Problem 1. For each discrete-time (DT) signal below, do the following.

(i) Sketch \( x(n) \) by hand, i.e. do not use Matlab. Carefully label the plots.

(ii) Calculate its energy, i.e. \( \sum_{n=-\infty}^{+\infty} |x(n)|^2 \). Do not use Matlab.

(iii) Calculate the convolution of the signal with itself. In other words, evaluate the following:
\[ x \ast x(n) = \sum_{k=-\infty}^{+\infty} x(k)x(n-k) \]. Do not use Matlab.

(iv) Find the smallest number \( L \) which satisfies the following inequality for every integer \( n \):
\[ |x(n)| \leq L \).

(v) Find the smallest number \( M \) which satisfies the following inequality for every integer \( n \):
\[ |x \ast x(n)| \leq M \).

a. \( x(n) = 4^{-|n|} \).

b. \( x(n) = n \cdot [u(n - 1) - u(n - 6)] \).

Here, as usual, \( u(n) \) is the DT unit step.

Hints.

1. Recall that the expression \( \sum_{k=-\infty}^{+\infty} y(k) \) means: \( \lim_{m \to -\infty, p \to +\infty} \left( \sum_{k=m}^{p} y(k) \right) \).

2. In order to compute the convolution for the first signal, it could be helpful to consider two cases: \( n < 0 \) and \( n \geq 0 \), and in each case, to sketch both \( x(k) \) and \( x(n-k) \) as functions of \( k \).

3. The numbers \( L \) and \( M \) in parts (iv) and (v) are the global maxima of the signals \( |x(n)| \) and \( |x \ast x(n)| \), respectively.