the system at a frequency slightly below the maximum frequency of the array's 'optic' band and subsequently chirping the drive to a slightly higher frequency, the authors were able to observe a number of interesting dynamical phenomena [264, 265, 267]. First, the authors noted, throughout the duration of the chirp, the formation of multiple localized modes, well dispersed across the array at seemingly random locations. These ILMs were observed to 'hop' around the array, interacting with one another throughout the interval of transient excitation. This validated, in part, previous investigations of ILMs, which emphasized the independence of *intrinsic* energy localization and structural impurity. As the excitation reached a constant frequency state, those ILMs vibrating at the frequency of the excitation signal were observed to persist, while those oscillating at alternative frequencies decayed. The persistent modes would remain dominant until the excitation was terminated, whereafter the localized oscillations became 'unpinnned', 'hopped' to various sites within the array interacting constructively and destructively with their counterparts, and subsequently decayed.

Following the success of their initial work, Sato and coworkers proceeded to investigate various extensions of their ILM research. Notable milestones from these latter works include: (i) the realization of ILMs in microbeam arrays with softening nonlinearities (acquired through electrostatic tuning) [266]; (ii) a demonstration of ILMs within the acoustic spectrum – a feat previously deemed to be improbable due to the influence of higher-frequency spectral components on localized responses [268]; and (iii) the manipulation of ILMs through the use of optically-induced impurity [266]. Of these works, the last is of particular note, as it demonstrated the ability to manipulate the location of energy confinement within a spatially-periodic array through the use of local, laser heating. Specifically, the work demonstrated that reducing the linear natural frequency of a single resonator near an existing ILM results in ILM repulsion, if the system is operating in a hardening response regime, and ILM attraction, if the system is operating in a softening response regime. Such spatial control opens doors to a number of micro/nanoscale targeted energy transfer applications.

Apart from the works of Sato et al., detailed above, there have been a number of ILM-related research efforts that have approached the topic from alternative perspectives. Maniadis and Flach, for example, utilized nonlinear invariant manifold theories to predict the optimal operating conditions for ILM emergence, and to predict that ILMs can be induced through paths other than frequency modulation [263]. A recent effort by Chen, et al. extended this further by demonstrating that ILMs can be induced through chaos [260]. Within the traditional nonlinear dynamics community, recent works by Dick et al. have addressed the existence of ILMs using the theory of nonlinear normal modes [261, 262]. This analytical approach, in comparison to those employed in prior works, facilitates the derivation of analytical expressions for ILM amplitude profiles, which should prove invaluable in future design efforts.

The final, well-defined class of literature emphasizing nonlinear behaviors which arise in coupled micro/nanoresonator arrays is that concerned with the synchronization of coupled resonators. While macroscale investigations of this phenomenon date to Huygen's mid-1600s observations of weakly-coupled pendulum clocks, investigations of synchronicity in microsystems date only to the early-2000s [270, 271]. Earliest amongst the various works on MEMS/NEMS synchronization is Hoppensteadt and Izhikevich's speculative effort of 2001 [270], which proposed the use of globally-coupled, limit-cycle oscillators as the functional backbone of MEMS-based neurocomputers. This work highlighted the distinct parallels between microelectromechanical resonators driven via positive feedback loops, phase-locked loops (PLLs), and lasers, and utilized theory the authors had previously developed for the latter two systems to convey the potential of MEMS-based autocorrelative associative memories. Cross and collaborators, building upon their earlier efforts related to coupled micro/nanosystems, would further this work, by considering, in appreciable depth, the synchronization of globally-coupled, limit-cycle oscillators with distributed frequencies [272, 273]. The latter efforts' emphasis on frequency mistunings is of particular note, as it addressed a common concern associated with micro/nanoresonator technologies – the potentially debilitating effects of process-induced variations.

While the works referenced above adopted a largely device-independent approach to micro/nanoscale synchronization research, two recent efforts have approached the problem with specific devices in mind. The first of these [190], by Sahai and Zehnder, considers the synchronization of elastically- and electrothermally-coupled, self-excited dome oscillators similar to those previously detailed in [274]. In this work, the authors utilized numerical methods to investigate the various operating conditions that result in the synchronization and entrainment of a representative two-resonator system. The second effort [275], by Shim et al., potentially represents the first experimental demonstration of synchronization in a micro/nanomechanical array. In this work, the authors utilize a two-element array of elastically-coupled, magnetomotively-excited resonators to show some degree of frequency locking. While the acquired results are promising, their interpretation has been debated within the nonlinear dynamics community.

CONCLUSION

With this review the authors have attempted to provide a thorough, yet brief, account of the history and literature to date in the area of nonlinear dynamics in micro/nanoresonators. The exploitation of nonlinearity has significant potential for improving the performance of some devices, but thoughtful modeling and analysis must be carried out in order to achieve such designs. This is especially true in applications involving dynamic behavior, where nonlinearity can lead to interesting results – or wreak havoc, if one is not careful. It is still a challenge to convince

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device builders to consider designs based on nonlinear behavior, since the goal for many decades has been to steer clear of nonlinearity. However, this trend is changing, and there are several devices based on nonlinear dynamic behavior under development that have real promise of reaching fruition.

Progress in this field is taking place on two broad, not unrelated, fronts. Engineering-oriented research is being carried out with specific targeted applications in mind, such as mass sensors, rate gyros, filters, etc. In this area one can use existing fabrication techniques and basic knowledge of nonlinear dynamics to design devices that have desired nonlinear behavior. Nonlinear design is crucial here, but one still must face all the considerations inherent in device design, such as fabrication tolerances, robustness, reliability, etc. Also working in this area are nonlinear dynamicists, who have found a number of interesting problems to consider, and this has led to a number of studies in which the goal is simply to understand the dynamics of nonlinear models for these devices. The other front is more physics-oriented and is geared towards fundamental experimental research. One example of such work is that of mesoscale vibration systems, that is, systems that exhibit quantum effects even though they can be modeled as continua. A typical illustration of such an effect would be to measure quantum energy levels in a nanobeam. Of course, this requires that one isolates the beam as much as possible from the environment thermally, mechanically, and electronically (although, of course, one must couple the beam to something in order to make measurements). The driving force behind these developments is the desire for devices that have unprecedented sensitivities, for example, for the detection of gravity waves. This is a very active area of research in physics, and the reader is referred to the review of Blencowe [276] for more information.

As devices become smaller, and yet are required to operate in ambient environments, the effects of noise will become an increasingly important consideration. There are several sources of noise, and these will limit the ultimate resolution of nanoscale sensors [277]. There is also growing interest in so-called "bifurcation amplifiers" that make use of nonlinear response branch jumping, as described at the end of the section on directly-excited systems, for targeted types of detection, and it is interesting to note that these systems actually *require* noise to function effectively. Likewise, the non-stationary effects of controlled parameter variations, for example, frequency sweeps used in sensors, is of interest, since sensor response specifications will depend on the determination of reliable sweep rates. This general topic has received considerable attention in the nonlinear vibrations and physics communities under various subject names, including "passage through resonance" and "non-stationary oscillations", and it has now found a new set of applications. The combination of these effects, namely nonlinearity, noise, and parameter sweeps, is a class of challenging, fundamental problems that will play an important role in the development of MEMS/NEMS resonators. It is encouraging that people working on these topics are talking to one another and collaborating. This approach, which brings together device engineers, physicists, and nonlinear dynamicists, will become increasingly important as devices shrink to the nanoscale and beyond.

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REFERENCES

[1] Feynman, R. P., 1992. "There's plenty of room at the bottom (reprint)". *Journal of Microelectromechanical Systems,* **1**(1), pp. 60–66.

[2] Feynman, R. P., 1993. "Infinitesimal machinery (reprint)". *Journal of Microelectromechanical Systems,* **2**(1), pp. 4–14.

[3] Nathanson, H. C., and Wickstrom, R. A., 1965. "A resonant-gate silicon surface transistor with high-Q bandpass properties". *Applied Physics Letters,* **7**(4), pp. 84–86.

[4] Nathanson, H. C., Newell, W. E., Wickstrom, R. A., and Davis Jr., J. R., 1967. "The resonant gate transistor". *IEEE Transactions on Electron Devices,* **14**(3), pp. 117–133.

[5] Petersen, K., 1982. "Silicon as a mechanical material". *Proceedings of the IEEE,* **70**(5), pp. 420–457.

[6] Holmes, P., 2005. "Ninety plus thirty years of nonlinear dynamics: Less is more and more is different". *International Journal of Bifurcation and Chaos,* **15**(9), pp. 2703–2716.

[7] Lorenz, E. N., 1963. "Deterministic nonperiodic flow". *Journal of the Atmospheric Sciences,* **20**, pp. 130–141.

[8] Smale, S., 1998. "Finding a horseshoe on the beaches of Rio". *Mathematical Intelligencer,* **20**, pp. 39–44.

[9] Brand, O., and Baltes, H., 1998. "Micromachined resonant sensors - an overview". *Sensors Update,* **4**(1), pp. 3–51.

[10] Schmidt, M. A., and Howe, R. T., 1987. "Silicon resonant microsensors". *Ceramic Engineering and Science Proceedings,* **8**(9-10), pp. 1019–1034.

[11] Stemme, G., 1991. "Resonant silicon sensors". *Journal*

Copyright © 2008 by ASME

of Micromechanics and Microengineering, **1**(2), pp. 113–125.

[12] Garcia, R., and Perez, R., 2002. "Dynamic atomic force microscopy methods". *Surface Science Reports,* **47**(6-8), pp. 197–301.

[13] Raman, A., Melcher, J., and Tung, R., 2008. "Cantilever dynamics in atomic force microscopy". *Nano Today,* **3**(1-2), pp. 20–27.

[14] Lifshitz, R., and Cross, M. C., 2008. "Nonlinear dynamics of nanomechanical and micromechanical resonators". In *Review of Nonlinear Dynamics and Complexity*, H. G. Schuster, ed., Vol. 1. pp. 1–52.

[15] Younis, M. I., and Nayfeh, A. H., 2007. "Simulation of squeeze-film damping of microplates actuated by large electrostatic load". *Journal of Computational and Nonlinear Dynamics,* **2**(3), pp. 232–240.

[16] Braginsky, V. B., Mitrofanov, V. P., and Panov, V. I., 1985. *Systems with Small Dissipation*. The University of Chicago Press, Chicago, IL.

[17] Cleland, A. N., and Roukes, M. L., 1999. "External control of dissipation in a nanometer-scale radiofrequency mechanical resonator". *Sensors and Actuators,* **72**(3), pp. 256–261.

[18] Hutchinson, A. B., Truitt, P. A., Schwab, K. C., Sekaric, L., Parpia, J. M., Craighead, H. G., and Butler, J. E., 2004. "Dissipation in nanocrystalline-diamond nanomechanical resonators". *Applied Physics Letters,* **84**(6), pp. 972–974.

[19] Lifshitz, R., 2002. "Phonon-mediated dissipation in micro- and nano-mechanical systems". *Physica B: Condensed Matter,* **316-317**, pp. 397–399.

[20] Mohanty, P., Harrington, D. A., Ekinci, K. L., Yang, Y. T., Murphy, M. J., and Roukes, M. L., 2002. "Intrinsic dissipation in high-frequency micromechanical resonators". *Physical Review B,* **66**, 085416.

[21] Yang, J., Ono, T., and Esashi, M., 2002. "Energy dissipation in submicron thick single-crystal silicon cantilevers". *Journal of Microelectromechanical Systems,* **11**(6), pp. 775–783.

[22] Hosaka, H., Itao, K., and Kuroda, S., 1995. "Damping characteristics of beam-shaped micro-oscillators". *Sensors and Actuators A: Physical,* **49**(1-2), pp. 87–95.

[23] Lifshitz, R., and Roukes, M. L., 2000. "Thermoelastic damping in micro- and nanomechanical systems". *Physical Review B,* **61**(8), pp. 5600–5609.

[24] Srikar, V. T., and Senturia, S. D., 2002. "Thermoelastic damping in fine-grained polysilicon flexural beam resonators". *Journal of Microelectromechanical Systems,* **11**(5), pp. 499–504.

[25] Ye, W., Wang, X., Hemmert, W., Freeman, D., and White, J., 2003. "Air damping in laterally oscillating microresonators: A numerical and experimental study". *Journal of Microelectromechanical Systems,* **12**(5), pp. 557–566.

[26] Zhang, W., and Turner, K., 2007. "Frequency dependent fluid damping of micro/nano flexural resonators: Experiment, model and analysis". *Sensors and Actuators A: Physical,* **134**(2), pp. 594–599.

[27] Yasumura, K. Y., Stowe, T. D., Chow, E. M., Pfafman, T., Kenny, T. W., Stipe, B. C., and Rugar, D., 2000. "Quality factors in micron- and submicron-thick cantilevers". *Journal of Microelectromechanical Systems,* **9**(1), pp. 117–125.

[28] Clark, J. V., and Pister, K. S. J., 2007. "Modeling, simulation, and verification of an advanced micromirror using SUGAR". *Journal of Microelectromechanical Systems,* **16**(6), pp. 1524–1536.

[29] Batra, R. C., Porfiri, M., and Spinello, D., 2007. "Review of modeling electrostatically actuated microelectromechanical systems". *Smart Materials and Structures,* **16**(6), pp. R23–R31.

[30] Fargas-Marques, A., Costa Castello, R., and Shkel, A. M., 2005. Modelling the electrostatic actuation of MEMS: State of the art 2005. Tech. Rep. IOC-DT-P-2005-18, Universitat Politecnica De Catalunya.

[31] De, S. K., and Aluru, N. R., 2004. "Full-Lagrangian schemes for dynamic analysis of electrostatic MEMS". *Journal of Microelectromechanical Systems,* **13**(5), pp. 737–758.

[32] Lin, R. M., and Wang, W. J., 2006. "Structural dynamics of microsystems - current state of research and future directions". *Mechanical Systems and Signal Processing,* **20**(5), pp. 1015–1043.

[33] Wittwer, J. W., Baker, M. S., and Howell, L. L., 2006. "Robust design and model validation of nonlinear compliant micromechanisms". *Journal of Microelectromechanical Systems,* **15**(1), pp. 33–41.

[34] Gabbay, L. D., Mehner, J. E., and Senturia, S. D., 2000. "Computer-aided generation of nonlinear reduced-order dynamic macromodels - I: Non-stress-stiffened case". *Journal of Microelectromechanical Systems,* **9**, pp. 262–269.

[35] Mehner, J. E., Gabbay, L. D., and Senturia, S. D., 2000. "Computer-aided generation of nonlinear reduced-order dynamic macromodels - II: Stress-stiffened case". *Journal of Microelectromechanical Systems,* **9**(2), pp. 270–278.

[36] Younis, M. I., Abdel-Rahman, E. M., and Nayfeh, A., 2003. "A reduced-order model for electrically actuated microbeam-based MEMS". *Journal of Microelectromechanical Systems,* **12**(5), pp. 672–680.

[37] Xie, W. C., Lee, H. P., and Lim, S. P., 2003. "Nonlinear dynamic analysis of MEMS switches by nonlinear modal analysis". *Nonlinear Dynamics,* **31**, pp. 243–256.

[38] Hung, E. S., and Senturia, S. D., 1999. "Generating efficient dynamical models for microelectromechanical systems from a few finite-element simulation runs". *Journal of Microelectromechanical Systems,* **8**(3), pp. 280–289.

[39] Liang, Y. C., Lin, W. Z., Lee, H. P., Lim, S. P., Lee, K. H., and Sun, H., 2002. "Proper orthogonal decomposition and its applications - part II: Model reduction for

 Copyright © 2008 by ASME

MEMS dynamical analysis". *Journal of Sound and Vibration,* **256**(3), pp. 515–532.

[40] Nayfeh, A. H., Younis, M. I., and Abdel-Rahman, E. M., 2005. "Reduced-order models for MEMS applications". *Nonlinear Dynamics,* **41**(1-3), pp. 211–236.

[41] Senturia, S. D., 2000. *Microsystem Design*, Kluwer Academic Publishers, Dordrecht.

[42] Pelesko, J. A., and Bernstein, D. H., 2002. *Modeling MEMS and NEMS*. Chapman & Hall/CRC, Boca Raton, FL.

[43] Lobontiu, N. O., 2006. *Mechanical Design of Microresonators: Modeling and Applications*. Nanoscience and Technology Series. McGraw-Hill, New York, NY.

[44] Cleland, A. N., 2003. *Foundations of Nanomechanics: From Solid-State Theory to Device Applications*. Advanced Texts in Physics. Springer, Berlin.

[45] Turner, K. L., Miller, S. A., Hartwell, P. G., MacDonald, N. C., Strogatz, S. H., and Adams, S. G., 1998. "Five parametric resonances in a microelectromechanical system". *Nature,* **396**(6707), pp. 149–152.

[46] Adams, S. G., Bertsch, F., and MacDonald, N. C., 1998. "Independent tuning of linear and nonlinear stiffness coefficients". *Journal of Microelectromechanical Systems,* **7**(2), pp. 172–180.

[47] DeMartini, B. E., Butterfield, H. E., Moehlis, J., and Turner, K. L., 2007. "Chaos for a microelectromechanical oscillator governed by the nonlinear Mathieu equation". *Journal of Microelectromechanical Systems,* **16**(6), pp. 1314–1323.

[48] DeMartini, B. E., Rhoads, J. F., Turner, K. L., Shaw, S. W., and Moehlis, J., 2007. "Linear and nonlinear tuning of parametrically excited MEMS oscillators". *Journal of Microelectromechanical Systems,* **16**(2), pp. 310–318.

[49] Jensen, B. D., Mutlu, S., Miller, S., Kurabayashi, K., and Allen, J. J., 2003. "Shaped comb fingers for tailored electromechanical restoring force". *Journal of Microelectromechanical Systems,* **12**(3), pp. 373–383.

[50] Chan, H. B., Aksyuk, V. A., Kleiman, R. N., Bishop, D. J., and Capasso, F., 2001. "Nonlinear micromechanical Casimir oscillator". *Physical Review Letters,* **87**(21), 211801.

[51] Buks, E., and Roukes, M. L., 2001. "Metastability and the Casimir effect in micromechanical systems". *Europhysics Letters,* **54**(2), pp. 220–226.

[52] Fargas-Marques, A., Casals-Terre, J., and Shkel, A. M., 2007. "Resonant pull-in condition in parallel-plate electrostatic actuators". *Journal of Microelectromechanical Systems,* **16**(5), pp. 1044–1053.

[53] Krylov, S., and Bernstein, Y., 2006. "Large displacement parallel plate electrostatic actuator with saturation type characteristic". *Sensors and Actuators A: Physical,* **130-131**, pp. 497–512.

[54] Krylov, S., Ilic, B. R., Schreiber, D., Serentensky, S., and Craighead, H., 2008. "The pull-in behavior of electrostatically actuated bistable microstructures". *Journal of Micromechanics and Microengineering,* **18**(5), 055026.

[55] Krylov, S., and Serentensky, S., 2006. "Higher order correction of electrostatic pressure and its influence on the pull-in behavior of microstructures". *Journal of Micromechanics and Microengineering,* **16**(7), pp. 1382–1396.

[56] Krylov, S., Serentensky, S., and Schreiber, D., 2008. "Pull-in behavior and multistability of a curved microbeam actuated by a distributed electrostatic force". In Proceedings of MEMS 2008: The 21st IEEE International Conference on Micro Electro Mechanical Systems, pp. 499–502.

[57] Kalyanaraman, R., Packirisamy, M., and Bhat, R. B., 2005. "Nonlinear pull-in study of electrostatically actuated MEMS structures". In Proceedings of ICISIP 2005: The Third International Conference on Intelligent Sensing and Signal Processing, pp. 195–199.

[58] Krylov, S., and Maimon, R., 2004. "Pull-in dynamics of an elastic beam actuated by continuously distributed electrostatic force". *Journal of Vibration and Acoustics,* **126**(3), pp. 332–342.

[59] Krylov, S., 2007. "Lyapunov exponents as a criterion for the dynamic pull-in instability of electrostatically actuated microstructures". *International Journal of Non-Linear Mechanics,* **42**(4), pp. 626–642.

[60] Krylov, S., Serentensky, S., and Schreiber, D., 2007. "Pull-in behavior of electrostatically actuated multistable microstructures". In Proceedings of IDETC/CIE 2007: The 2007 ASME International Design Engineering Technical Conferences & Computers and Information in Engineering Conference, 1st International Conference on Micro- and Nanosystems (MNS).

[61] Guckenheimer, J., and Holmes, P., 2002. *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*. Springer, New York, NY.

[62] Ashhab, M., Salapaka, M. V., Dahleh, M., and Mezic, I., 1999. "Dynamic analysis and control of microcantilevers". *Automatica,* **35**(10), pp. 1663–1670.

[63] Ashhab, M., Salapaka, M. V., Dahleh, M., and Mezic, I., 1999. "Melnikov-based dynamical analysis of microcantilevers in scanning probe microscopy". *Nonlinear Dynamics,* **20**(3), pp. 197–220.

[64] Basso, M., Giarre, L., Dahleh, M., and Mezic, I., 2000. "Complex dynamics in a harmonically excited Lennard-Jones oscillator: Microcantilever-sample interaction in scanning probe microscopes". *Journal of Dynamic Systems, Measurement, and Control,* **122**(1), pp. 240–245.

[65] Luo, A. C. J., and Wang, F.-Y., 2004. "Nonlinear dynamics of a micro-electro-mechanical system with time-varying capacitors". *Journal of Vibration and Acoustics,* **126**(1), pp. 77–83.

[66] Lenci, S., and Rega, G., 2006. "Control of pull-in dynamics in a nonlinear thermoelastic electrically actuated microbeam". *Journal of Micromechanics and Microengi-*

Copyright © 2008 by ASME

neering, **16**(2), pp. 390–401.

[67] Gottlieb, O., and Champneys, A. R., 2003. "Global bifurcations of nonlinear thermoelastic microbeams subject to electrodynamic actuation". In Proceedings of IUTAM Symposium on Chaotic Dynamics and Control of Systems and Processes in Mechanics, G. Rega and F. Vestroni, eds.

[68] Nayfeh, A. H., Younis, M. I., and Abdel-Rahman, E. M., 2007. "Dynamic pull-in phenomenon in MEMS resonators". *Nonlinear Dynamics,* **48**(1-2), pp. 153–163.

[69] Rhoads, J. F., Shaw, S. W., and Turner, K. L., 2006. "The nonlinear response of resonant microbeam systems with purely-parametric electrostatic actuation". *Journal of Micromechanics and Microengineering,* **16**(5), pp. 890–899.

[70] Ai, S., and Pelesko, J. A., 2008. "Dynamics of a canonical electrostatic MEMS/NEMS system". *Journal of Dynamics and Differential Equations,* **20**(3), pp. 609–641.

[71] Krylov, S., Harari, I., and Cohen, Y., 2005. "Stabilization of electrostatically actuated microstructures using parametric excitation". *Journal of Micromechanics and Microengineering,* **15**(6), pp. 1188–1204.

[72] Andres, M. V., Foulds, K. W. H., and Tudor, M. J., 1987. "Nonlinear vibrations and hysteresis of micromachined silicon resonators designed as frequency-out sensors". *Electronics Letters,* **23**(18), pp. 952–954.

[73] Ikeda, K., Kuwayama, H., Kobayashi, T., Watanabe, T., Nishikawa, T., Yoshida, T., and Harada, K., 1989. "Study of nonlinear vibration of silicon resonant beam strain gauges". In Proceedings of the 8th Sensor Symposium, pp. 21–24.

[74] Tilmans, H. A. C., Elwenspoek, M., and Fluitman, J. H. J., 1992. "Micro resonant force gauges". *Sensors and Actuators A: Physical,* **30**(1), pp. 35–53.

[75] Gui, C., Legtenberg, R., Tilmans, H. A. C., Fluitman, J. H. J., and Elwenspoek, M., 1998. "Nonlinearity and hysteresis of resonant strain gauges". *Journal of Microelectromechanical Systems,* **7**(1), pp. 122–127.

[76] Nguyen, C. T.-C., and Howe, R. T., 1993. "CMOS micromechanical resonator oscillator". In Proceedings of the IEEE International Electron Devices Meeting, pp. 199–202.

[77] Legtenberg, R., and Tilmans, H. A. C., 1994. "Electrostatically driven vaccum-encapsulated polysilicon resonators, part I: Design and fabrication". *Sensors and Actuators A: Physical,* **45**(1), pp. 57–66.

[78] Legtenberg, R., and Tilmans, H. A. C., 1994. "Electrostatically driven vacuum-encapsulated polysilicon resonators, part II: Theory and performance". *Sensors and Actuators A: Physical,* **45**(1), pp. 67–84.

[79] Bourouina, T., Garnier, A., Fujita, H., Masuzawa, T., and Peuzin, J.-C., 2000. "Mechanical nonlinearities in a magnetically actuated resonator". *Journal of Micromechanics and Microengineering,* **10**(2), pp. 265–270.

[80] Piekarski, B., DeVoe, D., Dubey, M., Kaul, R., Conrad, J., and Zeto, R., 2001. "Surface micromachined piezoelectric resonant beam filters". *Sensors and Actuators A: Physical,* **91**(3), pp. 313–320.

[81] Ayela, F., and Fournier, T., 1998. "An experimental study of anharmonic micromachined silicon resonators". *Measurement Science and Technology,* **9**(11), pp. 1821–1830.

[82] Camon, H., and Larnaudie, F., 2000. "Fabrication, simulation and experiment of a rotating electrostatic silicon mirror with large angular deflection". In Proceedings of MEMS 2000: The 13th Annual International Conference on Micro Electro Mechanical Systems, pp. 645–650.

[83] Abdel-Rahman, E. M., and Nayfeh, A. H., 2003. "Secondary resonances of electrically actuated resonant microsensors". *Journal of Micromechanics and Microengineering,* **13**(3), pp. 491–501.

[84] Abdel-Rahman, E. M., Younis, M. I., and Nayfeh, A. H., 2002. "Characterization of the mechanical behavior of an electrically actuated microbeam". *Journal of Micromechanics and Microengineering,* **12**(6), pp. 759–766.

[85] Najar, F., Choura, S., El-Borgi, S., Abdel-Rahman, E. M., and Nayfeh, A. H., 2005. "Modeling and design of variable-geometry electrostatic microactuators". *Journal of Micromechanics and Microengineering,* **15**(3), pp. 419–429.

[86] Nayfeh, A. H., and Younis, M. I., 2005. "Dynamics of MEMS resonators under superharmonic and subharmonic excitations". *Journal of Micromechanics and Microengineering,* **15**(10), pp. 1840–1847.

[87] Younis, M. I., and Nayfeh, A. H., 2003. "A study of the nonlinear response of a resonant microbeam to an electric actuation". *Nonlinear Dynamics,* **31**(1), pp. 91–117.

[88] Kaajakari, V., Mattila, T., Oja, A., and Seppa, H., 2004. "Nonlinear limits for single-crystal silicon microresonators". *Journal of Microelectromechanical Systems,* **13**(5), pp. 715–724.

[89] Jeong, H.-M., and Ha, S. K., 2005. "Dynamic analysis of a resonant comb-drive micro-actuator in linear and nonlinear regions". *Sensors and Actuators A: Physical,* **125**(1), pp. 59–68.

[90] Agarwal, M., Chandorkar, S. A., Mehta, H., Candler, R. N., Kim, B., Hopcroft, M. A., Melamud, R., Jha, C. M., Bahl, G., Yama, G., Kenny, T. W., and Murmann, B., 2008. "A study of electrostatic force nonlinearities in resonant microstructures". *Applied Physics Letters,* **92**, 104106.

[91] Agarwal, M., Mehta, H., Candler, R. N., Chandorkar, S. A., Kim, B., Hopcroft, M. A., Melamud, R., Bahl, G., Kenny, T. W., and Murmann, B., 2007. "Scaling of amplitude-frequency-dependence nonlinearities in electrostatically transduced microresonators". *Journal of Applied Physics,* **102**, 074903.

[92] Shao, L. C., Palaniapan, M., Tan, W. W., and Khine, L., 2008. "Nonlinearity in micromechanical free-free beam resonators: Modeling and experimental verification". *Journal of Micromechanics and Microengineering,*

 Copyright © 2008 by ASME

18(2), 025017.

[93] Dick, A. J., Balachandran, B., DeVoe, D. L., and Mote Jr., C. D., 2006. "Parametric identification of piezoelectric microscale resonators". *Journal of Micromechanics and Microengineering,* **16**(8), pp. 1593–1601.

[94] Li, H., Preidikman, S., Balachandran, B., and Mote Jr., C. D., 2006. "Nonlinear free and forced oscillations of piezoelectric microresonators". *Journal of Micromechanics and Microengineering,* **16**(2), pp. 356–367.

[95] Mahmoodi, S. N., and Jalili, N., 2007. "Non-linear vibrations and frequency response analysis of piezoelectrically driven microcantilevers". *International Journal of Non-Linear Mechanics,* **42**(4), pp. 577–587.

[96] Mahmoodi, S. N., Jalili, N., and Daqaq, M. F., 2008. "Modeling, nonlinear dynamics, and identification of a piezoelectrically actuated microcantilever sensor". *IEEE/ASME Transactions on Mechantronics,* **13**(1), pp. 58–65.

[97] Evoy, S., Carr, D. W., Sekaric, L., Olkhovets, A., Parpia, J. M., and Craighead, H. G., 1999. "Nanofabrication and electrostatic operation of single-crystal silicon paddle oscillators". *Journal of Applied Physics,* **86**(11), pp. 6072–6077.

[98] Sazonova, V., Yaish, Y., Ustunel, H., Roundy, D., Arias, T. A., and McEuen, P. L., 2004. "A tunable carbon nanotube electromechanical oscillator". *Nature,* **431**(7006), pp. 284–287.

[99] Huang, X. M. H., Feng, X. L., Zorman, C. A., Mehregany, M., and Roukes, M. L., 2005. "VHF, UHF and microwave frequency nanomechanical resonators". *New Journal of Physics,* **7**(247), pp. 1–15.

[100] Husain, A., Hone, J., Postma, H. W. C., Huang, X. M. H., Drake, T., Barbic, M., Scherer, A., and Roukes, M. L., 2003. "Nanowire-based very-high-frequency electromechanical resonator". *Applied Physics Letters,* **83**(6), pp. 1240–1242.

[101] Feng, X. L., He, R., Yang, P., and Roukes, M. L., 2007. "Very high frequency silicon nanowire electromechanical resonators". *Nano Letters,* **7**(7), pp. 1953–1959.

[102] Zaitsev, S., Almog, R., Shtempluck, O., and Buks, E., 2005. "Nonlinear dynamics in nanomechanical oscillators". In Proceeding of ICMENS'05: The 2005 International Conference on MEMS, NANO, and Smart Systems, pp. 387–391.

[103] Li, T. F., Pashkin, Y. A., Astafiev, O., Nakamura, Y., Tsai, J. S., and Im, H., 2008. "High-frequency metallic nanomechanical resonators". *Applied Physics Letters,* **92**, 043112.

[104] Postma, H. W. C., Kozinsky, I., Husain, A., and Roukes, M. L., 2005. "Dynamic range of nanotube- and nanowire-based electromechanical systems". *Applied Physics Letters,* **86**, 223105.

[105] Erbe, A., Krommer, H., Kraus, A., Blick, R. H., Corso, G., and Richter, K., 2000. "Mechanical mixing in nonlinear nanomechanical resonators". *Applied Physics Letters,* **77**(19), pp. 3102–3104.

[106] Alastalo, A. T., and Kaajakari, V., 2005. "Intermodulation in capactitively coupled microelectromechanical filters". *IEEE Electron Device Letters,* **26**(5), pp. 289–291.

[107] Alastalo, A. T., and Kaajakari, V., 2006. "Third-order intermodulation in microelectromechanical filters coupled with capacitive transducers". *Journal of Microelectromechanical Systems,* **15**(1), pp. 141–148.

[108] Koskenvuori, M., and Tittonen, I., 2008. "GHz-range FSK-reception with microelectromechanical resonators". *Sensors and Actuators A: Physical,* **142**(1), pp. 346–351.

[109] Koskenvuori, M., and Tittonen, I., 2008. "Towards micromechanical radio: Overtone excitations of a microresonator through the nonlinearities of the second and third order". *Journal of Microelectromechanical Systems,* **17**(2), pp. 363–369.

[110] Almog, R., Zaitsev, S., Shtempluck, O., and Buks, E., 2006. "High intermodulation gain in a micromechanical Duffing resonator". *Applied Physics Letters,* **88**, 213509.

[111] Almog, R., Zaitsev, S., Shtempluck, O., and Buks, E., 2006. "Noise squeezing in a nanomechanical Duffing resonator". *Physical Review Letters,* **98**, 078103.

[112] Buks, E., and Yurke, B., 2006. "Mass detection with nonlinear nanomechanical resonator". *Physical Review E,* **74**, 046619.

[113] Greywall, D. S., 2005. "Sensitive magnetometer incorporating a high-Q nonlinear mechanical resonator". *Measurement Science and Technology,* **16**(12), pp. 2473–2482.

[114] Choi, S., Kim, S.-H., Yoon, Y.-K., and Allen, M. G., 2007. "Exploitation of nonlinear effects for enhancement of the sensing performance of resonant sensors". In Proceedings of Transducers 2007: The 2007 International Solid-State Sensors, Actuators, and Microsystems Conference, pp. 1745–1748.

[115] Agarwal, M., Chandorkar, S. A., Candler, R. N., Kim, B., Hopcroft, M. A., Melamud, R., Jha, C. M., Kenny, T. W., and Murmann, B., 2006. "Optimal drive condition for nonlinearity reduction in electrostatic microresonators". *Applied Physics Letters,* **89**, 214105.

[116] Agarwal, M., Park, K., Candler, R., Hopcroft, M., Jha, C., Melamud, R., Kim, B., Murmann, B., and Kenny, T. W., 2005. "Non-linearity cancellation in MEMS resonators for improved power handling". In Proceedings of IEDM 2005: The 2005 IEEE International Electron Devices Meeting, pp. 286–289.

[117] Shao, L. C., Palaniapan, M., and Tan, W. W., 2008. "The nonlinearity cancellation phenomenon in micromechanical resonators". *Journal of Micromechanics and Microengineering,* **18**, 065014.

[118] Kozinsky, I., Postma, H. W. C., Bargatin, I., and Roukes, M. L., 2006. "Tuning nonlinearity, dynamic range, and frequency of nanomechanical resonators". *Applied Physics Letters,* **88**, 253101.

[119] Scheible, D. V., Erbe, A., Blick, R. H., and Corso, G.,

Copyright © 2008 by ASME

2002. “Evidence of a nanomechanical resonator being driven into chaotic response via the Ruelle-Takens route”. *Applied Physics Letters,* **81**(10), pp. 1884–1886.

[120] Gottlieb, O., Gemintern, A., and Blick, R. H., 2007. “Bifurcations and chaos in an experimental based quasi-continuum nonlinear dynamical system for the clapper nanoresonator”. In Proceedings of IDETC 2007: The 2007 ASME International Design Engineering Technical Conferences, 1st Conference on Micro- and Nanosystems.

[121] Liu, S., Davidson, A., and Lin, Q., 2004. “Simulation studies on nonlinear dynamics and chaos in a MEMS cantilever control system”. *Journal of Micromechanics and Microengineering,* **14**(7), pp. 1064–1073.

[122] De, S. K., and Aluru, N. R., 2006. “Complex nonlinear oscillations in electrostatically actuated microstructures”. *Journal of Microelectromechanical Systems,* **15**(2), pp. 355–369.

[123] De, S. K., and Aluru, N. R., 2005. “Complex oscillations and chaos in electrostatic microelectromechanical systems under superharmonic excitations”. *Physical Review Letters,* **94**, 204101.

[124] Park, K., Chen, Q., and Lai, Y., 2008. “Energy enhancement and chaos control in microelectromechanical systems”. *Physical Review E,* **77**, 026210.

[125] Aldridge, J. S., and Cleland, A. N., 2005. “Noise-enabled precision measurements of a Duffing nanomechanical resonator”. *Physical Review Letters,* **94**, 156403.

[126] Stambaugh, C., and Chan, H. B., 2006. “Supernarrow spectral peaks near a kinetic phase transition in a driven nonlinear micromechanical oscillator”. *Physical Review Letters,* **97**, 110602.

[127] Stambaugh, C., and Chan, H. B., 2006. “Noise-activated switching in a driven nonlinear micromechanical oscillator”. *Physical Review E,* **73**, 172302.

[128] Kozinsky, I., Postma, H. W. C., Kogan, O., Husain, A., and Roukes, M. L., 2007. “Basins of attraction of a nonlinear nanomechanical resonator”. *Physical Review Letters,* **99**(20), 207201.

[129] Badzey, R. L., Zolfagharkhani, G., Gaidarzhy, A., and Mohanty, P., 2004. “A controllable nanomechanical memory element”. *Applied Physics Letters,* **85**(16), pp. 3587–3589.

[130] Badzey, R. L., Zolfagharkhani, G., Gaidarzhy, A., and Mohanty, P., 2005. “Temperature dependence of a nanomechanical switch”. *Applied Physics Letters,* **86**, 023106.

[131] Guerra, D. N., Imboden, M., and Mohanty, P., 2008. “Electrostatically actuated silicon-based nanomechanical switch at room temperature”. *Applied Physics Letters,* **93**, 033515.

[132] Chan, H. B., and Stambaugh, C., 2006. “Fluctuation-enhanced frequency mixing in a nonlinear micromechanical oscillator”. *Physical Review B,* **73**, 224301.

[133] Almog, R., Zaitsev, S., Shtempluck, O., and Buks, E., 2007. “Signal amplification in a nanomechanical Duffing resonator via stochastic resonance”. *Applied Physics Letters,* **90**, 013508.

[134] Nayfeh, A. H., and Mook, D. T., 1995. *Nonlinear Oscillations*. Wiley-Interscience.

[135] Carr, D. W., Evoy, S., Sekaric, L., Craighead, H. G., and Parpia, J. M., 2000. “Parametric amplification in a torsional microresonator”. *Applied Physics Letters,* **77**(10), pp. 1545–1547.

[136] Olkhovets, A., Carr, D. W., Parpia, J. M., and Craighead, H. G., 2001. “Non-degenerate nanomechanical parametric amplifier”. In Proceedings of MEMS 2001: The 14th IEEE International Conference on Micro Electro Mechanical Systems, pp. 298–300.

[137] Ataman, C., Kaya, O., and Urey, H., 2004. “Analysis of parametric resonances in comb-driven microscanners”. *Proceedings of SPIE: MEMS, MOEMS, and Micromachining,* **5455**, pp. 128–136.

[138] Ataman, C., and Urey, H., 2006. “Modeling and characterization of comb-actuated resonant microscanners”. *Journal of Micromechanics and Microengineering,* **16**(1), pp. 9–16.

[139] Ataman, C., and Urey, H., 2004. “Nonlinear frequency response of comb-driven microscanners”. *Proceedings of SPIE: MOEMS Displays and Imaging Systems II,* **5348**, pp. 166–174.

[140] Urey, H., Kan, C., and Ataman, C., 2004. “Dynamic modeling of comb-driven microscanners”. In Proceedings of the International Conference on Optical MEMS and Their Applications (IEEE/LEOS Optical MEMS 2004), pp. 186–187.

[141] Mahboob, I., and Yamaguchi, H., 2008. “Bit storage and bit flip operations in an electromechanical oscillator”. *Nature Nanotechnology,* **3**(5), pp. 275–279.

[142] Kaajakari, V., and Lal, A., 2004. “Parametric excitation of circular micromachined silicon disks”. *Applied Physics Letters,* **85**(17), pp. 3923–3925.

[143] Zalalutdinov, M., Olkhovets, A., Zehnder, A., Ilic, B., Czaplewski, D., Craighead, H. G., and Parpia, J. M., 2001. “Optically pumped parametric amplification for micromechanical oscillators”. *Applied Physics Letters,* **78**(20), pp. 3142–3144.

[144] Yu, M.-F., Wagner, G. J., Ruoff, R. S., and Dyer, M. J., 2002. “Realization of parametric resonances in a nanowire mechanical system with nanomanipulation inside a scanning electron microscope”. *Physics Review B: Condensed Matter,* **66**, 073406.

[145] Ahmad, A., and Tripathi, V. K., 2005. “Parametric excitation of higher-order electromechanical vibrations of carbon nanotubes”. *Physical Review B,* **72**, 193409.

[146] Liu, C. S., and Tripathi, V. K., 2004. “Observational consequences of parametrically driven vibrations of carbon nanotubes”. *Physical Review B,* **70**, 115414.

[147] Zhang, W., Baskaran, R., and Turner, K. L., 2003. “Tun-

Copyright © 2008 by ASME

ing the dynamic behavior of parametric resonance in a micromechanical oscillator". *Applied Physics Letters,* **82**(1), pp. 130–132.
[148] Zhang, W., Baskaran, R., and Turner, K. L., 2002. "Effect of cubic nonlinearity on auto-parametrically amplified resonant MEMS mass sensor". *Sensors and Actuators A: Physical,* **102**(1-2), pp. 139–150.
[149] Rhoads, J. F., Shaw, S. W., Turner, K. L., and Baskaran, R., 2005. "Tunable microelectromechanical filters that exploit parametric resonance". *Journal of Vibration and Acoustics,* **127**(5), pp. 423–430.
[150] Napoli, M., Baskaran, R., Turner, K., and Bamieh, B., 2003. "Understanding mechanical domain parametric resonance in microcantilevers". In Proceedings of MEMS 2003: The IEEE 16th Annual International Conference on Micro Electro Mechanical Systems, pp. 169–172.
[151] Napoli, M., Olroyd, C., Bamieh, B., and Turner, K., 2005. "A novel sensing scheme for the displacement of electrostatically actuated microcantilevers". In Proceedings of the 2005 American Control Conference, pp. 2475–2480.
[152] Zhang, W., and Meng, G., 2005. "Nonlinear dynamical system of micro-cantilever under combined parametric and forcing excitations in MEMS". *Sensors and Actuators A: Physical,* **119**(2), pp. 291–299.
[153] Zhang, W.-M., and Meng, G., 2007. "Nonlinear dynamic analysis of electrostatically actuated resonant MEMS sensors under parametric excitation". *IEEE Sensors Journal,* **7**(3), pp. 370–380.
[154] Rhoads, J. F., Shaw, S. W., Turner, K. L., Moehlis, J., DeMartini, B. E., and Zhang, W., 2006. "Generalized parametric resonance in electrostatically actuated microelectromechanical oscillators". *Journal of Sound and Vibration,* **296**(4-5), pp. 797–829.
[155] Rhoads, J. F., Shaw, S. W., Turner, K. L., DeMartini, B. E., Moehlis, J., and Zhang, W., 2005. "Generalized parametric resonance in electrostatically-actuated microelectromechanical systems [abstract]". In Proceedings of ENOC 2005: The Fifth EUROMECH Nonlinear Dynamics Conference.
[156] Rhoads, J. F., Shaw, S. W., Turner, K. L., Moehlis, J., DeMartini, B. E., and Zhang, W., 2005. "Nonlinear response of parametrically-excited MEMS". In Proceedings of IDETC/CIE 2005: The 2005 ASME International Design Engineering Technical Conferences, 20th Biennial Conference on Mechanical Vibration and Noise.
[157] Baskaran, R., and Turner, K. L., 2003. "Mechanical domain coupled mode parametric resonance and amplification in a torsional mode micro electro mechanical oscillator". *Journal of Micromechanics and Microengineering,* **13**(5), pp. 701–707.
[158] Shaw, S. W., Turner, K. L., Rhoads, J. F., and Baskaran, R., 2003. "Parametrically excited MEMS-based filters". In Proceedings of the IUTAM Symposium on Chaotic Dynamics and Control of Systems and Processes in Mechanics, G. Rega and F. Vestroni, eds., Vol. 122 of *Solid Mechanics and its Applications*, Springer, pp. 137–146.
[159] Zhang, W., Baskaran, R., and Turner, K. L., 2002. "Nonlinear dynamics analysis of a parametrically resonant MEMS sensor". In Proceedings of the 2002 SEM Annual Conference and Exposition on Experimental and Applied Mechanics.
[160] Adams, S. G., Bertsch, F., Shaw, K. A., Hartwell, P. G., Moon, F. C., and MacDonald, N. C., 1998. "Capacitance based tunable resonators". *Journal of Microelectromechanical Systems,* **8**(1), pp. 15–23.
[161] Zhang, W., and Turner, K. L., 2005. "Application of parametric resonance amplification in a single-crystal silicon micro-oscillator based mass sensor". *Sensors and Actuators A: Physical,* **122**(1), pp. 23–30.
[162] Requa, M. V., and Turner, K. L., 2006. "Electromechanically driven and sensed parametric resonance in silicon microcantilevers". *Applied Physics Letters,* **88**, 263508.
[163] Requa, M. V., 2006. "Parametric resonance in microcantilevers for applications in mass sensing". Ph.D. Dissertation, University of California, Santa Barbara.
[164] Cleland, A. N., 2005. "Thermomechanical noise limits on parametric sensing with nanomechanical resonators". *New Journal of Physics,* **7**(235), pp. 1–16.
[165] Requa, M. V., and Turner, K. L., 2007. "Precise frequency estimation in a microelectromechanical parametric resonator". *Applied Physics Letters,* **90**, 173508.
[166] Wang, Y. C., Adams, S. G., Thorp, J. S., MacDonald, N. C., Hartwell, P., and Bertsch, F., 1998. "Chaos in MEMS, parameter estimation and its potential application". *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications,* **45**(10), pp. 1013–1020.
[167] Rhoads, J. F., 2007. "Exploring and exploiting resonance in coupled and/or nonlinear microelectromechanical oscillators". Ph.D. Dissertation, Michigan State University.
[168] Yazdi, N., Ayazi, F., and Najafi, K., 1998. "Micromachined inertial sensors". *Proceedings of the IEEE,* **86**(8), pp. 1640–1659.
[169] Oropeza-Ramos, L. A., and Turner, K. L., 2005. "Parametric resonance amplification in a MEMgyroscope". In Proceedings of IEEE Sensors 2005: The Fourth IEEE Conference on Sensors.
[170] Oropeza-Ramos, L. A., 2007. "Investigations on novel platforms of micro electro mechanical inertial sensors: Analysis, construction and experimentation". Ph.D. Dissertation, University of California, Santa Barbara.
[171] Miller, N. J., Shaw, S. W., Oropeza-Ramos, L. A., and Turner, K. L., 2008. "A MEMS-based rate gyro based on parametric resonance". In Proceedings of ESDA 2008: The 9th Biennial ASME Conference on Engineering Systems Design and Analysis.
[172] Miller, N. J., Shaw, S. W., Oropeza-Ramos, L. A., and Turner, K. L., 2008. "Analysis of a novel MEMS gyro-

 Copyright © 2008 by ASME

scope actuated by parametric resonance”. In Proceedings of ENOC 2008: The 6th EUROMECH Nonlinear Dynamics Conference.

[173] Chan, H. B., and Stambaugh, C., 2007. “Activation barrier scaling and crossover for noise-induced switching in a micromechanical parametric oscillator”. *Physical Review Letters,* **99**, 060601.

[174] Chan, H. B., Dykman, M. I., and Stambaugh, C., 2008. “Paths of fluctuation induced switching”. *Physical Review Letters,* **100**, 130602.

[175] Howson, D. P., and Smith, R. B., 1970. *Parametric Amplifiers*. European Electrical and Electron Engineering Series. McGraw-Hill, London.

[176] Louisell, W. H., 1960. *Coupled Mode and Parametric Electronics*. John Wiley & Sons, New York, NY.

[177] Mumford, W. W., 1960. “Some notes on the history of parametric transducers”. *Proceedings of the IRE,* **48**(5), pp. 848–853.

[178] Rugar, D., and Grutter, P., 1991. “Mechanical parametric amplification and thermomechanical noise squeezing”. *Physical Review Letters,* **67**(6), pp. 699–702.

[179] Gallacher, B. J., Burdess, J. S., Harris, A. J., and Harish, K. M., 2005. “Active damping control in MEMS using parametric pumping”. In Proceedings of Nanotech 2005: The 2005 NSTI Nanotechnology Conference and Trade Show, Vol. 7, pp. 383–386.

[180] Gallacher, B. J., Burdess, J. S., and Harish, K. M., 2006. “A control scheme for a MEMS electrostatic resonant gyroscope excited using combined parametric excitation and harmonic forcing”. *Journal of Micromechanics and Microengineering,* **16**(2), pp. 320–331.

[181] Raskin, J.-P., Brown, A. R., Khuri-Yakub, B. T., and Rebeiz, G. M., 2000. “A novel parametric-effect MEMS amplifier”. *Journal of Microelectromechanical Systems,* **9**(4), pp. 528–537.

[182] Koskenvuori, M., and Tittonen, I., 2008. “Parametrically amplified microelectromechanical mixer”. In Proceedings of MEMS 2008: The 21st IEEE International Conference on Micro Electro Mechanical Systems, pp. 1044–1047.

[183] Dana, A., Ho, F., and Yamamoto, Y., 1998. “Mechanical parametric amplification in piezoresistive gallium arsenide microcantilevers”. *Applied Physics Letters,* **72**(10), pp. 1152–1154.

[184] Roukes, M. L., Ekinci, K. L., Yang, Y. T., Huang, X. M. H., Tang, H. X., Harrington, D. A., Casey, J., and Artlett, J. L., 2004. An apparatus and method for two-dimensional electron gas actuation and transduction for GaAs NEMS. International Patent.

[185] Ono, T., Wakamatsu, H., and Esashi, M., 2005. “Parametrically amplified thermal resonant sensor with pseudo-cooling effect”. *Journal of Micromechanics and Microengineering,* **15**(12), pp. 2282–2288.

[186] Mahboob, I., and Yamaguchi, H., 2008. “Parametrically pumped ultrahigh Q electromechanical resonator”. *Applied Physics Letters,* **92**, 253109.

[187] Rhoads, J. F., and Shaw, S. W., 2008. “The effects of nonlinearity on parametric amplifiers”. In Procedings of IDETC/CIE 2008: The ASME 2008 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, 2nd International Conference on Micro- and Nanosystems, to appear.

[188] Aubin, K., Zalalutdinov, M., Alan, T., Reichenbach, R., Rand, R., Zehnder, A., Parpia, J., and Craighead, H., 2004. “Limit cycle oscillations in CW laser-driven NEMS”. *Journal of Microelectromechanical Systems,* **13**(6), pp. 1018–1026.

[189] Pandey, M., Rand, R., and Zehnder, A., 2007. “Perturbation analysis of entrainment in a micromechanical limit cycle oscillator”. *Communications in Nonlinear Science and Numerical Simulation,* **12**(7), pp. 1291–1301.

[190] Sahai, T., and Zehnder, A. T., 2008. “Modeling of coupled dome-shaped microoscillators”. *Journal of Microelectromechanical Systems,* **17**(3), pp. 777–786.

[191] Zalalutdinov, M., Aubin, K. L., Pandey, M., Zehnder, A. T., Rand, R. H., Craighead, H. G., Parpia, J. M., and Houston, B. H., 2003. “Frequency entrainment for micromechanical oscillator”. *Applied Physics Letters,* **83**(16), pp. 3281–3283.

[192] Pandey, M., Aubin, K., Zalalutdinov, M., Reichenbach, R. B., Zehnder, A. T., Rand, R. H., and Craighead, H. G., 2006. “Analysis of frequency locking in optically driven MEMS resonators”. *Journal of Microelectromechanical Systems,* **15**(6), pp. 1546–1554.

[193] Sung, S., Lee, J. G., Lee, B., and Kang, T., 2003. “Design and performance test of an oscillation loop for a MEMS resonant accelerometer”. *Journal of Micromechanics and Microengineering,* **13**(2), pp. 246–253.

[194] Sung, S., Lee, J. G., and Kang, T., 2003. “Development and test of MEMS accelerometer with self-sustatined oscillation loop”. *Sensors and Actuators A: Physical,* **109**(1-2), pp. 1–8.

[195] Bedair, S. S., and Fedder, G. K., 2004. “CMOS MEMS oscillator for gas chemical detection”. In Proceedings of IEEE Sensors 2004: The 3rd IEEE Conference on Sensors, pp. 955–958.

[196] Feng, Z. C., Shih, A., and Miller, W. H., 2003. “Self-excited oscillations of structures by particle emission”. *Nonlinear Dynamics,* **32**(1), pp. 15–32.

[197] di Bernardo, M., Budd, C. J., Champneys, A. R., and Kowalczyk, P., 2007. *Piecewise-Smooth Dynamical Systems: Theory and Applications*. Applied Mathematics Series No. 163. Springer, New York, NY.

[198] Mita, M., Aria, M., Tensaka, S., Kobayashi, D., and Fujita, H., 2003. “A micromachined impact microactuator driven by electrostatic force”. *Journal of Microelectromechanical Systems,* **12**(1), pp. 37–41.

[199] Zhao, X., Dankowicz, H., Reddy, C. K., and Nayfeh, A. H., 2004. “Modeling and simulation methodology for

Copyright © 2008 by ASME

impact microactuators". *Journal of Micromechanics and Microengineering,* **14**(6), pp. 775–784.

[200] Zhao, X., Reddy, C. K., and Nayfeh, A. H., 2005. "Nonlinear dynamics of an electrically driven impact microactuator". *Nonlinear Dynamics,* **40**(3), pp. 227–239.

[201] Dankowicz, H., and Zhao, X., 2005. "Local analysis of co-dimension-one and co-dimension-two grazing bifurcations in impact microactuators". *Physica D,* **202**(3-4), pp. 238–257.

[202] Shaw, S. W., 1985. "The dynamics of a harmonically excited system having rigid amplitude constraints, part I: Subharmonic motions and local bifurcations". *Journal of Applied Mechanics,* **52**(2), pp. 453–458.

[203] Shaw, S. W., 1985. "The dynamics of a harmonically excited system having rigid amplitude constraints, part II: Chaotic motions and global bifurcations". *Journal of Applied Mechanics,* **52**(2), pp. 459–464.

[204] Lin, Y., Li, W.-C., Ren, Z., and Nguyen, C. T.-C., 2008. "The micromechanical resonant switch ("resoswitch")". In Proceedings of Hilton Head 2008: The 2008 Solid-State Sensors, Actuators, and Microsystems Workshop, pp. 40–43.

[205] Zhang, W., Zhang, W., and Turner, K. L., 2005. "Nonlinear dynamics of micro impact oscillators in high frequency MEMS switch application". In Proceedings of Transducers 2005: the 13th International Conference on Solid-State Sensors and Actuators.

[206] Bucher, I., Avivi, G., and Velger, M., 2004. "Design and analysis of multi degrees of freedom micro-mirror for triangular-wave scanning". In Proceedings of Smart Structures and Materials 2004: Smart Structures and Integrated Systems, A. B. Flatau, ed., Vol. 5390, pp. 410–420.

[207] Krylov, S., and Barnea, D. I., 2005. "Bouncing mode electrostatically actuated scanning micromirror for video application". *Journal of Micromechanics and Microengineering,* **14**(6), pp. 1281–1296.

[208] Conley, W., Raman, A., Krousgrill, C. M., and Mohammadi, S., 2008. "Nonlinear and non-planar dynamics of suspended nanotube and nanowire resonators". *Nano Letters,* **8**(6), pp. 1590–1595.

[209] Daqaq, M. F., Abdel-Rahman, E. M., and Nayfeh, A. H., 2008. "Towards a stable low-voltage torsional microscanner". *Microsystem Technologies,* **14**(6), pp. 725–737.

[210] Daqaq, M. F., Abdel-Rahman, E. M., and Nayfeh, A. H., 2006. "Modal interactions in resonant microscanners". In Proceedings of IMECE 2006: The 2006 ASME International Mechanical Engineering Congress and Exposition.

[211] Zhao, J. P., Chen, H. L., Huang, J. M., and Liu, A. Q., 2005. "A study of dynamic characteristics and simulation of MEMS torsional micromirrors". *Sensors and Actuators A: Physical,* **120**(1), pp. 199–210.

[212] Vyas, A., and Bajaj, A. K., 2005. "Nonlinear modeling of novel MEMS resonators". In Proceedings of ENOC-2005: The Fifth EUROMECH Nonlinear Dynamics Conference.

[213] Vyas, A., Peroulis, D., and Bajaj, A. K., 2008. "A microresonator design based on nonlinear 1:2 internal resonance in flexural structural modes". *Journal of Microelectromechanical Systems*, to appear.

[214] Lin, L., Nguyen, C. T.-C., Howe, R. T., and Pisano, A. P., 1992. "Micro electromechanical filters for signal processing". In Proceedings of IEEE Microelectromechanical Systems Workshop, pp. 226–231.

[215] Johnson, R. A., Borner, M., and Konno, M., 1971. "Mechanical filters - a review of progress". *IEEE Transactions of Sonics and Ultrasonics,* **SU-18**(3), pp. 155–170.

[216] Nguyen, C. T.-C., 1997. "High-Q microelectromechanical oscillators and filters for communications". In Proceedings of the 1997 IEEE International Symposium on Circuits and Systems, pp. 2825–2828.

[217] Wang, K., and Nguyen, C. T.-C., 1997. "High-order micromechanical electronic filters". In Proceedings of the 1997 IEEE International Micro Electro Mechanical Workshop, pp. 25–30.

[218] Bannon III, F. D., Clark, J. R., and Nguyen, C. T.-C., 2000. "High-Q HF microelectromechanical filters". *IEEE Journal of Solid-State Circuits,* **35**(4), pp. 512–526.

[219] Demirci, M. U., and Nguyen, C. T.-C., 2006. "Mechanically corner-coupled square microresonator array for reduced series motional resistance". *Journal of Microelectromechanical Systems,* **15**(6), pp. 1419–1436.

[220] Demirci, M. U., and Nguyen, C. T.-C., 2003. "Higher-mode free-free beam micromechanical resonators". In Proceedings of the 2003 IEEE International Frequency Control Symposium, pp. 810–818.

[221] Li, S.-S., Lin, Y.-W., Ren, Z., and Nguyen, C. T.-C., 2006. "Disk-array design for suppression of unwanted modes in micromechanical composite-array filters". In Proceedings of MEMS 2006: The 19th IEEE International Conference on Micro Electro Mechanical Systems, pp. 866–869.

[222] Lin, Y.-W., Hung, L.-W., Li, S.-S., Ren, Z., and Nguyen, C. T.-C., 2007. "Quality factor boosting via mechanically-coupled arraying". In Proceedings of Transducers 2007: The 14th International Conference on Solid-State Sensors, Actuators, and Microsystems, pp. 2453–2456.

[223] Bannon III, F. D., Clark, J. R., and Nguyen, C. T.-C., 1996. "High frequency microelectromechanical IF filters". In Proceedings of the IEEE International Electron Devices Meeting.

[224] Arellano, N., Quevy, E. P., Provine, J., Maboudian, R., and Howell, L. L., 2008. "Silicon nanowire coupled microresonators". In Proceedings of MEMS 2008: The 21st IEEE International Conference on Micro Electro Mechanical Systems, pp. 721–724.

[225] Clark, J. R., Bannon III, F. D., Wong, A.-C., and Nguyen, C. T.-C., 1997. "Parallel-resonator hf micromechanical bandpass filters". In Proceedings of the 1997 International Conference on Solid-State Sensors and Actuators, pp. 1161–1164.

Copyright © 2008 by ASME

[226] Pourkamali, S., and Ayazi, F., 2005. "Electrically coupled MEMS bandpass filters, part I: With coupling elements". *Sensors and Actuators A: Physical,* **122**(2), pp. 307–316.

[227] Pourkamali, S., and Ayazi, F., 2005. "Electrically coupled MEMS bandpass filters, part II: Without coupling elements". *Sensors and Actuators A: Physical,* **122**(2), pp. 317–325.

[228] Ho, G. K., Abdolvand, R., and Ayazi, F., 2004. "Through-support-coupled micromechanical filter array". In Proceedings of MEMS 2004: The 17th IEEE International Conference on Micro Electro Mechanical Systems, pp. 769–772.

[229] Greywall, D. S., and Busch, P. A., 2002. "Coupled micromechanical drumhead resonators with practical application as electromechanical bandpass filters". *Journal of Micromechanics and Microengineering,* **12**(6), pp. 925–938.

[230] Judge, J. A., Houston, B. H., Photiadis, D. M., and Herdic, P. C., 2005. "Effects of disorder in one- and two-dimensional micromechanical resonator arrays for filtering". *Journal of Sound and Vibration,* **290**(3-5), pp. 1119–1140.

[231] Zalalutdinov, M. K., Baldwin, J. W., Marcus, M. H., Reichenbach, R. B., Parpia, J. M., and Houston, B. H., 2006. "Two-dimensional array of coupled nanomechanical resonators". *Applied Physics Letters,* **88**, 143504.

[232] Weinstein, D., Bhave, S. A., Tada, M., Mitarai, S., Morita, S., and Ikeda, K., 2007. "Mechanical coupling of 2D resonator arrays for MEMS filter applications". In Proceedings of the 2007 IEEE International Frequency Control Symposium, pp. 1362–1365.

[233] Li, S.-S., Lin, Y.-W., Ren, Z., and Nguyen, C. T.-C., 2007. "A micromechanical parallel-class disk-array filter". In Proceedings of the 2007 IEEE International Frequency Control Symposium, pp. 1356–1361.

[234] Acar, C., and Shkel, A. M., 2006. "Inherently robust micromachined gyroscopes with 2-DOF sense-mode oscillator". *Journal of Microelectromechanical Systems,* **15**(2), pp. 380–387.

[235] Trusov, A. A., Schofield, A. R., and Shkel, A. M., 2008. "New architectural design of a temperature robust MEMS gyroscope with improved gain-bandwidth characteristics". In Proceedings of Hilton Head 2008: A Solid-State Sensors, Actuators, and Microsystems Workshop.

[236] Anderson, P. W., 1958. "Absence of diffusion in certain random lattices". *Physical Review,* **109**(5), pp. 1492–1505.

[237] Spletzer, M., Raman, A., Reifenberger, R., Wu, A. Q., and Xu, X., 2005. "Biochemical mass detection using mode localization in microcantilever arrays". In Proceedings of IMECE 2005: The 2005 ASME International Mechanical Engineering Congress and Exposition.

[238] Spletzer, M., Raman, A., Sumali, H., and Sullivan, J. P., 2008. "Highly sensitive mass detection and identification using vibration localization in coupled microcantilever arrays". *Applied Physics Letters,* **92**, 114102.

[239] Spletzer, M., Raman, A., Wu, A. Q., Xu, X., and Reifenberger, R., 2006. "Ultrasensitive mass sensing using mode localization in coupled microcantilevers". *Applied Physics Letters,* **88**, 254102.

[240] DeMartini, B. E., Rhoads, J. F., Shaw, S. W., and Turner, K. L., 2007. "A resonant single input - single output mass sensor based on a coupled array of microresonators". *Sensors and Actuators A: Physical,* **137**(1), pp. 147–156.

[241] DeMartini, B. E., Rhoads, J. F., Shaw, S. W., and Turner, K. L., 2006. "A resonant SISO sensor based on a coupled array of microelectromechanical oscillators". In Proceedings of Hilton Head 2006: A Solid-State Sensor, Actuator, and Microsystems Workshop.

[242] DeMartini, B. E., Rhoads, J. F., Zielke, M. A., Owen, K. G., Shaw, S. W., and Turner, K. L., 2008. "A single input-single output coupled microresonator array for the detection and identification of multiple analytes". *Applied Physics Letters*, to appear.

[243] Shaw, S. W., Rhoads, J. F., DeMartini, B. E., and Turner, K. L., 2006. Sensor with microelectro-mechanical oscillators. US Patent. Application no. 20080110247.

[244] Porfiri, M., 2008. "Vibrations of parallel arrays of electrostatically actuated microplates". *Journal of Sound and Vibration,* **315**(4-5), pp. 1071–1085.

[245] Zhu, J., Ru, C. Q., and Mioduchowski, A., 2006. "Stuctural instability of a parallel array of mutually attracting identical microbeams". *Journal of Micromechanics and Microengineering,* **16**(10), pp. 2220–2229.

[246] Gaidarzhy, A., Imboden, M., Mohanty, P., Rankin, J., and Sheldon, B. W., 2007. "High quality factor gigahertz frequencies in nanomechanical diamond resonators". *Applied Physics Letters,* **91**, 203503.

[247] Gaidarzhy, A., Zolfagharkhani, G., Badzey, R. L., and Mohanty, P., 2005. "Spectral reponse of a gigahertz-range nanomechanical oscillator". *Applied Physics Letters,* **86**, 254103.

[248] Abdel-Rahman, E. M., Hammad, B. K., and Nayfeh, A. H., 2005. "Simulation of a MEMS RF filter". In Proceedings of IDETC/CIE 2005: The 2005 ASME International Design Engineering Technical Conferences & Computers and Information in Engineering Conference.

[249] Hammad, B. K., Abdel-Rahman, E. M., and Nayfeh, A. H., 2006. "Characterization of a tunable MEMS RF filter". In Proceedings of IMECE 2006: The 2006 ASME International Mechanical Engineering Congress and Exposition.

[250] Hammad, B. K., Nayfeh, A. H., and Abdel-Rahman, E., 2007. "A discretization approach to modeling capacitive MEMS filters". In Proceedings of IMECE 2007: The 2007 ASME International Mechanical Engineering Congress and Exposition.

[251] Hammad, B. K., Nayfeh, A. H., and Abdel-Rahman, E.,

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2007. "A subharmonic resonance-baesd MEMS filter". In Proceedings of IMECE 2007: The 2007 ASME International Mechanical Engineering Congress and Exposition.

[252] Buks, E., and Roukes, M. L., 2002. "Electrically tunable collective response in a coupled micromechanical array". *Journal of Microelectromechanical Systems,* **11**(6), pp. 802–807.

[253] Napoli, M., Zhang, W., Turner, K., and Bamieh, B., 2005. "Characterization of electrostatically coupled microcantilevers". *Journal of Microelectromechanical Systems,* **14**(2), pp. 295–304.

[254] Bromberg, Y., Cross, M. C., and Lifshitz, R., 2006. "Response of discrete nonlinear systems with many degrees of freedom". *Physical Review E,* **73**, 016214.

[255] Lifshitz, R., and Cross, M. C., 2003. "Response of parametrically driven nonlinear coupled oscillators with application to micromechanical and nanomechanical resonator arrays". *Physical Review B,* **67**, 134302.

[256] Zhu, J., Ru, C. Q., and Mioduchowski, A., 2007. "High-order subharmonic parametric resonance of nonlinearly coupled micromechanical oscillators". *European Physical Journal B,* **58**(4), pp. 411–421.

[257] Gutschmidt, S., and Gottlieb, O., 2008. "Numerical analysis of a three element microbeam array subject to electrodynamical parametric excitation". In Proceedings of ESDA 2008: The 9th Biennial ASME Conference on Engineering Systems Design and Analysis.

[258] Gutschmidt, S., and Gottlieb, O., 2008. "Nonlinear internal resonance of a microbeam array near the pull-in point". In Proceedings of ENOC 2008: The Sixth EUROMECH Nonlinear Dynamics Conference.

[259] Gutschmidt, S., and Gottlieb, O., 2007. "Internal resonance in microbeam arrays subject to electrodynamical parametric excitation". In Proceedings of IDETC/CIE 2007: ASME 2007 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference, 6th International Conference on Multibody Systems, Nonlinear Dynamics, and Control (MSNDC).

[260] Chen, Q., Huang, L., and Lai, Y.-C., 2008. "Chaos-induced intrinsic localized modes in coupled microcantilever arrays". *Applied Physics Letters,* **92**, 241914.

[261] Dick, A. J., Balachandran, B., and Mote Jr., C. D., 2008. "Intrinsic localized modes in microresonator arrays and their relationship to nonlinear vibration modes". *Nonlinear Dynamics*, to appear.

[262] Dick, A. J., Balachandran, B., and Mote Jr., C. D., 2006. "Nonlinear vibration modes in micro-resonator arrays". In Smart Structures and Materials 2006: Modeling, Signal Processing, and Control, D. K. Lindner, ed., Vol. 6166.

[263] Maniadis, P., and Flach, S., 2006. "Mechanism of discrete breather excitation in driven micro-mechanical cantilever arrays". *Europhysics Letters,* **74**(3), pp. 452–458.

[264] Sato, M., Hubbard, B. E., English, L. Q., Sievers, A. J., Ilic, B., Czaplewski, D. A., and Craighead, H. G., 2003. "Study of intrinsic localized vibrational modes in micromechanical oscillator arrays". *Chaos,* **13**(2), pp. 702–715.

[265] Sato, M., Hubbard, B. E., and Sievers, A. J., 2006. "Nonlinear energy localization and its manipulation in micromechanical oscillator arrays". *Reviews of Modern Physics,* **78**(1), pp. 137–157.

[266] Sato, M., Hubbard, B. E., Sievers, A. J., Ilic, B., and Craighead, H. G., 2004. "Optical manipulation of intrinsic localized vibrational energy in cantilever arrays". *Europhysics Letters,* **66**(3), pp. 318–323.

[267] Sato, M., Hubbard, B. E., Sievers, A. J., Ilic, B., Czaplewski, D. A., and Craighead, H. G., 2003. "Observation of locked intrinsic localized vibrational modes in a micromechanical oscillator array". *Physical Review Letters,* **90**(4), 044102.

[268] Sato, M., and Sievers, A. J., 2007. "Driven localized excitations in the acoustic spectrum of small nonlinear macroscopic and microscopic lattices". *Physical Review Letters,* **98**, 214101.

[269] Campbell, D. K., Flach, S., and Kivshar, Y. S., 2004. "Localizing energy through nonlinearity and discreteness". *Physics Today,* **57**(1), pp. 43–49.

[270] Hoppensteadt, F. C., and Izhikevich, E. M., 2001. "Synchronization of MEMS resonators and mechanical neurocomputing". *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications,* **48**(2), pp. 133–138.

[271] Bennett, M., Schatz, M. F., Rockwood, H., and Wiesenfeld, K., 2002. "Huygen's clocks". *Proceedings of the Royal Society of London A,* **458**(2019), pp. 563–579.

[272] Cross, M. C., Rogers, J. L., Lifshitz, R., and Zumdieck, A., 2006. "Synchronization by reactive coupling and nonlinear frequency pulling". *Physical Review E,* **73**, 036205.

[273] Cross, M. C., Zumdieck, A., Lifshitz, R., and Rogers, J. L., 2004. "Synchronization by nonlinear frequency pulling". *Physical Review Letters,* **93**, 224101.

[274] Zalalutdinov, M., Aubin, K. L., Reichenbach, R. B., Zehnder, A. T., Houston, B., Parpia, J. M., and Craighead, H. G., 2003. "Shell-type micromechanical actuator and resonator". *Applied Physics Letters,* **83**(18), pp. 3815–3817.

[275] Shim, S.-B., Imboden, M., and Mohanty, P., 2007. "Syncronized oscillation in coupled nanomechanical oscillators". *Science,* **316**(5821), pp. 95–99.

[276] Blencowe, M., 2004. "Quantum electromechanical systems". *Physics Reports,* **395**(3), pp. 159–222.

[277] Ekinci, K. L., Yang, Y. T., and Roukes, M. L., 2004. "Ultimate limits to inertial mass sensing based upon nanoelectromechanical systems". *Journal of Applied Physics,* **95**(5), pp. 2682–2689.

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