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SOLUTION OF YAROSHEVSKII'S PLANETARY ENTRY EQUATION VIA A PERTURBATIVE METHOD

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An analytical solution for ballistic entry problems at circular speed is obtained for Yaroshevskii's planetary entry equation. Using the Poincaré-Lindstedt method, explicit expressions for planar motions—velocity, altitude, flight path angle. Generalized Yaroshevskii's planetary entry equations are derived and solved for ballistic entry at circular speed for the case of zero or small initial flight path angles. The solutions are verified to be highly accurate via numerical simulation.

INTRODUCTION

Yaroshevskii[1,2] was one of the early pioneers of analytical work related to earth returning space vehicles during the 1950s and 1960s. Some of his contemporaries are Allen and Eggers, Loh, Chapman, and Vinh who developed entry theories on their own[3, 4, 5, 6]. In these theories, either the gravity force or the "centrifugal" force or both are neglected to obtain the analytical solutions.

Yaroshevskii developed a generalized semi-analytical entry theory in 1964.[1,2] Yaroshevskii's theory is generalized in the sense that it allows a single trajectory solution for a given initial velocity and flight path angle to be computed which applies to every possible ballistic satellite, regardless of the mass, surface area, and drag coefficient. Yaroshevskii demonstrated that when the tangential gravitational acceleration is neglected, planar equations of entry can lead to a second-order nonlinear differential equation which can be integrated analytically by using series expansion for the case of entry at circular speed for zero and small initial flight path angles. Separate solutions are available for both ballistic and lifting (small lift-to-drag coefficients) entry. It was found that Yaroshevskii's theory is a distinct case of the more refined theory by Chapman.[5]

In this paper, we will use Poincaré-Lindstedt method to solve Yaroshevskii's second-order nonlinear differential equation from circular speed for both ballistic entry at circular speed for non-zero initial flight angles. The subsequent goal is to derive and solve generalized Yaroshevskii's equation applicable to super circular entry but solved for ballistic entry at circular speed for zero or small flight path angles.

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YAROSHEVSKII'S EQUATION OF PLANETARY ENTRY

For the problem, we consider the flight of a non-thrusting, non-lifting vehicle of mass *m* inside the atmosphere of a central body. The central body is assumed to be non-rotating with a universal gravitational field and stationary atmosphere. The trajectory variables r, θ, ϕ, V, γ, ψ, are defined in Figure 1, where *r* is the radial distance, θ the longitude, ϕ the latitude, V the velocity magnitude, γ the flight-path angle relative to the local horizontal, and ψ the heading angle relative to the local latitude line. With the above assumptions, the equations of planar motion are[6]

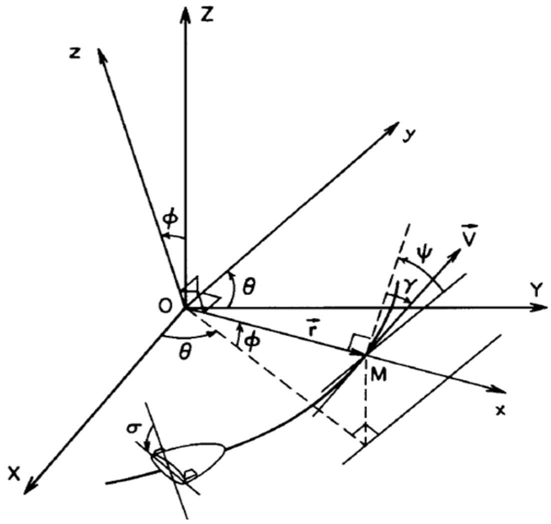

Figure 1. Coordinate systems for 3 degree-of-freedom motion of a spacecraft inside a planet's atmosphere, adapted from Vinh et al.[6]

$$\frac{dr}{dt} = V \sin\gamma \tag{1}$$

$$\frac{dV}{dt} = -\frac{\rho S C_D V^2}{2m} - g\sin\gamma \tag{2}$$

$$\frac{d\gamma}{dt} = -\frac{1}{V}\left(g - \frac{V^2}{r}\right)\cos\gamma \tag{3}$$

Where, we note that *g* in Eqs. (2) and (3) refers to the local gravitational acceleration of a given planet. S and C_D are the surface area and the drag coefficient of the spacecraft respectively. The atmosphere is assumed to be locally exponential and is given by

$$d\rho = -\beta(r)\rho dr \tag{4}$$

Yaroshevskii's Solutions for Planar Ballistic Spacecraft Entry

Yaroshevskii used some simplifying assumptions to derive a second-order nonlinear differential equation. Two separate analytical solutions were obtained—one for ballistic circular entry with $\gamma_i = 0°$, and the second for various values of γ_i. C_L and C_D are functions of the angle of attack α, Mach Number M, and the Reynolds Number R_e. For constant angle of attack, Yaroshevskii assumed that C_L and C_D are functions of the Mach Number. For an isothermal atmosphere, C_L and C_D just a function of speed, V. The tangential gravitational acceleration $-g\sin\gamma$ is neglected, and small flight path angle, γ is assumed. Thus, the equations of motion become

$$\frac{dr}{dt} = V\gamma \tag{5}$$

$$\frac{dV}{dt} = -\frac{\rho A C_D V^2}{2m} \tag{6}$$

$$\frac{d\gamma}{dt} = -\frac{1}{V}\left(g - \frac{V^2}{r}\right)\cos\gamma \tag{7}$$

Yaroshevskii further assumed that when C_D is independent of the Mach Number. The independent variable x and a dependent variable y are defined as

$$x \equiv \ln\frac{\sqrt{gr_0}}{V}; \quad y \equiv \frac{C_D S}{2m}\sqrt{\frac{r_0}{\beta}}\rho \tag{8}$$

therefore

$$\frac{dy}{dx} = \phi = -\sqrt{\beta r_0}\,\gamma \tag{9}$$

Thus, we have the Yaroshevskii's second-order nonlinear differential equation given by[6]

$$\frac{d^2y}{dx^2} = \frac{e^{2x}-1}{y} \tag{10}$$

Ballistic Decay of Spacecraft from High-Altitude Orbits

Thus for ballistic entry trajectory we have

$$\frac{d^2y}{dx^2} = \frac{e^{2x}-1}{y} \tag{11}$$

Though Eq. (9) looks simples, no closed form solution exists. Spacecraft at high altitude orbits, under the action of atmospheric drag predominantly at periapsis tends to circularize the orbit just before actual entry, i.e. $\gamma_i = 0°$. The initial conditions for such a case are given by[6]

$$x_i = 0; \quad y(0) = 0; \quad y'(0) = \frac{dy(0)}{dx} = 0 \tag{12}$$

Yaroshevskii found an elegant analytical solution given by

$$y=\sqrt{\frac{8}{3}}x^{\frac{3}{2}}\left(1+\frac{1}{6}x+\frac{1}{24}x^2+\frac{47}{4752}x^3\right) \tag{13}$$

$$\phi=\frac{dy}{dx}=\sqrt{6}x^{\frac{1}{2}}\left(1+\frac{5}{18}x+\frac{7}{72}x^2+\frac{47}{1584}x^3\right) \tag{14}$$

The variables of planar equations of motion namely, velocity, altitude, and flight path angle can be calculated by using Eqs. (13) and (14) in to Eqs. (8) and (9).

Circular Ballistic Entry of Spacecraft from High-Altitude Orbits for Various Nonzero Initial Flight Angles

When the initial flight path angle is nonzero, the nonlinear differential equation can be integrated for circular orbits. For entry from circular orbits, using Eq. (4), the initial conditions are

$$x_i=0; \quad y(0)=0; \quad y'(0)=c_1=\phi_i=-\sqrt{\beta r_0}\,\gamma_i \tag{15}$$

Yaroshevskii found series solution of the form

$$y=c_1x+c_2x^2+c_3x^3+c_4x^4+c_5x^5+\ldots \tag{16}$$

$$\phi=\frac{dy}{dx}=c_1+2c_2x+3c_3x^2+4c_4x^3+5c_5x^4+\ldots \tag{17}$$

where

$$c_0=0; \quad c_2=\frac{1}{c_1}; \quad c_3=\left(1-\frac{1}{c_1^2}\right)\frac{1}{3c_1} \tag{18}$$

$$c_4=\left(1-\frac{2}{c_1^2}+\frac{2}{c_1^4}\right)\frac{1}{9c_1}; \quad c_5=\left(3-\frac{10}{c_1^2}+\frac{20}{c_1^4}-\frac{17}{c_1^6}\right)\frac{1}{90c_1} \tag{19}$$

Yaroshevskii's solution, although elegant, suffers from certain drawbacks. First is the existence of a singularity at $y=0$. The approximate solutions, Eqs. (13) and (14) for zero initial flight path angle, and Eqs. (16), (17), (18) and (19) for nonzero initial flight path angle work well as compared to the numerical solutions. The disadvantage is that two separate solutions are needed for cases of zero and nonzero initial flight path angles. Thus, one objective is to seek a unified solution which applies to circular ballistic entry for any initial flight path angle. The trajectory variables, altitude, and flight path angle can be obtained from the analytical solutions using Eqs. (8) and (9).

NEW SOLUTION FO YAROSHEVSKII'S EQUATION

In this section, we use the Poincaré-Lindstedt method of solving differential equation for the case of circular ballistic spacecraft reentry problem. The independent variable x is a monotonically increasing function for all the possible atmospheric entry trajectory cases—direct reentry, aerocapture, and fly-through.

In order to use Poincaré-Lindstedt, we artificially insert a small parameter

$$\varepsilon = y(0) = \frac{C_D S}{2m}\sqrt{\frac{r_0}{\beta}}\rho_e \tag{20}$$

where $\rho_e = 0$ is the density of the atmosphere of any planet at entry interface. For a typical Earth entry problem from Low-Earth-Orbit, ε is of the order $\sim 10^{-5}$.

We introduce the following new variables which are functions of y and x

$$\eta = \frac{y}{\varepsilon}; \quad \tau = \frac{x}{\varepsilon}; \quad \frac{d\tau}{dx} = \frac{1}{\varepsilon}; \quad \frac{d(\ )}{d\tau} = (\ )' \tag{21}$$

Using Eq. (20), Eq. (9) is transformed in to

$$\frac{d^2\eta}{d\tau^2} = \frac{e^{2\varepsilon\tau} - 1}{\eta} \tag{22}$$

We seek solutions of the form

$$\eta = \eta_0 + \varepsilon\eta_1 + \varepsilon^2\eta_2 + \varepsilon^3\eta_3 + \ ... \tag{23}$$

With the initial conditions

$$\eta_0(0) = 1, \quad \eta_1(0) = \eta_2(0) = ... = 0 \tag{24}$$

$$\eta_0'(0) = -\sqrt{\beta r_0}\sin\gamma_i = b; \quad \eta_1'(0) = \eta_2'(0) = ... = 0 \tag{25}$$

Substituting Eq. (23) in to Eq. (22) and expanding the exponential series $e^{2\varepsilon\tau}$; and equating the coefficients of different powers of ε, after extensive algebra, we have the following system of equations

$$\varepsilon^0 : \qquad \eta_0'' = 0 \tag{26}$$

$$\varepsilon^1 : \qquad \eta_1'' - \frac{2\tau}{\eta_0} = 0 \tag{27}$$

$$\varepsilon^2 : \qquad \eta_2'' + \frac{2\eta_1\tau}{\eta_0^2} - \frac{2\tau^2}{\eta_0} = 0 \tag{28}$$

Eqs. (26), (27), and (28) must be sequentially solved. Using Eqs. (24) and (25) in Eq. (26) yields

$$\eta_0 = b\tau + a \tag{29}$$

Use Eq. (29) in Eq. (27) and solving yields

$$\eta_1 = \frac{1}{b^3}\left[b\tau(b\tau + 2) - 2(b\tau + 1)\ln(b\tau + 1)\right] \tag{30}$$

Substituting Eqs. (29) and (30) in to Eq. (228) and after series of algebra and integration steps yield

$$\eta_2 = -\frac{1}{3b^6}\left\{\begin{array}{l} -b^5\tau^3 + 3b^4\tau^2 + b^3\tau\left(\tau^2+15\right) + 9b^2\tau^2 - 6\ln\left(b\tau+1\right)\left[b^3\tau + b^2\left(\tau^2+1\right) + 5\left(b\tau+1\right)\right] \\ +33b\tau + 6\left(b\tau+1\right)\left[\ln\left(b\tau+1\right)\right]^2 + 12 \end{array}\right\} \tag{31}$$

Thus, the complete approximate solution is given by

$$\eta = \eta_0 + \varepsilon\eta_1 + \varepsilon^2\eta_2 + \ \ldots \tag{32}$$

where $e^{2\varepsilon\tau}$ and ϕ are obtained from Eqns. (12) and (21)

$$y = \varepsilon\eta_0 + \varepsilon^2\eta_1 + \varepsilon^3\eta_2 + \ \ldots \tag{33}$$

$$\phi = \frac{dy}{dx} \tag{34}$$

The trajectory variables, altitude, and flight path angle can be obtained from the analytical solutions using Eqs. (8) and (9).

GENERALIZED YAROSHEVSKII'S EQUATION FOR CIRCULAR BALLISTIC ENTRY

In Yaroshevskii's formulation the independent variable x, becomes negative for super-circular entry speed. Vinh et al.[6] derived a set of generalized Yaroshevskii's system of equations which is applicable to a wide variety of problems. We summarize the derivation and then use it to obtain a new robust solution for circular and sub-circular ballistic reentry problems for various initial flight path angles.

Because of the shallow-entry and high-altitude-deceleration characteristics specific to ballute aerocapture, Vinh's assumptions of small changes in radial position (i.e. altitude) are readily applied to Eqs. (1)-(3).

$$g\left(r\right) \approx g\left(R\right); \quad V^2/r \approx V^2/R; \quad and, -g\sin\gamma \approx 0 \tag{35}$$

where R is the radial distance at atmosphere interface (or entry). Vinh uses the following nondimensional variables, representing altitude, flight path angle, and speed, respectively

$$Z = \frac{\rho S C_D}{m}\sqrt{\frac{R}{\beta}}; \quad \phi = -\sqrt{\beta R}\sin\gamma \tag{36}$$

$$X = \log\left(V_e^2/V^2\right); \quad \alpha = gR/V_e^2 = V_c^2/V_e^2 \tag{37}$$

Where Z is a Yaroshevskii-type altitude variable similar to y. An additional non-dimensional variable, α, specifies the type of entry orbit (i.e. circular: $\alpha = 1$, hyperbolic: $\alpha < 0.5$, parabolic: $\alpha = 0.5$, elliptic: $\alpha > 0.5$. Note that the independent variable X is 0 regardless of any reentry trajectory type.

Dividing the time derivatives of Eqns. (1) and (3) by the time derivative of Eqn. (2), using the applying the small angle approximation and using Eqs. (36) and (37) provide new equations of motion

$$\frac{dZ}{dX} = \phi \tag{38}$$

$$\frac{d\phi}{dX} = \left(\alpha e^{X} - 1\right) Z^{-1} \tag{39}$$

Eqs. (38) and (39) represent the generalized Yaroshevskii's system of equations for ballistic planetary entry. Substituting Eq. (38) in to Eq. (39) we obtain the generalized Yaroshevskii's second-order nonlinear differential equation.

$$\frac{d^2 Z}{dX^2} = \frac{\left(\alpha e^{X} - 1\right)}{Z} \tag{40}$$

$$Z\left(0\right) = 0; \quad \Phi\left(0\right) = 0 \tag{41}$$

Approximate Solution of Generalized Yaroshevskii's Equation for Circular Ballistic Entry

Equation (42) cannot be solved by assuming a solution of the form $Z = a_0 + a_1 x + a_2 x^2 + ...$ because the initial conditions given by Eqn. (43) force a_m to be identically zero[6]. It is apparent that the first term is neither a constant nor a linear function of X. We will use Yaroshevskii's approach to find the first term of the series. For very small values of X

$$Z \frac{d^2 Z}{dX^2} \approx \alpha x \tag{42}$$

The solution of Eq. (42) is obtained by assuming $Z = cx^n$ and solving for c and n to obtain

$$Z \approx 2\sqrt{\frac{\alpha}{3}} x^{\frac{3}{2}} \tag{43}$$

$$\frac{dZ}{dX} = \Phi \approx \sqrt{3\alpha}\, x^{\frac{1}{2}} \tag{44}$$

Eqs. (43) and (44) satisfy the initial condition in Eqn. (41) as $X \to 0$. As in Yaroshevskii's solution, we can seek a solution of the form

$$Z \approx 2\sqrt{\frac{\alpha}{3}} x^{\frac{3}{2}} \left(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + ...\right) \tag{45}$$

Substituting Eq. (45) in to Eq. (42), the coefficients a_m can be obtained after equating coefficients of like powers of X and thus, the approximate solution are given by

$$Z = 2\sqrt{\frac{\alpha}{3}}x^{\frac{3}{2}}\left[1+\frac{1}{3}\left(\frac{x}{4}\right)+\frac{1}{6}\left(\frac{x}{4}\right)^2+\frac{47}{594}\left(\frac{x}{4}\right)^3+\frac{20021}{605880}\left(\frac{x}{4}\right)^4\right] \tag{46}$$

$$\Phi = \sqrt{3\alpha x}^{\frac{1}{2}}\left[1+\frac{5}{9}\left(\frac{x}{4}\right)+\frac{7}{18}\left(\frac{x}{4}\right)^2+\frac{47}{198}\left(\frac{x}{4}\right)^3+\frac{20021}{165240}\left(\frac{x}{4}\right)^4\right] \tag{47}$$

Approximate Solution of Generalized Yaroshevskii's Equation for Circular Ballistic Entry

Altitude: Instead of the actual altitude, from Eqn. (38), it is actually easier to compute,

$$\ln\left(\frac{Z}{Z_0}\right) = \beta(r_0 - r) \tag{48}$$

If r_{pl} is the radius of the planet, then altitude, *h*, can be obtained from the following expression

$$h = r - r_{planet} - \ln\sqrt[\beta]{\frac{Z}{Z_0}} \tag{49}$$

Flight Path Angle: The flight path angle can be readily calculated from the Eq. (36) given by Eq. (50)

$$\gamma = \sin^{-1}\left(-\frac{\Phi}{\sqrt{\beta R}}\right) \tag{50}$$

Velocity: The magnitude of the velocity can be obtained flight path angle can be readily calculated from Eqs. (36) and (37) given by Eq. (50)

$$V = \sqrt{gr}\, e^{-\frac{X}{2}} \tag{51}$$

NUMERICAL RESULTS

It is easy to see that both the classical and generalized Yaroshevskii's system of equations are universal in nature. That is, they are valid for all types of entry trajectories of any entry vehicle regardless of its mass, size, and aerodynamic characteristics. The atmospheric properties also do not show up in the equations unless we want to compute the physical variables like velocity, altitude, and the flight path angle. Nonetheless, universality does not mean that we can necessarily obtain analytical solutions for all entry trajectory types. In fact, we saw that the classical and new analytical solutions in fact are limited in their applications—ballistic, circular reentry problems for zero to small initial flight path angle cases. We now assess the accuracies of the analytical solutions both in the cases of Classical and Generalized Yaroshevskii's nonlinear differential equations as well the applications of the solutions to cases of planetary entry. The vehicle parameters used in the simulations to verify the accuracy of analytical solutions are summarized in Table 1. Table 2 contains the entry conditions and atmospheric constants at Earth.

Table 1. Vehicle Parameters for an Apollo 10 Spacecraft.[7]

Spacecraft Parameter	Value
Mass (kg)	5624
C_D	1.289
Surface Area, S (m^2)	12.03

Table 2. Entry Conditions and Atmospheric Constants at Earth.[7]

Parameter	Value
βR	900
g, m/s^2	9.86055
Reference density at Entry Interface, kg/m^2	2.22 x 10^{-8}
Entry Altitude, km	124
Inertial Entry Speed, km/s	7.8296

Assessment of Solutions of Yaroshevskii's Equation

We know that the analytical solutions work well for initial flight angle $\gamma_i = 0°$, and circular speed. Note that for supercircular entry speed, the solutions break down as *x* becomes negative at entry. Yaroshevskii's nonlinear differential equation is solved numerically using adaptive-step Runge-Kutta 4 algorithm and compared with the new analytical solutions and that of Yaroshevskii. For the case of Apollo 10 spacecraft without lift, and for $\gamma_i = -40°$, the solutions are plotted in Figure 2. We observed accuracy of the analytical solutions is reasonable as compared to the exact numerical solution even for non-zero initial flight path angles. The plot in Figure 1 also represents the particular solution of the nonlinear differential equation of the form given by Eq. (11) subjected to the initial conditions given by Eq. (12), which holds some mathematical significance.

In Figure 3, the relative error of the altitude variable *y* as a function of the independent dimensionalized velocity variable, *x*. The new solution obtained by the Poincaré-Lindstedt is less accurate than the classical Yaroshevskii's solution. The average relative error of the Poincaré-Lindstedt solution is found to be around 3%. However, it will be more relevant to see the accuracy and applicability of the solution to actual trajectory variables like altitude, velocity, and flight path angles.

The non-physical variables, x and y, can be suitably transformed in to the actual trajectory variables of altitude, velocity, and flight path angle. In Figure 4, the altitude of the entry spacecraft is plotted as a function of the speed. It can be seen that the order of magnitude of the error in altitude is same as that obtained using Yaroshevskii's solution. In fact better the new solution is better than Yaroshevskii's for portion of the trajectory where maximum deceleration and heating occurs. The actual relative error in altitude using the analytical solution can be seen from Figure 5. The average relative error for the case of new analytical solution is around 0.02% and that of Yaroshevskii's solution is 0.05%. However, it still represents an error of around 30 m and 70 m respectively. It is found that that the error in solution of altitude and flight path angle as a function of the velocity is still order of magnitude better than the classical solution due to Allen and Eggers.[3]

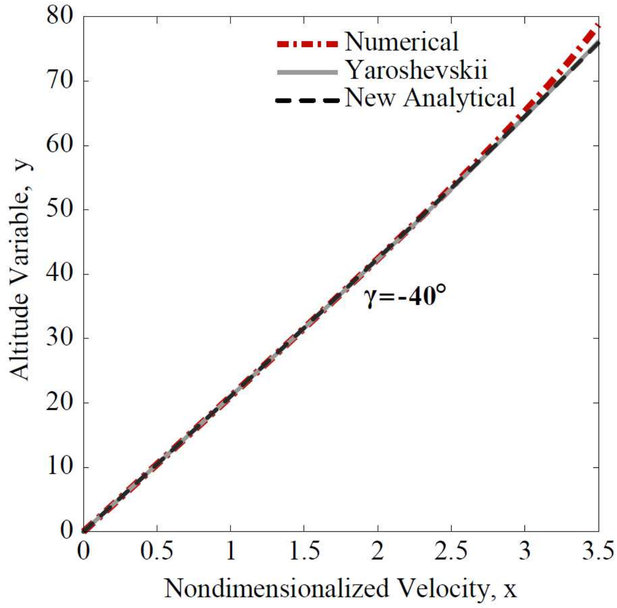

Figure 2. Variation of Altitude Variable, *y* as a Function of the Nondimensionalized Velocity, *x*

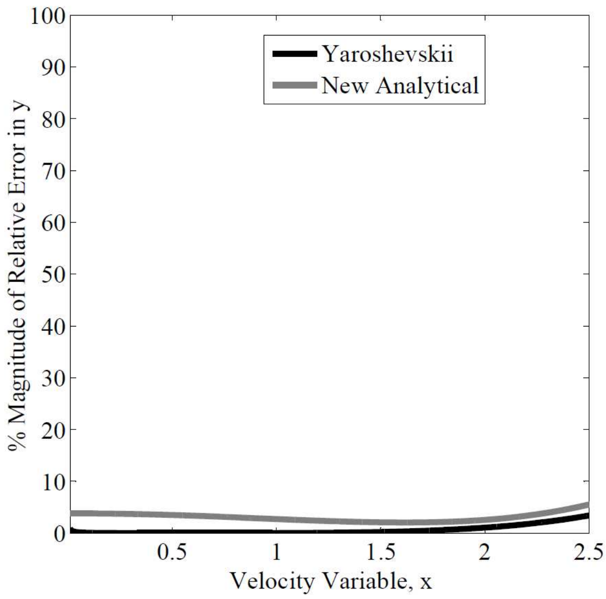

Figure 3. Plot of Relative Error of Altitude Variable as a Function of Dimensionalized Velocity Variable.

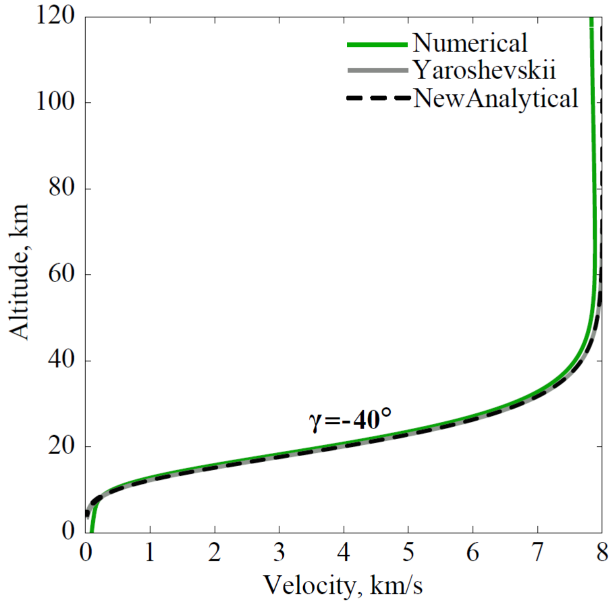

Figure 4. Variation of Altitude as a Function of Speed of the Spacecraft.

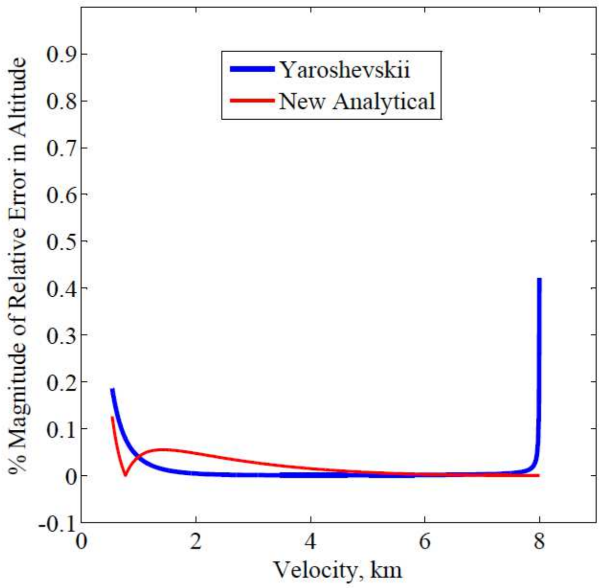

Figure 5. Plot of Relative Error of Altitude as a Function of Speed of the Entry Spacecraft.

The new solution work very well for any entry speed equal to or less than circular speed. This is due to the definition of the independent dimensionalized velocity variable, *x*, given by Eq. (8).

Similarly, the solution is also applicable to cases of non-zero entry flight angles constrained by Eq. (9).

Assessment of Solutions of Generalized Yaroshevskii's Equation

To cover the case of zero or nearly-zero (small) flight angle, we consider the solution obtained by solving the generalized Yaroshevskii's equation. We use the same spacecraft parameters as in Table 1, and same entry conditions and atmospheric constants as in Table 2 to obtain the solution for ballistic entry at circular speed for an entry flight path angle of $\gamma_i = 0°$. Instead of altitude, let the plot of flight path angle as function of the spacecraft speed is shown in Figure 6. The solution is obtained by using Eqs. (46)–(51). The solution of $\gamma_i = 0°$ is highly accurate and the average absolute error in flight path angle as a function of spacecraft speed is around 0.01°. However, the solution can be used for nearly-zero entry flight path angle up to around $\gamma_i = -2°$ beyond which solution starts to diverge.

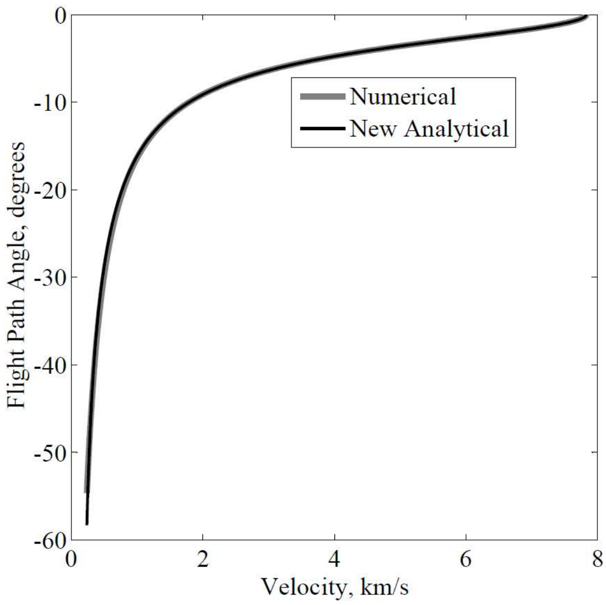

Figure 6. Variation of Flight Path Angle as a Function of Spacecraft Speed for Entry at Zero Flight Path Angle using Solution of Generalized Yaroshevskii's Equation.

The error for moderately large $(-\gamma_i > 2°)$ entry flight path angle is evident from Figure 7. For the case of $\gamma_i = -5°$, the error in flight path angle is too large to be acceptable for most part of the entry trajectory. It is noteworthy that the generalized Yaroshevskii's equation applies to ballistic entry at circular and super-circular speeds, denoted by the parameter α as seen in Eq. (40). However, the methodology used to solve the equation put a constraint on the parameter $\alpha = 1$, which denotes circular speed.

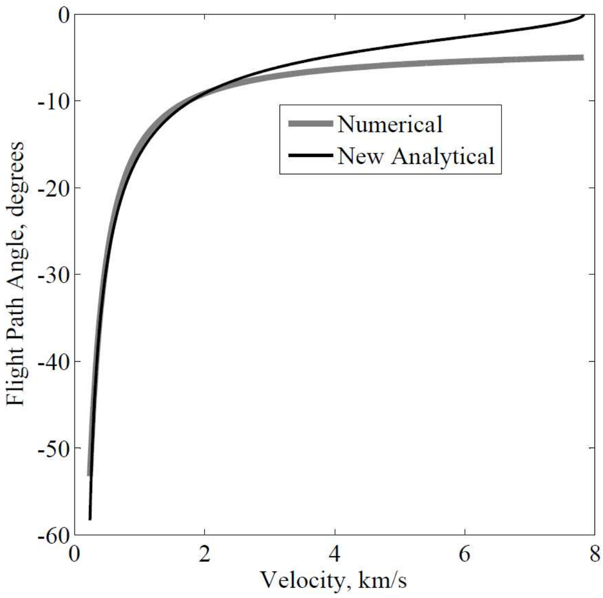

Figure 7. Variation of Flight Path Angle as a Function of Spacecraft Speed for Entry at Non-Zero Flight Path Angle using Solution of Generalized Yaroshevskii's Equation.

CONCLUSION

Based on prior analytic work by Yaroshevskii and Vinh, from the new analytic solutions provided in this paper, we are able to find explicit expressions for velocity, altitude, and flight path angle history of a spacecraft entering a planetary atmosphere. Yaroshevskii's solutions, although elegant, suffer from certain drawbacks. The disadvantage is that two separate solutions are needed for cases of zero and nonzero initial flight path angles. A new analytical solution was obtained using Poincaré-Lindstedt method that is applicable to ballistic entry at circular speed for non-zero initial flight path angles. Numerical solutions showed that these analytical solutions agree well with the exact numerical solutions for both zero and small nonzero initial flight path angles.

Generalized Yaroshevskii's system of equations and the resulting second order nonlinear differential equations are derived. A new approximate analytical solution was obtained for the case of ballistic, circular entry with zero or nearly zero initial flight angle. However, the solution was found to have significant errors for the case of large nonzero initial flight path angles. The new solutions will add to the repertoire of classical approximate analytical solutions. These solutions will help of rapid conceptual probe mission studies for the exploration of the solar system. In addition, the solution can provided initial guesses for the application of optimization to spacecraft entry problems.

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