ECE 438
Homework 6, due in class Friday, 10/3/2003.

Problem 1. A continuous-time signal $x_c(t)$ is a linear combination of sinusoids of frequencies 250 Hz, 450 Hz, and 1 kHz. The signal $x_c(t)$ is ideally sampled at the rate of 1.5 kHz, and the result (denoted $x_s(t)$ in class and in the notes) is passed through an ideal lowpass filter with cutoff frequency 750 Hz, generating a continuous-time signal $y_c(t)$. What are the frequency components present in the reconstructed signal $y_c(t)$?

Problem 2. The continuous-time signal $x_c(t) = 3\cos(400\pi t) + 5\sin(1200\pi t) + 6\cos(4400\pi t) + 2\sin(5200\pi t)$ is ideally sampled at the rate of 4 kHz generating a discrete-time signal $x(n)$. Find $x(n)$.

Problem 3.

Suppose the continuous-time signal $x_c(t)$ whose spectrum $X_c(f)$ is shown above is ideally sampled with the sampling rate $f_s = 2$ kHz using the ideal sampling scheme discussed in class and illustrated in Fig. 1.44 of the class notes.

(a) Sketch the spectrum $X_s(f)$ of the sampled signal.

(b) Argue that $x_c(t)$ can be perfectly reconstructed from $x_s(t)$ using an ideal bandpass filter. Sketch the frequency response of the filter.

Problem 4. DT interpolation with sinc functions.

In class, we considered the following system for interpolating a DT signal:

In this system, upsampling by a factor of $L$ is followed by an ideal low-pass filter. For this problem, we assume the following frequency response for the ideal low-pass filter:

$$H(e^{j\omega}) = \begin{cases} L, & |\omega| \leq \frac{\pi}{L} \\ 0, & \frac{\pi}{L} < |\omega| \leq \pi \end{cases}$$

(a) Use the inverse DTFT formula to show that the impulse response of the ideal low-pass filter is $h(n) = \text{sinc}\left(\frac{n}{L}\right)$. 
Argue that therefore interpolating a DT signal \( x(n) \) can be represented as a convolution of the up-sampled version of \( x(n) \) with a sinc:

\[
x_{\text{int}}(n) = x_u * h(n),
\]

(1)

where \( h(n) = \text{sinc} \left( \frac{n}{L} \right) \),

and where \( L \) is the upsampling factor. Therefore, interpolation is achieved by summing up scaled and shifted sinc functions.

(b) In MATLAB, create the following vector of time points from 0 to 32 seconds:

\[
t = 0:0.05:32
\]

Using this vector, create a sinusoid

\[
s(t) = \sin \left( \frac{\pi}{8} t \right).
\]

Create another signal, \( s_1(n) \), which is the sampling of \( s(t) \) with sampling period 1 second. Create \( s_2(n) \), which is the sampling of \( s(t) \) with sampling period 2 seconds.

Now interpolate \( s_2(n) \), using Eq. (1) with \( L = 2 \).

Step 1. Upsample \( s_2(n) \) by inserting a zero after each sample.

Step 2. Using MATLAB’s `sinc` function, create \( h(n) \):

\[
h = \text{sinc}([-A:A]/L);
\]

where \( A \) is some positive integer. Make sure that the value of \( A \) is large enough, so that the smallest sidelobe of the sinc is small compared to the mainlobe.

Step 3. Using MATLAB’s `conv` function, convolve the upsampled version of \( s_2(n) \) with \( h(n) \), to get \( s_{2\text{int}}(n) \).

Step 4. Truncate \( s_{2\text{int}}(n) \), to make its first sample correspond to \( s(t)|_{t=0} \), and its last sample correspond to \( s(t)|_{t=32} \).

Using `subplot`, display a plot of \( s(t) \) and stem plots of \( s_1(n) \), \( s_2(n) \), and \( s_{2\text{int}}(n) \) in one window. Label the horizontal axes correctly. Make sure that \( s_1(n) \) and \( s_{2\text{int}}(n) \) are similar. Submit your MATLAB code and the printout of the plots. (Use `orient tall` before you print.)

(c) Repeat part (b), with the same vector \( t \) and

\[
s(t) = \begin{cases} 
1, & 8 \leq t \leq 24 \\
0, & \text{otherwise}
\end{cases}
\]

Again, submit a plot of \( s(t) \) and stem plots of \( s_1(n) \), \( s_2(n) \), and \( s_{2\text{int}}(n) \), as well as your MATLAB code. Can you notice any dissimilarities between \( s_{2\text{int}}(n) \) and \( s_1(n) \)? If so, write a very brief explanation for these dissimilarities.
Problem 5. DT interpolation with box functions.

One alternative to the scheme considered in Problem 4 would be to convolve with a box function (instead of convolving with a sinc), namely, with

\[ h(n) = \begin{cases} 
1, & n = 0, 1, \ldots, L - 1 \\
0, & \text{otherwise} 
\end{cases} \]

Repeat Parts (b) and (c) of Problem 4 for this new \( h(n) \). You only need to submit the stem plots for the two new signals \( s_{2\text{int}}(n) \). Comment on the differences between the results you get and your results from Problem 4.