EE 302 Division 1.
Homework 11, due Wednesday, 4/24/2002 at 5pm.

Reading assignment: Sections 6.1-6.4, 7.1-7.4; accompanying practice problems with solutions distributed in class.

Problem 1. Consider the following three-state Markov chain.

\[
\begin{array}{ccc}
0.1 & 0.7 & 0.4 \\
0.3 & 0.1 & 0.2 \\
0.3 & 0.3 & 0.6
\end{array}
\]

Determine the three-step transition probabilities \( r_{11}(3) \), \( r_{12}(3) \), and \( r_{13}(3) \).

Problem 2. For the Markov process pictured here, the questions below may be answered by inspection.

\[
\begin{array}{ccccccc}
1 & 1/2 & 1/4 & 1/2 & 1 & \\
1/2 & 2 & 1/4 & 1/2 & 1/3 & \\
1/3 & 3 & 1/2 & 1/2 & 1/3 & \\
1/3 & 4 & 1/2 & 1/3 & 1/3 & \\
0 & 5 & & & & \\
\end{array}
\]

Given that this process is in state 0 just before the first transition, determine the probability that:

(a) The process enters state 2 for the first time as the result of the \( K \)-th transition.
(b) The process never enters state 4.
(c) The process enters state 2, and leaves state 2 on the transition immediately after it entered state 2.
(d) The process enters state 1 for the first time on the third transition.
(e) The process is in state 3 as a result of the \( N \)-th transition.
**Problem 3.** Consider the Markov chain below. For all parts of this problem, the process is in state 3 immediately before the first transition.

(a) Find the variance for $J$, the number of transitions up to and including the transition on which the process leaves state 3.

(b) Find the expectation for $K$, the number of transitions up to and including the transition on which the process enters state 4 for the first time.

(c) Find $\pi_i$ for $i = 1, 2, \ldots, 7$, the probability that the process is in state $i$ after $10^{10}$ transitions or explain why these probabilities cannot be found.

(d) Given that the process never enters state 4, find the $\pi_i$'s as defined in part (c) or explain why they cannot be found.