Errata:

Dynamics of Coupled Electromagnetically-Actuated Microbeams

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This document serves as an errata for the paper entitled Dynamics of Coupled Electromagnetically-Actuated Microbeams, which was presented by the aforementioned authors at the 2011 NSF Engineering Research and Innovation Conference in Atlanta, Georgia.

Correction 1. In Fig. 2, the orientation of the magnetic field was incorrectly defined. The correct version is given below.

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\[ \vec{B} \] is oriented at an angle \( \alpha \) with respect to the vertical reference.
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Figure 2: Model of the beam in three dimensions. As shown in the inset figure, the magnetic field \( \vec{B} \) is oriented at an angle \( \alpha \) with respect to the vertical reference.
Correction 2. Equations (1), (5), (11), (12), (13), (15), (21), (27) and (30) contained several mistakes. The correct versions are given below.

\[ \dot{L} = \frac{1}{2} p A (\dot{u}^2 + \dot{v}^2) - \frac{1}{2} E I \psi'^2, \]  

\[
\frac{\partial^2 \dot{v}}{\partial t^2} + \frac{\epsilon}{E} \frac{\partial^2 \dot{v}}{\partial s^2} + \frac{v_0^2}{E} \frac{\partial^2 \dot{v}}{\partial s^2} \frac{\partial^3 \dot{v}}{\partial s^3} + \frac{v_0^2}{E} \frac{\partial^2 \dot{v}}{\partial s^2} + \frac{v_0^2}{2 \psi^2} \frac{\partial}{\partial s} \int_0^s \left( \frac{\partial \psi}{\partial s} \right)^2 \partial s \text{d}s
\]

\[
+ \frac{1}{2} \frac{\partial^2 \dot{v}}{\partial s^2} \int_0^s \frac{\partial}{\partial s} \int_0^s \left( \frac{\partial \psi}{\partial s} \right)^2 \partial s \text{d}s \text{d}s + \frac{t^2}{E T} \frac{\partial \dot{v}}{\partial \tau} \int_0^s F_1 (\dot{t}) \psi (\dot{t} - 1) \text{d}s
\]

\[
+ \frac{3}{2} \frac{v_0^2}{E T} \left( \frac{\partial \psi}{\partial s} \right)^2 \frac{\partial^2 \dot{v}}{\partial s^2} \int_0^s F_1 (\dot{t}) \psi (\dot{t} - 1) \text{d}s + \frac{t^2}{E T} \frac{\partial \dot{v}}{\partial \tau} F_1 (\dot{t}) \psi (\dot{t} - 1) + \frac{v_0^2}{2 \psi^2} \left( \frac{\partial \psi}{\partial s} \right)^3 F_1 (\dot{t}) \psi (\dot{t} - 1)
\]

\[
= \frac{l^3}{v_0 E T} F_2 (\dot{t}) \psi (\dot{t} - 1),
\]

\[ \Phi (\dot{t}) = \left[ B g \sin \alpha_0 \int_0^1 \psi _0 \int_0^1 \psi d \frac{d s}{d s} \right] + B g \cos \alpha_0 - \left[ \frac{B g \cos \alpha_0^2 \int_0^1 \psi _0 \int_0^1 \psi d \frac{d s}{d s}}{2l} \right] z^2, \]

\[ V_{\text{emf}} = - \frac{\partial \Phi (t)}{\partial t}, \]

\[ \omega_0 \frac{\partial \Phi (\tau)}{\partial \tau}, \]

\[ V_{\text{emf}} = - \left( \frac{B g \sin \alpha_0 \omega_0}{T} \int_0^1 \psi _0 \int_0^1 \psi d \frac{d s}{d s} \right) z' + \left( \frac{B g \cos \alpha_0^2 \omega_0}{I T} \int_0^1 \psi _0 \int_0^1 \psi d \frac{d s}{d s} \right) z z', \]

\[ \alpha_2 = - \frac{(B g)^2 \sin^2 \alpha T}{Z \omega_0 \rho A l} \psi^2 |_{s=1}, \]

\[ V_o = V_i n + \sum_{p=1}^N \kappa_p \omega_0 \Omega \left[ \frac{z (i \Omega)}{\frac{1}{2}} \right], \]

\[ \kappa_p = - \frac{B g \sin \alpha_0 \psi _0}{T} \int_0^1 \psi _0 \int_0^1 \psi d \frac{d s}{d s}, \]

\[ R_{ij} = \begin{bmatrix} 18 & 4.75 \times 10^{-4} & 6.47 \times 10^{-4} \\ 5.92 \times 10^{-4} & 19.78 & 1.72 \times 10^{-4} \\ 5.23 \times 10^{-4} & 1.12 \times 10^{-4} & 16.44 \end{bmatrix} \frac{H z}{p g}, \]

\[ V_{\text{emf}} = \kappa_1 z' + \kappa_2 z z', \]

\[ \kappa_1 = - \frac{B g \sin \alpha_0 \omega_0}{T} \int_0^1 \psi _0 \int_0^1 \psi d \frac{d s}{d s}, \]

\[ \kappa_2 = \frac{B g \cos \alpha_0^2 \omega_0}{I T} \int_0^1 \psi _0 \int_0^1 \psi d \frac{d s}{d s}, \]
Correction 3. The expressions for $\epsilon_{\lambda_1}$ and $\epsilon_{\lambda_3}$ in Table 1 and $\epsilon_{(\alpha_2)_i}$ in Table 2 contained errors. The correct versions are given below.

$$
\epsilon_{\lambda_1} = \frac{gB\ell^2 \cos \alpha}{EI\omega_0^2} \int_0^1 \Psi^2 d\hat{s},
$$

$$
\epsilon_{\lambda_3} = \frac{v_0^2 gB \cos \alpha}{2EI\omega_0^2} \int_0^1 \Psi'\Psi d\hat{s},
$$

$$
\epsilon_{(\alpha_2)_i} = -\frac{(Bg_i)^2 \sin^2 \alpha T}{Z\omega_0 \rho A_i l_i} \Psi^2|_{\hat{s}=1}.
$$

Correction 4. Due to the aforementioned errors, Figs. 4, 5 and 7 were incorrect. Corrected versions are given below.

The authors apologize for any inconvenience caused by these typographical errors and technical discrepancies.

Figure 4: Plot of measured output voltage $V_o$ (arbitrary units) of an electromagnetically-actuated microcantilever array plotted against the normalized excitation frequency $\Omega$ for the parameter values shown in Table 3. The response shows three distinct antiresonances in the response corresponding to the coupled natural frequencies of the system.
Figure 5: Plot of measured output voltage $V_o$ (arbitrary units) of an electromagnetically-actuated microcantilever array against the normalized excitation frequency $\Omega$ for the parameter values shown in Table 3, with and without added mass. The red curve corresponds to the sensor without the added mass, while the blue curve corresponds to the sensor with the added mass on the third cantilever (the cantilever with the lowest uncoupled natural frequency). The natural frequency of the loaded cantilever is altered significantly while the resonance frequencies of the other cantilevers are largely unchanged, indicating that this sensor can be used as a single-input and single-output device capable of detecting multiple analytes.

Figure 7: Bifurcation diagram of the steady-state amplitude (arbitrary units) for the parameters in Table 3 and $\alpha = \pi/2$. 
