IE 581 — Simulation Design and Analysis  
(1 point) Name ______________________

Closed book, open notes.  120 minutes.
For all true/false questions, three points if correct, two points if blank;
if you wish, write an explanation of your thinking.

Score ________________________________
1. (3 points each) For most of this course, we discussed how to estimate the value of a performance measure \( \theta \) for a single system. For each part, state whether the method is relevant for studying only one system, two, or multiple systems.

(a) Common Random Numbers

(b) Antithetic Variates

(c) Stochastic Optimization

2. Consider output data \( Y_1, Y_2, \ldots, Y_n \) from one long simulation run. If the data are steady state, then

\[
V(\bar{Y}) = \frac{V(Y)}{n} \left[ 1 + 2 \sum_{h=1}^{n} \left( 1 - \frac{h}{n} \right) \rho_h \right],
\]

where \( \rho_h \) is the lag-\( h \) autocorrelation. To estimate the standard error, we discussed the formula

\[
\hat{V}(\bar{Y}) = \frac{\hat{V}(Y)}{n} \left[ 1 + 2 \sum_{h=1}^{m} \left( 1 - \frac{h}{m} \right) \hat{\rho}_h \right].
\]

(a) T F The point here is to to choose a value of \( m \) to make the experiment more efficient by reducing the value of \( V(\bar{Y}) \) without changing the value of \( E(\bar{Y}) \).

(b) T F The asymptotic term \( \gamma_0 = \sum_{h=-\infty}^{\infty} \rho_h \) can be rewritten exactly as \( \gamma_0 = 1 + 2 \sum_{h=1}^{\infty} \rho_h \).

(c) T F Typically we would use common random numbers to induce the correlation values \( \rho_h \) to be positive.

(d) T F Multiple random number streams are necessary to obtain the correct values of the many correlations \( \rho_h \).

(e) (5 points) Suppose you and a colleague watch an animation of a manufacturing process. After a couple of minutes, your colleague suggests that the system is under utilized, since you are observing little congestion, including many machines being idle. You suggest turning off the animation for a while, simulating into the future, and then examining another two-minute period. Why is this procedure better than simply watching the two minutes that follow the first couple of minutes?
3. Random-variate generation. Consider the random-variate generation algorithm
\[ x = a u + (1 - a) u^2, \] where \( a \) is a constant satisfying \( 0 \leq a \leq 1 \).

(a) (4 points) Which of the four random-variate generation concepts (inverse
transformation, composition, acceptance-rejection, and special properties) does this
generator use? (Possibly more than one.)

(b) (5 points) What is the cumulative distribution function of the random variable \( X \)? Be
specific about the range of \( X \).

4. Importance Sampling. For given input model \( f \) and logic model \( g \), we use a function \( h(x) \)
to rewrite the integral
\[
\theta = E(g(X)) = \int g(x) f(x) \, dx = \int \left[ g(x) f(x) / h(x) \right] h(x) \, dx
\]
to provide an alternate simulation experiment that is (it is hoped) either easier to
implement or statistically more efficient. (If the random variables \( X \) are discrete, replace
the integral with a sum.)

(a) T F This idea works for random vectors \( X \).

(b) (5 points) State the restrictions on the choice of \( h \) so that the value of \( \theta \) is unchanged.
5. In the last homework assignment, the cleaning and patching times \((X_1, X_2)\) were generated with exponential marginal distributions using the NORTA (NORmal To Anything) method. Correlations ranging from \(\rho = -1\) to \(\rho = 1\) were specified as model parameters so that the effect of correlation could be studied.

(a) T F \((X_1, X_2)\) is bivariate normal.

(b) T F \((X_1, X_2)\) is part of the logic model.

(c) T F In this application, the independent normal random variates can be generated using the Box-Muller method.

(d) T F \(\text{corr}(X_1, X_2) = \rho\).

6. Recall: Stochastic simulation experiments, as studied in this class, are used to analyze a given probability model. In particular, a performance measure \(\theta\) is estimated by generating random variates \(X\) from a known input model, transforming them to output data \(Y\) using a known logic model, and from \(Y\) computing a point estimator \(\hat{\theta}\). The standard error of the point estimator, \(\text{sfe}(\hat{\theta})\), depends upon how long the model is simulated.

(a) (5 points) What is the ideal value of \(\hat{\theta}\)?

(b) (5 points) What is the ideal value of \(\text{sfe}(\hat{\theta})\)?

(c) T F The choice of random-number seed is part of the input-model specification.
7. Consider the nonhomogeneous Poisson process (nhpp) with rate function $\lambda(\tau)$ for $0 \leq \tau < \infty$. The cdf for a nhpp is

$$F_{T_i | T_{i-1} = t_{i-1}} (t) = 1 - \exp(-\Lambda(t) - \Lambda(t_{i-1})),$$

where $\Lambda(t) = \int_0^t \lambda(\tau) d\tau$.

Suppose that $\lambda(\tau) = 2$ for every time $\tau$.

(a) (5 points) Suppose that the current time is $t_{i-1} = 5$. If the inverse cdf returns the time $t_i = 7$, what is the value of the random number?

(b) (6 points) Suppose that the rate function above is thinned to create observations from the "seasonal" rate $d + a \sin(b + c \tau)$ for given constants $a$, $b$, $c$ and $d$. (That is, the rate is oscillating between $d - a$ and $d + a$.) What are the valid values of the parameters $d$ and $a$?
8. Control variates. Consider the linear control-variate estimator \( \hat{\theta}(\alpha) = \hat{\theta} + \alpha(C - \mu_C) \). This notation differs a bit from that used in class.

(a) (3 points) What is the meaning of \( \hat{\theta} \)? Is its value known before the simulation experiment is run?

(b) (3 points) What is the meaning of \( \mu_C \)? Is its value known before the simulation experiment is run?

c) (3 points) How many control variates are being used?

d) (3 points) Suppose the first control variate is positively correlated with the control-variate estimator. What should be the sign of \( \alpha \)?

(e) (5 points) Suppose the control variate is number of breakdowns and that we are estimating expected throughput per shift. Do we require common random numbers to induce the desired correlation? Why?

No Yes