1. Consider the following piece of a SERVO output report. Here the model is of a single-server queueing system. The random variable \( \text{ibusy} \) is the indicator variable, one when the server is busy and zero when idle. The purpose of the simulation is to estimate \( p \), the steady-state probability that the server is busy.

<table>
<thead>
<tr>
<th>Distribution Property</th>
<th>Point Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.796</td>
<td>( 0.0154647 )</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>0.496645</td>
<td>( 0.0107279 )</td>
</tr>
<tr>
<td>Variance</td>
<td>0.246661</td>
<td>( 0.008485 )</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.231791</td>
<td>( 0.116534 )</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.5659</td>
<td>( 0.377775 )</td>
</tr>
</tbody>
</table>

(a) (6 points) Based on these results, what is your guess of \( \frac{\text{E} \left( \text{ibusy} - p \right)^3}{\text{std}(\text{ibusy})} \)?

(b) (6 points) If only "meaningful" digits were being reported, how would you write the point estimate 0.796? Why?
2. Consider again the SERVO results from Problem 1. Suppose that the corresponding service-time distribution is from a Bezier family with mean 2.3 minutes and standard deviation 1.2 minutes. Let $S_i$ denote the service time of the $i$th customer.

We are given $(\hat{p}_j, \bar{x}_j)$ for $j = 1, 2, ..., 20$. Here $\hat{p}_j$ is the fraction of time that the server is busy during the $j$th macroreplication. Similarly, $\bar{x}_j$ is the sample service-time average of the $j$th macroreplication.

From Problem 1 we know that the observed fraction of time busy is $\hat{\rho} = \frac{\sum_{j=1}^{20} \hat{p}_j}{20} \approx 0.796$.

(a) (3 points each) $\hat{\rho} \approx 0.796$ is only an approximation because...

(i) T F the random variable is continuous, so the probability of any particular value is zero.

(ii) T F the random variable is continuous, so the observed value requires additional digits to be stated completely.

(iii) T F the standard error 0.0154647 is not zero.

(b) (3 points each) Generically, the linear control-variate point estimate is

$$\hat{\theta}(\hat{\alpha}^*) = \hat{\theta} - \hat{\alpha}^* \times (\bar{c} - \text{E}(C))$$

For each generic object below, provide the equivalent from the context above.

(i) $\hat{\theta}$

(ii) $\bar{c}$

(iii) $\text{E}(C)$

(iv) $\alpha^*$

(v) $\hat{\alpha}^*$

(c) (3 points each) In the world view $G \rightarrow U \rightarrow V \rightarrow Y \rightarrow \hat{\theta}$ to estimate the performance measure $\theta$, which letter corresponds to...

(i) $S_{16}$?

(ii) $i_{\text{busy}}$?

(iii) $\rho$?
3. Consider again the SERVO results from Problem 1.

   (a) (4 points) Sketch a realization of \( \text{ibusy} \) that corresponds to a single macroreplication.

   (b) (3 points) On your sketch, show the location of \( p \).

   (c) (3 points) Which SimLib routine is designed to estimate \( p \)? (choose one)

      sampst  spacet  timest

   (d) (5 points) Discuss how \( \hat{p} \) is computed in the routine that you chose in Part (c).
4. Early in the semester we considered the tolerancing problem of determining the distribution of 
\( Y = X_3 - X_1 - X_2 \), where the joint distribution of \((X_1, X_2, X_3)\) is given.

Assume that the output data are \( Y_{i,j} \), arranged in \( k \) macroreplications, each composed of \( m \) microreplications. The purpose is to estimate \( P(Y \leq 10) \).

(a) (6 points) Jamie discussed gradient estimation. In the context of the tolerancing problem, provide an example of such a gradient. (Do not discuss estimation.)

(b) (6 points) We discussed several variance-reduction techniques, including common random numbers, antithetic variates, importance sampling, and control variates. Discuss the relationship of any one of these with gradient estimation, including how it can be used to reduce variance.

(c) (3 points each)

(i) T  F  The data \( Y_{i,j} \) must be independent of each other.

(ii) T  F  To obtain a single random variate \( y_{i,j} \), any of the four random-variate generation methods (inverse cdf, composition, acceptance/rejection, special properties) is used (although inverse cdf is preferred if it is easy to implement).

(iii) T  F  A reasonable point estimator of \( P(Y \leq 10) \) is to sum all output data \( Y_{i,j} \), then divide by \( km \), and then return one if less than ten and return zero otherwise.

(iv) T  F  If the length \( m \) of each macroreplication is doubled and the number of macroreplications \( k \) is doubled, then the standard error of the point estimator is (approximately or exactly) halved.
5. Clueless colleagues.

(a) (6 points) Given a particular random-number generator with valid seed values \{1, 2, \ldots, m - 1\}, give advice to a (clueless) colleague about how to choose an initial seed value.

(b) (6 points) In fitting real-world data to many families of distributions, your (clueless) colleague asks for advice because goodness-of-fit tests strongly reject every family that he/she has tried. What do you ask or say to help?