ECE 302
Homework 5, due in class Wednesday, 2/25/2004.

Reading: Sections 2.5-3.3 and accompanying end-of-chapter problems.

Problem 1. A fair four-sided die (with faces labeled 0, 1, 2, 3) is thrown once to determine how
many times a fair coin is to be flipped: if \( N \) is the number that results from throwing the die, we flip
the coin \( N \) times. Let \( K \) be the total number of heads resulting from the coin flips. Determine and
sketch each of the following probability mass functions for all values of their arguments:

(a) \( p_N(n) \).
(b) \( p_{K|N}(k|2) \).
(c) \( p_{N|K}(n|2) \).
(d) \( p_K(k) \).
(e) Also determine the conditional PMF for random variable \( N \), given that the experimental value \( k \)
of random variable \( K \) is an odd number.

Problem 2. Let \( X \) and \( Y \) be independent random variables. Random variable \( X \) has a discrete
uniform distribution over the set \{1, 2, 3\}, and \( Y \) has a discrete uniform distribution over the set
\{1, 3\}. Let \( V = X + Y \), and \( W = X - Y \).

(a) Are \( V \) and \( W \) independent? Explain without calculations.
(b) Find and plot \( p_V(v) \). Also, determine \( E[V] \) and \( \text{var}(V) \).
(c) Find and show in a diagram \( p_{V,W}(v,w) \).
(d) Find \( E[V|W>0] \).
(e) Find the conditional variance of \( W \) given the event \( V = 4 \).
(f) Find and plot the conditional PMF \( p_{X|V}(x|v) \), for all values.

Problem 3. A pair of fair four-sided dice is thrown once. Each die has faces labeled 1, 2, 3, and
4. Discrete random variable \( X \) is defined to be the product of the down-face values. Determine the
conditional variance of \( X^2 \) given that the sum of the down-face values is greater than the product of
the down-face values.

Problem 4. Evaluate the following summation without too much work:

\[
\sum_{n=0}^{N} \binom{N}{n} n^2 A^n(1 - A)^{N-n},
\]
where $0 < A < 1$ and $N > 2$.

**Problem 5.** The joint PMF of discrete random variables $X$ and $Y$ is given by:

$$p_{X,Y}(x, y) = \begin{cases} Cx^2 \sqrt{y}, & \text{for } x = -5, -4, \ldots, 4, 5 \text{ and } y = 0, 1, \ldots, 10. \\ 0, & \text{otherwise.} \end{cases}$$

Here, $C$ is some constant. What is $E[XY^3]$? (*Hint:* This question admits a short answer/explanation. Do not spend time doing calculations.)

**Problem 6.** A random variable $X$ is called a *shifted exponential* when its PDF has the following form:

$$f_X(x) = \begin{cases} \frac{1}{\theta} e^{-\frac{x-\alpha}{\theta}}, & x > \alpha \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the CDF of $X$.

(b) Calculate the mean and the variance of $X$.

(c) Find the real number $\mu$ that satisfies: $F_X(\mu) = 1/2$. This number $\mu$ is called the *median* of the random variable.